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“On D-brane configurations and AdS/CFT duality”

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RESUMEN DEL CONTENIDO DE TESIS DOCTORAL

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RESUMEN (en español)

La teoría de cuerdas ha sido un considerable avance conceptual y matemático hacia la posible teoría física más fundamental de todas. La idea comenzó en los años 60, considerando los constituyentes más elementales de la materia como objetos unidimensionales representados por cuerdas vibrando. De este modo se trató, sin éxito, de describir la interacción fuerte. Sin embargo, años más tarde, la teoría de cuerdas fue recuperada y se ha ido desarrollado como una posible teoría de unificación en la cual numerosos avances continúan llevándose a cabo a día de hoy, y la cual podría describir todas las partículas de materia e interacciones conocidas.

En ésta tesis se presentan dos trabajos de investigación independientes en el área de la teoría de cuerdas. El primero se enmarca en el contexto de las configuraciones no-BPS inestables, mientras que el segundo generaliza ciertas "particle-like branes" que aparecen en relación a la conjetura de AdS/CFT. Tras un capítulo de introducción a las cuerdas, branas y dualidades, cada trabajo de investigación aparece en una parte de la tesis (partes I, II). Las conclusiones se presentan al final de cada una de estas partes.

En la primera parte introducimos ciertas configuraciones no-BPS inestables, que son las D*p*-branas no-BPS y los sistemas de (D*p*,anti-D*p*)-brana. Su inestabilidad se refleja en la presencia de modos taquiónicos en su espectro de cuerdas. Estos modos taquiónicos pueden decaer (''condensar'') dando lugar a una nueva configuración, que puede a su vez ser estable o no. El formalismo de "boundary state" es introducido, así como un enfoque de potencial efectivo, con el fin de sentar las bases para el trabajo de investigación presentado en esta primera parte de la tesis. El trabajo de investigación se presenta como una adaptación de [1]. En dicho trabajo presentamos una acción de "worldvolume" efectiva apropiada para el estudio de la fase confinante de un sistema (D*p*,anti-D*p*) en acoplo débil. Identificamos el mecanismo por el cual la cuerda fundamental aparece a partir de ésta acción cuando la D*p* y la anti-D*p* se aniquilan. También construimos una acción dual explícita, más adecuada para el estudio del régimen de acoplamiento fuerte. Nuestra descripción dual indica que los objetos taquiónicos que se condensan se originan a partir de D(*p*-2)-branas extendidas entre la brana y la antibrana.

En la segunda parte de la tesis presentamos los más relevantes resultados relativos a las "particle-like branes" que aparecen en relación a AdS/CFT. Estas configuraciones se componen de branas que viven en el interior del espacio AdS y un cierto número de cuerdas que se extienden hasta la frontera de este espacio, donde son vistas como quarks. Repasamos el estudio de estabilidad del vértice bariónico en $AdS_5 \times S^5$ y cómo estas configuraciones fueron generalizadas introduciendo un flujo magnético. También repasamos las configuraciones de di-bariones que aparecen en este mismo contexto. Finalmente y antes de presentar [2], explicamos las bases de la teoría de ABJM, una propuesta de AdS/CFT realizada sobre un espacio AdS_4 y relacionada con una teoría de Chern-Simons (de materia) supersimétrica en tres dimensiones. En [2] estudiamos el efecto de añadir carga de D-brana de inferior dimensionalidad, generalizando las configuraciones de "particle-like branes" que aparecen en $AdS_4 \times \mathbf{P}^3$. Mostramos que dichas configuraciones requieren cuerdas fundamentales adicionales con el fin de cancelar ciertos "tadpoles" de "worldvolume" que aparecen. Un estudio dinámico revela que las cargas deben pertenecer a un cierto intervalo para encontrar configuraciones bien definidas, y para el vértice bariónico y el di-barion, el número de cuerdas fundamentales también debe restringirse a un rango. Adicionalmente discutimos cómo estas configuraciones son modificadas en presencia de una masa de Romans no nula.

[1] N. Gutierrez, Y. Lozano, Phys. Rev. D79 (2009) 046010, arXiv:0809.1005 [hep-th]

[2] N. Gutierrez, Y. Lozano, D. Rodriguez-Gomez, JHEP 1009 (2010) 101, arXiv:1004.2826 [hep-th].

RESUMEN (en Inglés)

String theory has been a considerable conceptual and mathematical advance in the search for the most fundamental theory of physics. The idea started in the 1960's, by considering the smaller constituents of matter as one-dimensional objects represented by vibrant strings, in an attempt to describe the strong interaction. The attempt failed, but years later, string theory was recovered and has been developed as a possible unification theory in which numerous advances continue to take place nowadays, and which would encompass all known matter particles and interactions.

In this thesis two independent research works in the area of string theory are presented. The first one falls in the context of the unstable non-BPS configurations, while the other has to do with the particle-like branes appearing in relation to the AdS/CFT conjecture. After a general introductory chapter to the strings, branes and dualities, each research work appears in a separate part of the thesis (parts I, II). Conclusions are given at the end of each part.

In the first part we introduce certain unstable non-BPS brane configurations, the non-BPS D*p*-branes and the (D*p*,anti-D*p*) systems. Their instability is reflected in the presence of tachyonic modes in their string spectra. These tachyonic modes can decay (`condense') giving rise to a new configuration, which can in turn be stable or not. The boundary state formalism is introduced, as well as an effective potential approach, in order to tackle the problem and lay the basis for the research work presented in this first part. That research is presented as an adaptation of [1]. In this work we present a worldvolume effective action suitable for the study of the confined phase of a (D*p*,anti-D*p*) system at weak coupling. We identify the mechanism by which the fundamental string arises from this action when the D*p* and the anti-D*p* annihilate. We also construct an explicit dual action, more suitable for the study of the strong coupling regime. Our dual description indicates that the condensing tachyonic objects originate from open D(*p*-2)-branes stretched between the brane and the antibrane.

In the second part of the thesis we present the most relevant results from the particle-like branes appearing in relation to AdS/CFT. These configurations are made of branes living in the bulk of AdS and a certain number strings stretched all the way to the boundary, where they are seen as external quarks. We review the stability study of the baryon vertex in the AdS_5 x S^5 background and how this configuration was generalized by introducing a magnetic flux. We also comment on the Di-baryon configurations that appear in the same context. Finally and before presenting [2], we explain the basis of the ABJM theory, an AdS/CFT proposal realized over an AdS_4 space and related to a three dimensional supersymmetric Chern-Simons matter theory. In [2] we study the effect of adding lower dimensional brane charges, generalizing the particle-like brane configurations that appears in AdS_4 x $\mathbf{P}^3$. We show that these configurations require additional fundamental strings in order to cancel certain worldvolume tadpoles appearing. A dynamical study reveals that the charges must lie inside some interval in order to find well defined configurations, and for the baryon vertex and the di-baryon, the number of fundamental strings must also lie inside an allowed interval. We also discuss how these configurations are modified in the presence of a non-zero Romans mass.

[1] N. Gutierrez, Y. Lozano, Phys. Rev. D79 (2009) 046010, arXiv:0809.1005 [hep-th]

[2] N. Gutierrez, Y. Lozano, D. Rodriguez-Gomez, JHEP 1009 (2010) 101, arXiv:1004.2826 [hep-th].

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To those who made this possible,
thank you.

Contents

Chapter 0

Introduction

Unification has been a highly important challenge in modern physics. It would mean going one step forward towards interlacing and probably simplifying our most basic knowledge about our world. In fact, after being successfully advancing in rather different directions, it seems logical to try to put it all together and verify its consistency as a whole. This reasoning has already lead to new physical discoveries in the past. In the middle of the XIX century, thanks to prominent figures such as Faraday and Maxwell, the electromagnetic unification was carried out. At that time, electricity and magnetism were elegantly reduced to the so-called Maxwell equations. Let us briefly describe the main accomplishments of XX century fundamental physics and motivate the appearance of *string theory* in this context, a unification theory in which numerous advances continue to take place nowadays and that would encompass all matter particles and interactions.

The Standard Model

In the beginning of the XX century, an authentic revolution took place in physics: with the birth of special relativity and quantum physics completely new horizons were open. In a few years the world became much more complex and strange than it was thought to be. The new theories managed to explain the strange phenomena that started to be observed[1], but on the other hand these theories left lots of new open questions and, in some occasions, they continued advancing in slightly different directions without an apparent connection between them.

Some years after, both general relativity (GR) and quantum mechanics were subject of study for some of the most celebrated physicists of that time. Huge advances were achieved in those fields while some links between quantum physics, electromagnetism (EM) and special relativity appeared, giving rise to the development of *quantum field theory* (QFT)

[1] The Michelson-Morley experiment, the black-body radiation spectrum, the photoelectric effect and the atomic spectra.

between the '40s and '60s. That was another big step in physics, starting by the enormous success achieved by *quantum electrodynamics* (QED), which satisfactorily described the interaction between charged particles[2], including their creation and annihilation processes. The Lamb-Retherford experiment verified its implications up to a precision never reached before in any other field. It was little time after that, in the '60s, when Glashow, Salam, and Weinberg unified the EM and the weak interaction, using the ideas with which Yang and Mills had failed in their attempt to unify the strong interaction and the EM in the '50s.

We had to wait until the late 70's for the establishment of *quantum chromodynamics* (QCD), the QFT describing the (strong) interaction among quarks and gluons, responsible for binding protons and neutrons in the atomic nucleus. That (strong) interaction successfully explains the phenomenon of *asymptotic freedom*[3] observed for all particles possessing the new charge called color. This theory has had excellent results, even though it is impossible to apply it directly in order to understand complex many-particle systems such as the atomic nucleus, having problems even when describing "simple" quark-composed particles (hadrons). The problem resides in the fact that the effective coupling constant becomes extremely large at low energies, preventing any kind of perturbative treatment to be done; computational *lattice* methods are used instead[4]. Furthermore, the potential between quark and antiquark is linear in the distance between them, such that if one tries to separate them there is a distance at which it is energetically favored that a new quark-antiquark pair from the vacuum appears and recombines with the original particles. This effect, known as *hadronization*, is the reason why we cannot observe any isolated colored elemental particle (i.e. a quark or gluon) in nature. This is a low energy effect, and accordingly to what we have said, it cannot be studied by any known analytical method. This *color confinement* occurs at energies of the order of $\Lambda_{QCD} \sim 200MeV$, where a phase transition takes place. Below that energy we should use an effective theory with hadrons as fundamental degrees of freedom (dof), instead of working with elementary particles.

With the success of these new QFTs the *standard model* (SM) was born, covering the EM, weak and strong interactions. Those interactions are mediated by the *vectorial gauge bosons*, particles with spin 1 ruled by a *gauge group*, in this case $SU(3)\times SU(2)\times U(1)$. This model also includes 12 fermions of spin 1/2 as matter particles, 6 of them being leptons and 6 being quarks (in addition to their corresponding antiparticles) divided in tree gener-

[2]As in quantum field theory particles are described by fields, we are not going to make any distinction between these terms in the following.

[3]As the effective coupling constant of this theory decreases when increasing the energy, the shorter the distance between particles, the weakest their interaction, being effectively free at very small distances (smaller than quark-composed particles, as protons and neutrons).

[4]Lattice models discretize the space or space-time, and are common in QCD, condensed matter physics and even in polymers studies. The finiteness, as opposed to the continuous point of view, makes computational calculations an ideal tool for these models.

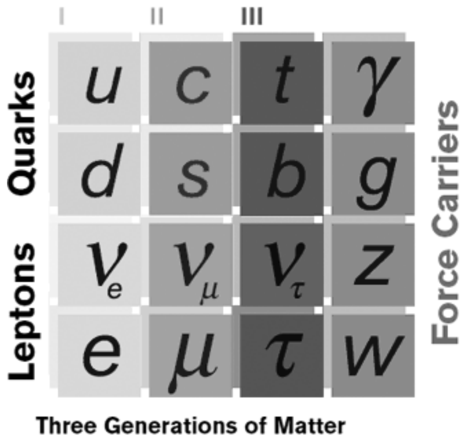

Figure 1: The SM particle content is given by 6 quarks, 6 leptons, their corresponding 12 antiparticles, 4 intermediate bosons and the Higgs particle (not included in the figure).

ations, as it is showed in fig. 1. But we must add one extra boson, the Higgs field, in order to provide mass to the particles in a consistent way[5]. Up to now this mechanism is the only known consistent way of providing mass to the particles without spoiling the theory. Once this Higgs particle is taken into account, the internal consistency of the theory is warranted.

One of the key concepts used by this theory is *gauge invariance*, which already appeared in classic EM, i.e. the fact that there is a certain freedom in choosing the potentials without modifying the dynamics of the system. This concept was generalized in field theory in order to account for those transformations of the fields with more dof than relevant physical variables. In this way the gauge invariance reflects a new kind of symmetry in the system, but not only that, using that principle we are able to derive realistic interacting field theories as QED and QCD (once the symmetries are made local, interactions appear in order to preserve them).

Nevertheless, important questions still exist. Although the SM provides nice particle-physics explanations and predictions from nearly 10^{-19}m, corresponding to the TeV scale, to cosmological distances[6], it seems impossible to include gravitational interactions in the model in a consistent way. This topic still awaits a satisfactory explanation, pointing in the direction of a new theory.

[5]Results of the observation of a new boson at a mass of 125 GeV were recently reported from both ATLAS and CMS experiments at the LHC at CERN (Geneva, Switzerland) [1, 2], which could correspond to this Higgs field.

[6]Even though what seem to be the main elements of the universe have not yet been identified.

In need of a new theory?

If the SM explains most of the experiments done so far, and in fact makes some predictions about what could be found at higher energies, then why should we expend resources considering other theories? First of all there is the possibility that the SM could not be the right answer, and as experimental layouts cost so much time, personal and money, a wide phenomenological exploration should be made in order to consider other reasonable possibilities that could be observed in the following. On the other hand, one has to take into account the fact that the SM itself does not contain gravity. A reason for why it is possible for both theories to coexist nowadays, out from a common theoretical framework, is the negligible effect of gravity in the processes that we can generate in particle colliders. Since the required scale in order to study gravity is too large compared to the typical one of these experiments, we can neglect its contribution, and it is nowadays practically impossible to test both theories simultaneously. What is more, GR is often considered a beautiful closed theory in opposition to the SM construction, but we have lots of experimental data testing the later meanwhile we are not able to prove GR much more beyond the classical perturbative tests already made, apart from certain supernova studies. For some time now, black holes have become a usual playground for this kind of studies, as long as both gravity and particle physics play an important role on their description. However, we cannot ignore the fact that QFT becomes ambiguous and eventually breaks down when it is applied to some systems where gravitation is too strong[7].

But, why it is not possible to combine GR and the SM in a satisfactory way? The main problem is that the Einstein-Hilbert action of GR turns out to be *non-renormalizable*, and once the gravitational interaction between elementary particles is put to test, divergences show up. Let us think of the following situation. Consider two gravity-interacting particles propagating as in fig. 2. If we try to carry out a perturbative treatment as it is done with the SM interactions, it must be taken into account that the coupling constant corresponding to what we should call graviton will be G_N. According to that, the ratio of exchange of one of these gravitons with respect to the free propagation situation will be governed by the only adimensional expression that can be created using the parameters of the problem, $G_N E^2 \hbar^{-1} c^{-5}$, where E stands for the characteristic energy of the process. Therefore the results diverge for $E > \sqrt{\frac{\hbar c^5}{G_N}}$ and the approach is no longer valid. This problem can be compared to trying to extrapolate Fermi's theory to arbitrarily high energies.

Lots of mechanisms and models that would consistently coexist within the actual framework have been studied with the aim of developing a new theory. However the energy that is required in order to test most of them is well beyond the capacities of the last particle

[7]Some advances in QFT in curved spaces have been made, although with no hope of completely solving the problem.

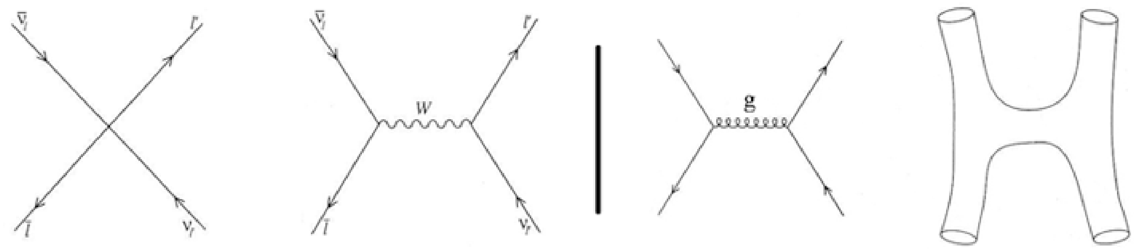

Figure 2: The evolution from one graviton interchange in QFT to the string theory proposal is shown in a similar fashion to moving from the Fermi's theory to the GSW model. In string theory the trajectories (*worldlines*) that the particles describe in the space-time become cylinders (*worldsheets*). The interaction becomes nonlocal and is no longer punctual, eliminating the ultraviolet divergences that appear when interaction points are brought too close to each other. The apparent divergences shown in QFT would only be due to not having enough resolution to see the real diagram at the current energies.

accelerators. These sets of proposals comprise what is usually referred as *beyond standard model* physics, which include ideas that range from *supersymmetry* (SUSY) to small compact additional dimensions. However, most of these ideas are just proposed as possible corrections to the SM without providing an explicit consistent frame for their realization. Furthermore, both the SM and its possible BSM "upgrades" have a lot of parameters without any dynamical origin, and are all in that sense, fundamental constants of the theory. This is not "de facto" a problem in a certain theory or model but, as we already mentioned, the search for "naturalness" and a possible hidden simplicity have lead to great discoveries in the past, and this might well be the case.

String theory to the rescue

String theory has been a considerable conceptual and mathematical advance in the search for a more fundamental theory. The idea started by considering the smaller constituents of matter as one-dimensional objects represented by vibrant strings. The first step was taken by Veneziano [3] who, in the 1960's and followed by Susskind [4], wrote a phenomenological amplitude for the strong interaction. At that time, before QCD appeared, these kind of approaches were the only possible way of trying to describe the strong interaction[8]. Virasoro and Shapiro [5, 6] later improved the early work of Veneziano, being able

[8]In the 50's many resonances and excited states of hadrons were found, and in the 60's *Regge theory*, based only in very general assumptions about the S-matrix, allowed to perform certain asymptotic scattering calculations.

to describe some features of that hadronic spectrum. They explained the relation between the observed mass of the lightest hadron with a given spin and its angular momentum, given by $m^2 \sim TJ^2 + \text{const.}$, such that both the mass and the angular momentum would come from a rotating quantum relativistic string of tension T living in $\mathbb{R}^D$. Nevertheless QCD turned out to be right answer, ruling the idea out for years. It was later on, in 1974, when Schwarz and Scherk ([7]) discovered a possible connection between string theory and GR. The theory indeed predicted a massless spin-two particle, which was identified with the graviton, and became in a few years not only a theory of gravitation but also a unification theory that could be capable of describing all known interactions. This was called the *first string revolution*.

As we are going to see string theory is only consistent in spacetimes with additional dimensions. The first precursor to the idea of proposing additional dimensions was the German physicist Theodor Kaluza. In 1921 he presented a work by which he was trying to unify EM and gravitation, including the first through the use of an extra fifth dimension [8]. Few years later the mathematician Oscar Klein provided an explanation to that new dimension of Kaluza, in such a way that any point of the tetradimensional space-time would have a small circumference associated to it representing that new dimension [9]. This mechanism is actually named Kaluza-Klein compactification in their honor.

The idea of extra dimensions was recovered with string theory. An initial theory of 26 dimensions with only bosonic strings was considered at first, in which twenty-two of the dimensions would be twisted around themselves in a little space of size comparable to the *Plank length*[9], being only able to notice our tetradimensional world at energies far smaller than the Planck's scale. In the next chapter, we are going to first elaborate on this initial bosonic theory in order to get the basic ideas, introducing later the so-called *superstring theories* which include fermionic states and live in 10 dimensions. We will partially follow the approach taken in [10], where a quite broad introduction to string theory is given. Nevertheless, for a more detailed introduction the classics [11] and [12] can be read.

[9]By dimensional analysis, the length that can be obtained by combining the speed of light c, Newton's gravitational constant G, and Planck's constant $\hbar$ is known as Planck length, given by $l_p = \sqrt{\hbar G/C^3} \approx 10^{-35}m$

Chapter 1

Strings, branes and duality

1.1 The bosonic string

As well as the trajectory of one particle in a N-dimensional space-time is given by a *worldline* parametrized by a certain real parameter τ, we can parametrize the worldsheet spanned by a one-dimensional spacial extended object, a *string* or *fundamental string*, with two coordinates τ and σ, where σ localizes a point along the length of the string. These strings are basic objects in the theory and can consistently be taken to be open or closed, orientable or not, depending on their topology, and to wind around topological defects a certain number of times (*winding number*).

Imposing global Poincaré invariance

$$X^\mu \quad \rightarrow \quad X'^\mu = \Lambda^\mu{}_\nu X^\nu + a^\mu \tag{1.1}$$

and worldsheet reparametrization invariance

$$(\tau, \sigma) \quad \rightarrow \quad (\tau'(\tau, \sigma), \sigma'(\tau, \sigma)) \tag{1.2}$$

in a Minkowski type space, the simplest action that one can construct depends on the area swept by the string, and it is referred as the Nambu-Goto action

$$\mathcal{S}_{NG} = -\frac{1}{2\pi\alpha'} \int_M d\tau \; d\sigma \sqrt{|g|}. \tag{1.3}$$

Here we have denoted $|det(g)|$ as $|g|$, and the integral runs over the entire worldsheet M. In turn $g_{ab} = \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$ represents the metric induced on M by the metric of the ambient space $\eta_{\mu\nu} = (-1, 1, ..., 1)$ via the pullback $\partial_a X^\mu \partial_b X^\nu$, where $X^\mu(\tau, \sigma)$ stand for the N coordinates of the worldsheet in that space[1]. Therefore, when no interactions are

[1]These $X^\mu(\tau, \sigma)$ embedding functions describe the string configuration in the ambient space. We will use the notation x^μ only when referring to the space-time coordinates themselves. In this way Greek indices run through ambient space coordinates $\mu, \nu = 0, ..., D-1$, whereas Latin indices run through worldvolume coordinates, in this case $a, b = \tau, \sigma$.

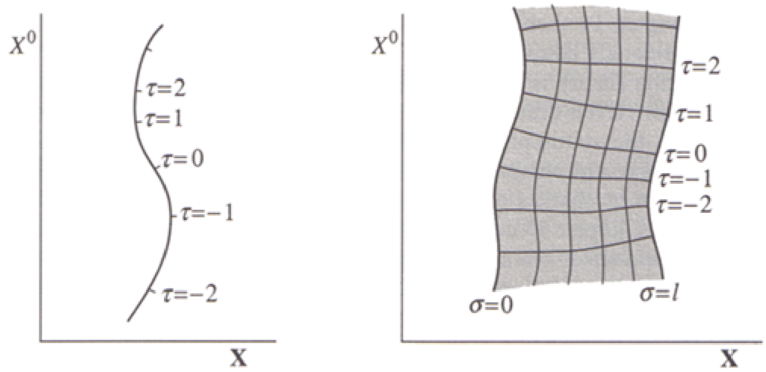

Figure 1.1: As a point particle describes a line in the space-time, a string span a bidimensional surface.

considered, the string will propagate in such a way that the area spanned will be minimal. This theory would have no free fundamental parameters other than α', the *Regge slope*[2], which is a merely scale parameter whose order of magnitude would be l_p^2. The coefficient in front $T = 1/(2\pi\alpha')$ is usually known as *string tension*, as it gives information about the energy per unit length of the string.

The Nambu-Goto action is classically equivalent to Polyakov's action[3] [13], first introduced in [14]

$$\mathcal{S}_P = -\frac{1}{4\pi\alpha'}\int d\tau\ d\sigma\sqrt{|\gamma|}\ \gamma^{ab}\ \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}, \tag{1.4}$$

although (1.4) presents less difficulties for quantization. This Polyakov action additionally exhibits local Weyl invariance

$$\gamma_{ab}(\tau,\sigma) \quad\rightarrow\quad \gamma'_{ab}(\tau,\sigma) = e^{2\omega(\tau,\sigma)}\gamma_{ab}(\tau,\sigma), \quad \forall\ \omega(\tau,\sigma), \tag{1.5}$$

an invariance of the metric tensor under local changes of scale of the worldsheet metric γ_{ab}. In this action the worldsheet metric is an independent variable (although non dynamical) and will in general differ from the ambient space metric.

Action (1.4) defines a two-dimensional scalar field theory in the worldsheet, with energy-momentum tensor given by ("$\cdot$" means contraction with the flat spacetime metric)

$$T_{ab} = -4\pi\alpha'\frac{1}{\sqrt{|\gamma|}}\frac{\delta S}{\delta\gamma^{ab}} = \partial_a X\cdot\partial_b X - \frac{1}{2}\gamma_{ab}\gamma^{cd}\partial_c X\cdot\partial_d X = 0, \tag{1.6}$$

[2]In Regge theory the Regge slope appears exponentiating the Mandelstam variable s in the amplitude.
[3]Using the equations of motion for the metric γ in the Polyakov action one obtains the Nambu-Goto one.

leading to the constraints

$$T_{ab} = 0, \qquad T^a_a = 0. \tag{1.7}$$

By exploiting reparametrizations and Weyl rescalings the auxiliary field can be gauge fixed to be $\gamma_{ab} = \eta_{ab}$. In this way and by ignoring interactions, it is possible to obtain a more handy action for the free bosonic string

$$\mathcal{S}_P = -\frac{1}{4\pi\alpha'}\int_M d\tau\, d\sigma\, \eta^{ab}\, \partial_a X^\mu \partial_b X_\mu. \tag{1.8}$$

Indeed the variation of (1.8) provides a bidimensional wave equation

$$(\partial_\tau^2 - \partial_\sigma^2)X^\mu = 0 \tag{1.9}$$

with solutions given in terms of free waves depending on a pair of arbitrary functions, representing the possible oscillation modes of the string, going to the left or to the right

$$X^\mu(\tau,\sigma) = X^\mu_L(\tau+\sigma) + X^\mu_R(\tau-\sigma). \tag{1.10}$$

Imposing boundary conditions we can now Fourier expand these for closed strings, satisfying $X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma+2\pi)$ [4], this gives

$$X^\mu_\pm(\tau\pm\sigma) = \frac{1}{2}x^\mu + \alpha' p^\mu(\tau\pm\sigma) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}\alpha_n^{\mu(\pm)} e^{-in(\tau\pm\sigma)}. \tag{1.11}$$

x^i and p^μ denote, respectively, the position and momentum of the center of mass, while the oscillatory last term provides additional dof to the strings as compared to point-like particles. Here $\alpha_n^{\mu(+)}$ and $\alpha_n^{\mu(-)}$ are respectively the amplitudes of the left and right handed modes of the string, and satisfy

$$\alpha_{-n}^{\mu(\pm)} = (\alpha_n^{\mu(\pm)})^\star \tag{1.12}$$

as the $X^\mu_\pm$ are real functions. We are going to alternatively use the $+/-$ or the L/R notation for left- and right-movers in the following.

On the other hand, variations of the action (1.8) with the condition

$$\delta X^\mu(\tau_{initial}) = \delta X^\mu(\tau_{final}) = 0 \tag{1.13}$$

allow open strings satisfying either Neumann-type boundary conditions

$$\partial_\sigma X^\mu|_{\sigma=0,\,\sigma=\pi} = 0, \tag{1.14}$$

[4] For the closed strings it is typical to take a parametrization such that $\sigma \in [0, 2\pi]$, whereas for the open strings $\sigma \in [0,\pi]$.

in which case no momentum can flow through the string endpoints, or Dirichlet-type ones

$$\partial_\tau X^\mu|_{\sigma=0,\,\sigma=\pi} = 0, \tag{1.15}$$

for which the string end-points are fixed to hypersurfaces of different dimensionalities, the so-called *Dirichlet branes* or for short *D-branes* ([47]). In this context, Neumann boundary conditions are also understood in term of those hyperplanes, indicating directions filled by a D-brane, along which the string end-points can move freely.

The mode expansion for the open string satisfying Neumann boundary conditions at both extrema is given by

$$X^\mu_{NN}(\tau,\sigma) = x^\mu + 2\alpha' p^\mu \tau + i\sqrt{2\alpha'}\sum_{m\neq 0}\frac{1}{m}\alpha^\mu_m e^{-im\tau} cos(m\sigma), \tag{1.16}$$

whereas the modes for the open string corresponding to Dirichlet boundary conditions at both extrema are given by

$$X^\mu_{DD}(\tau,\sigma) = x^\mu + \frac{1}{\pi}(y^\mu - x^\mu)\sigma + i\sqrt{2\alpha'}\sum_{m\neq 0}\frac{1}{m}\alpha^\mu_m e^{-im\tau} sin(m\sigma). \tag{1.17}$$

Finally, the last possibility consists in mixing Dirichlet and Neumann conditions. This can be achieved by imposing the Dirichlet condition $X^\mu(\tau,0) = x^\mu$ at one end, and the Neumann condition $\partial_\sigma X^\mu(\tau,\sigma)|_{\sigma=\pi} = 0$ at the other

$$X^\mu_{ND}(\tau,\sigma) = x^\mu + \sqrt{\frac{\alpha'}{2}}\sum_{r\neq 0}\frac{1}{r}\alpha^\mu_r e^{-ir\tau} sin(r\sigma), \tag{1.18}$$

where $r = n + 1/2, n \in \mathbb{Z}$.

The following natural step is the canonical (first) quantization of the oscillation modes of the string, from which the graviton will emerge as a massless spin 2 state of the closed string. As usual, we can replace the theory's Poisson brackets by commutators $[...]_{P.B.} \to i[...]$. This, by inserting the mode expansion for both coordinates and conjugate momenta, leads to

$$\left[\alpha^{\mu(\pm)}_m, \alpha^{\nu(\pm)}_n\right] = m\eta^{\mu\nu}\delta_{m+n}, \qquad \left[\alpha^{\mu(\pm)}_m, \alpha^{\nu(\mp)}_n\right] = 0, \qquad \alpha^{\mu(\pm)}_{-n} = (\alpha^{\mu(\pm)}_n)^\dagger \tag{1.19}$$

for the closed string, while a similar result holds for the open string. In this way we can treat the α_m's as creation and annihilation operators, m indicating the number of particles created ($m < 0$) or destroyed ($m > 0$). By definition, the ground state $|0>$ is annihilated by the lowering operators

$$\alpha^\mu_m|0\rangle = 0 \qquad \text{for} \qquad m > 0, \tag{1.20}$$

meanwhile a string state with momentum k^μ is created by applying the rising operators

$$|\phi\rangle = \alpha_{m_1}^{\mu_1\dagger}\alpha_{m_2}^{\mu_2\dagger}...\alpha_{m_n}^{\mu_n\dagger}|0;k\rangle, \qquad p^\mu|\phi\rangle = k^\mu|\phi\rangle, \qquad m_i > 0. \tag{1.21}$$

Note that from (1.19) commutators of time components get a minus sign from Minkowski metric

$$[\alpha_m^0, \alpha_m^{0\dagger}] < 0, \qquad m > 0, \tag{1.22}$$

in such a way that negative norm states (in principle) appear in the bosonic string spectrum, as

$$\left|\alpha_m^{0\dagger}|0\rangle\right|^2 = \ \langle 0|\alpha_m^0\alpha_m^{0\dagger}|0\rangle \ < 0. \tag{1.23}$$

Besides, it is possible to check that for physical closed string states the number of excitations in each direction is the same, i.e. $N_L = N_R = N$. This is called *level matching condition*, and forces closed string states to be generated by using the same number of $\alpha^{(+)}$ and $\alpha^{(-)}$ operators.

The constraints (1.7) can be written as an infinite number of conditions of the form $L_n = 0$, with $n \in \mathbb{Z}$. These L_n are the Fourier coefficients of the energy-momentum tensor, called the *Virasoro generators*, and give rise to the so-called *Virasoro algebra*. In the quantized theory those constraints are promoted to operator conditions on the states $L_n|\phi >= 0$, for $n > 0$. An arbitrariness in the ordering prescription of L_0 leads to the inclusion of a constant a, ending up in the appearance of a new quantum-mechanical term in the algebra.

At this point in time we should remark that the number of space-time dimensions D has, up to now, been taken to be arbitrary. Nevertheless, the spectrum generated turns out to be free of negative-norm states only for $a \leq 1$ and $D \leq 26$. Indeed, different analysis exist all leading to the same critical conditions: $a = 1$ and $D = 26$. For seeing this it is necessary to use the so-called *light-cone gauge*, a particular non-covariant gauge choice. In this gauge, the Fock space generated can be manifestly free of negative-norm states and all the Virasoro conditions are solved explicitly, instead of being imposed as constraints. Light cone coordinates can be introduced as

$$X^\pm = \frac{1}{2}(X^0 \pm X^{D-1}), \tag{1.24}$$

leaving the $D-2$ transverse coordinates X^i unchanged. In the light-cone gauge all the excitations are generated by the action of the transverse modes α_n^i over the vacuum. An infinite tower of oscillator states is finally obtained in this way as can be observed in fig. 1.2, with masses given by an on-shell condition. Notice that the first massless state obtained belongs to a $(D-2)$-component vector representation of the rotation group $SO(D-2)$, and as a general rule, Lorentz invariance implies that this kind of states must be massless. This condition implies $a = 1$ and, after certain considerations, $D = 26$. This avoids a Lorentz

$$
\begin{array}{lcl}
|0;k\rangle & \longrightarrow & scalar \\
\cancel{\alpha_{-1}^{i(+)}|0;k\rangle} & & \\
\cancel{\alpha_{-1}^{i(-)}|0;k\rangle} & & \\
\alpha_{-1}^{i(+)}\alpha_{-1}^{j(-)}|0;k\rangle & \longrightarrow & symmetric\ tensor + antisym.\,tensor + trace(scalar) \\
\cdots & &
\end{array}
$$

Figure 1.2: Bosonic closed string spectrum. An scalar is represented by the ground state, which is tachyonic (i.e. possesses a negative (mass)^2). The next non-zero object, a $(D-2)\times(D-2)$ matrix, can be decomposed into a symmetric tensor, an antisymmetric one and a trace. Notice that due to the level matching condition only states with the same number of excitations propagating in each direction are allowed. On the other hand, the open string spectrum also contains a tachyonic state, in addition to a massless vector boson as well as the correspondent tower of massive states.

anomaly in the light-cone gauge, that would lead to a conformal anomaly in a covariant gauge. Alternatively, the same constraint can be derived by imposing the Lorentz generators to satisfy the Lorentz algebra $[J^{i-}, J^{j-}] = 0$, which is not manifest in the light-cone gauge.

In order to study the low energy regime of the theory, only the lowest mass modes compatible with the boundary conditions have to be considered. Those massless modes join up into certain group representations, as physical states must appear in complete Lorentz multiplets[5]. More concretely, for the closed string a symmetric traceless tensor of spin 2 g_{ij}, an antisymmetric tensor of spin 2 B_{ij} and an scalar particle ϕ are obtained (fig. 1.2). The massless spin 2 particle g_{ij} couples to the energy-momentum tensor and provides space-time general covariance to the theory. This state is going to be identified with the graviton, whereas the massless antisymmetric tensor B_{ij} is on the other hand associated to an space-time gauge symmetry. In the open string sector, the first state obtained turns out to be tachyonic, with the implications previously remarked. The only massless state coming from this sector corresponds to a vector boson. Finally, higher harmonics of both sectors resulting in massive excitations would represent different (massive) elementary particles.

It is possible to generalize the action (1.4) by including couplings to background fields

[5]This is because Lorentz generators, seeing as quantum operators, and the L_m commute $[L_m, J^{\mu\nu}] = 0$, and so the physical-state condition is invariant under Lorentz transformations.

associated to the just obtained massless bosonic fields as follows

$$\mathcal{S} = -\frac{1}{4\pi\alpha'}\int d\tau\, d\sigma\sqrt{|\gamma|}\Big(\gamma^{ab}\,\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}(X) + \epsilon^{ab}\partial_a X^\mu \partial_b X^\nu B_{\mu\nu}(X) + \alpha' R^{(2)}(\gamma)\,\phi(X)\Big). \tag{1.25}$$

Imposing conformal invariance to this theory[6] the ambient space metric $g_{\mu\nu}$ is forced to satisfy the Einstein equations in 26 dimensions. This justifies our previous statement about the graviton, and in this way gravity is naturally contained in string theory. We should remark that the coupling to $B_{\mu\nu}$ in (1.25) is only present in bosonic theories with oriented strings, as reversal orientation invariance does not allow for it. This coupling can be regarded as a 2-form generalization of a 1-form Maxwell field coupling to the world line of a charged particle $q\int d\tau A_\mu \dot{x}^\mu$. Finally, the dilaton couples to $R^{(2)}(\gamma)$, the scalar curvature of the 2-dimensional string worldsheet.

In order to account for interactions among strings we use the fact that the worldsheet metric and the ambient space metric are independent in (1.4). In this way we can allow for non-trivial topologies by using the general metric $g_{\mu\nu}(x^\mu)$ instead of $\eta_{\mu\nu}$. In order to see how this can be done, let us consider a constant dilaton $\phi = \phi_0$. In this case the dilaton term becomes a total derivative, and so its integral is determined by the global topology of the worldsheet and does not contribute to the classical equations of motion. More concretely, its integral is given by the *Euler characteristic* of the worldsheet M

$$\chi(M) = \frac{1}{4\pi}\int_M d\tau\, d\sigma\sqrt{|\gamma|}R^{(2)}(h), \tag{1.26}$$

a topological invariant. The Euler characteristic of a bidimensional surface can be computed as $\chi = 2(1-h) - b - c$, being h, b and c its number of handles, holes and cross-caps respectively. The simplest case would then be given by a 2-sphere, with $\chi = 2$; $\chi = 1$ would be the case of a disk ($b = 1$) or a projective plane ($c = 1$), and so on and so forth. For a non-constant dilaton, its *vacuum expectation value* (vev) $<\phi>$ can also be treated this way. At this point we can define a low energy perturbative expansion in $g_s = e^{<\phi>}$, where each diagram would come with a weight $g_s^{-\chi}$ (χ running over all the different possible topologies of the worldsheet). The existence of this new parameter g_s does not contradict what we said before about α', i.e. that this is the only adjustable parameter of the theory, as this coupling constant is actually dynamically generated from the dilaton field.

Notice that both α' and g_s can be used to make perturbative expansions in string theory. At low energies we can expand in α' if $g_s << 1$ (weak coupling regime), whereas we can sometimes explore the $g_s >> 1$ region (strong coupling regime) if we use duality relations

[6]This can be achieved by requiring the β functions of the 2D theory to vanish.

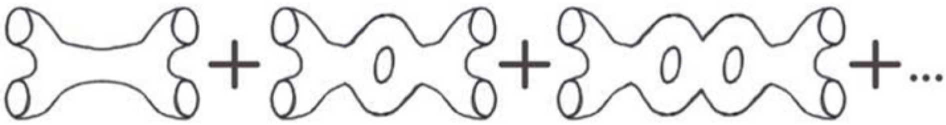

Figure 1.3: Before having elaborated a QFT of strings, interaction was proposed in terms of breakings and unions among them. It is possible to resum the perturbative series created by the different γ metrics of the Polyakov action, in an analogous way to the Feynman diagrams in perturbative QFT. Note that no string theory with open but not closed strings does exist; this would not be consistent with string interactions, as open strings would lead to the appearance of closed ones.

that map weak and strong coupling regions of the same or different string theories, as we are going to see later in certain superstring theories. In the middle a non perturbative regime is left with $g_s \sim 1$, which is much more difficult to explore.

The main problem of the bosonic string theory, besides the absence of fermionic states, is that the lowest energy propagating state both in the open and closed string spectra is tachyonic, as we previously mentioned. Nowadays tachyonic states are seen as an instability of the system. In the case at hand the vacuum of the theory itself seems to be unstable, a problem that remains unsolved. Let us now introduce the different superstring theories that exist once SUSY is imposed which, besides adding fermionic excitations[7], eliminate the tachyonic modes of the spectrum.

1.2 Superstring theories

SUSY is introduced in order to allow the appearance of fermions in string theory. Two basic approaches exist, the *Ramond-Neveu-Schwarz* (RNS) and the *Green-Schwarz* (GS) formalisms, leading to an explicit SUSY invariance on the worldsheet or in the space-time respectively, although other proposals do exist. These two formalisms lead to the same results in 10-dimensional Minkowski space-time. We are going to make use of the first one and we will see that, in 10 dimensions, a total of five different superstring theories are consistent.

In the following, the $X^\mu(\sigma,\tau)$ bosonic fields will be paired up with the new fermionic fields $\psi^\mu(\sigma,\tau)$, worldsheet spinors which transform as vectors under the Lorentz group

[7]SUSY relates bosons and fermions. It can be described by an invariance under an infinitesimal transformation which mixes both types of fields, and implies the existence of the same number of dof of each kind. As this is not observed in nature, SUSY must be broken at low energies.

$SO(1, D-1)$ of the ambient space-time. We can add a total of D Majorana fermions to the initial D massless bosons as follows

$$\mathcal{S} = -\frac{1}{4\pi\alpha'} \int d\tau d\sigma \left(\partial_i X_\mu \partial^i X^\mu + \bar{\psi}^\mu \gamma^i \partial_i \psi_\mu\right), \tag{1.27}$$

where the γ^i's ($i = 0, 1$) represent the two-dimensional Dirac matrices. This action is invariant under the following worldsheet SUSY transformations

$$\delta X^\mu = i\bar{\epsilon}\psi^\mu, \qquad \delta\psi^\mu = \gamma^i \partial_i X^\mu \epsilon. \tag{1.28}$$

Here ϵ represents a constant infinitesimal Majorana spinor. It is now possible to split the spinor ψ into two chiral components, and in doing so, the component with positive/negative chirality will be the right/left-moving respectively

$$(\partial_\tau - \partial_\sigma)\psi_+^\mu = 0, \qquad (\partial_\tau + \partial_\sigma)\psi_-^\mu = 0. \tag{1.29}$$

In the open string case, boundary conditions are

$$\left(\psi_-^\mu \delta\psi_{-\,\mu} - \psi_+^\mu \delta\psi_{+\,\mu}\right)\Big|_{\sigma=0}^{\sigma=\pi} = 0, \tag{1.30}$$

which implies

$$\psi_+^\mu = \pm\psi_-^\nu, \qquad \delta\psi_+^\mu = \pm\delta\psi_-^\nu, \qquad \sigma = 0, \pi. \tag{1.31}$$

From here two different fermionic sectors appear for the open strings. The sector corresponding to a relative plus sign in equation (1.31), which is called the *Ramond* (R) sector, and the one corresponding to a relative minus sign which is called the *Neveu-Schwarz* (NS) sector.

In the closed string case, we have the following choice in the periodicity of the fermions

$$\psi_+^\mu(\sigma = 2\pi) = \pm\psi_+^\mu(\sigma = 0), \qquad \psi_-^\mu(\sigma = 2\pi) = \pm\psi_-^\mu(\sigma = 0), \tag{1.32}$$

where the plus sign defines the R-sector and the minus defines the NS-sector. Thereby, four different possibilities exist when combining left- and right-movers in the closed string. This results in the NS - NS, R - R, R - NS and NS - R sectors.

When quantizing the theory, the bosonic states are the same as the ones previously obtained, and the fermionic sector can be expanded in fermionic oscillators also interpreted as rising and lowering operators. Without entering in much detail, let us just consider the massless sector of the superstring. As we previously remarked, massless states can be classified into representations of the $SO(D-2) = SO(8)$ group. This group has one vector and two chiral representations, denoted $\mathbf{8}_v$, $\mathbf{8}_+$ and $\mathbf{8}_-$ respectively. It turns out that the R-sector is in either $\mathbf{8}_+$ or $\mathbf{8}_-$ representations, while the NS-sector is in the $\mathbf{8}_v$.

We should mention that in order to identify the physical propagating states, one must previously perform the so-called *GSO projection* introduced by Gliozzi, Scherk and Olive [25]. This projection is necessary to obtain a consistent worldsheet conformal field theory (CFT). Space-time SUSY (induced by the just introduced worldsheet SUSY) requires the same number of bosonic and fermionic dof, and applying GSO projection ensures that. In addition, a tachyonic mode in the superstring spectrum is eliminated by this projection and does not propagate, eliminating the instability endemic to the bosonic string. Concretely, GSO projection discards all NS states created by an even number of fermionic creation operators, as well as fixes the chirality of the R sector's ground state. In order to derive this projection, 1 and 2-loop *modular invariance*[8] can be demanded, which is in fact automatically satisfied in the GS formalism.

Now, in order to construct the massless states that will finally show up in the strings, we have to glue left and right-movers. This can be made consistently in only five ways, resulting in five different superstring theories, one denoted *type I* superstring theory, two *type II* theories, and two *heterotic* superstring theories. Let us now briefly introduce them, presenting their decomposition in representations as well as the corresponding low energy supergravity limits. Fermionic counterparts can be obtained by imposing SUSY although will generally be ignored throughout this thesis.

1.2.1 Type II string theories

The two type II theories are obtained by initially considering just closed strings with $\mathcal{N}=2$ space-time SUSY, constructed as tensor products of two open strings with $\mathcal{N}=1$ space-time SUSY. Fermionic 10-dimensional Dirac spinors with 16 components[9] of the initial effective theory of the open string gives rise to 32-component spinors in the closed string type II theory. As GSO projection allows for two different choices of the relative chirality between left- and right-movers in the R-sectors, two consistent theories arise. The *type IIA* superstring theory, with R-states of opposite chirality and thus non-chiral (i.e. left-right symmetric), and the *type IIB* superstring theory, with R-states of the same chirality and therefore a chiral theory (i.e. left-right asymmetric). The NS - NS sector (NSNS) is then common to both theories, but not the R - R (RR), NS - R and R - NS (RNS) sectors. The first two sectors are composed by space-time bosons whereas the last ones contain the fermions.

Type IIA theory

[8] invariance under the $SL(2,\mathbb{Z})$ group of large diffeomorphisms of the torus.

[9] They match with the 8 bosonic dof of the gauge vector field as the corresponding Dirac equation reduces the number of independent fermionic components by half.

The type IIA theory has R-sectors of opposite chirality, resulting in a non-chiral space-time theory and making anomalies to trivially cancel. This theory can be decomposed into $SO(8)$ representations in the following way

$$\begin{aligned} (\mathbf{8_v} + \mathbf{8_+}) \otimes (\mathbf{8_v} + \mathbf{8_-}) \ &= \\ (\mathbf{1}\ +\ \mathbf{28}\ +\ \mathbf{35_v})_{\mathrm{NS-NS}} + (\mathbf{8_v} + \mathbf{56_v})_{\mathrm{R-R}} + (\mathbf{8_+} + \mathbf{56_-})_{\mathrm{NS-R}} &+ (\mathbf{8_-} + \mathbf{56_+})_{\mathrm{R-NS}}. \end{aligned} \tag{1.33}$$

In the NSNS sector we find the same massless states as in the closed bosonic string, although living in 10 dimensions, i.e. the dilaton scalar field ϕ (one state), the antisymmetric 2-form gauge field B_2 (28 states) and the traceless symmetric graviton g of spin 2 (35 states). In the RR sector two new antisymmetric tensor fields appear, a 1-form denoted C_1 (8 states) and a 3-form C_3 (56 states). Finally, the RNS sector contains the space-time fermions. It is also convenient to introduce the Hodge duals of the RR forms in this case, which are a 7-form C_7 and a 5-form C_5 respectively.

The low energy limit of the Type IIA superstring theory is a Type IIA *supergravity* (SUGRA) [31, 32]. Supergravity theories are QFTs constructed as supersymmetric extensions of general relativity in a different number of dimensions. In these theories SUSY is a local symmetry, and is combined with the usual Poincaré algebra giving rise to the so-called super-Poincaré group[10]. It is remarkable that superstring theories indeed provide finite ultraviolet completions of these SUGRA theories. In the case of IIA SUGRA, it is described by the effective action (the following low energy effective actions are all given in the string frame)

$$\begin{aligned} \mathcal{S}_{IIA} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{|g|}\Bigg\{ & e^{-2\phi}\left(\mathcal{R} - 4(\partial\phi)^2 - \frac{1}{2\cdot 3!}H^2\right) - \frac{1}{4}\left(G_2\right)^2 \\ & - \frac{1}{2\cdot 4!}\left(G_4\right)^2\Bigg\} \ - \frac{1}{4\kappa^2}\int d^{10}x\, G_4\wedge G_4\wedge B, \end{aligned} \tag{1.34}$$

where $G_2 = dC_1$, $H = dB$, $G_4 = dC_3 - H\wedge C_1$, are the RR and NSNS field strengths and κ is related to the gravitational constant in 10 dimensions G_D by $\kappa = \sqrt{8\pi G_D}$. This low energy SUGRA action admits classical solutions which are interpreted as extended p-dimensional objects known as *branes* or *p-branes*. These branes couple to the RR and NSNS fields, in such a way that an electric p-brane couples to a $(p+1)$-form potential whereas a magnetic one couples to a $(7-p)$-form potential. In type IIA we find NS-1 and NS-5 branes[11], as well as RR p-branes for $p = 0, 2, 4, 6$. Surprisingly, in 1995, Polchinski

[10] For a string theory introduction with an exhaustive self-contained introduction to SUGRA theories one can read [33].

[11] An NS-string (NS-1) exists in the type II and heterotic superstring theories and is identified with the corresponding fundamental string (F1) [34]. The NS-string is electrically charged under the B_2 field,

identified black p-brane solutions (those RR branes) with the aforementioned D-branes on which open strings can end[12] [28]. Therefore open strings can also appear in type II theories, with ends attached to D-branes. In the case at hand, Dp-branes appear for $p = 0, 2, 4, 6, (8)$, where the D8-brane appears as charged to a RR 9-form C_9 [28, 30] with no much dynamics. Apart from that, in any superstring effective action we also expect to find both gravitational waves (GW) and Kaluza-Klein (KK) monopole solutions[13].

Romans massive Type IIA SUGRA

The C_9 RR form is the EM dual of an F_0 RR flux that can be introduced in the type IIA theory. This field is the so-called *Romans mass* parameter, and the corresponding extension of type IIA is referred as *Romans massive type IIA SUGRA* [26, 27]. D8 branes couple magnetically to this F_0 field, and electrically to its EM dual, a C_9 RR potential. In this massive version, one gives mass to the NS 2-form of standard IIA SUGRA through the deformation $G_2 \to G_2 + mB_2$, and the RR 1-form C_1 is then gauged away. The massive deformation gives rise aswell to an effective cosmological constant, of undetermined magnitude in the case of IIA string theory [28]. Romans massive IIA describes part of the space-time when a D8-brane is present in the type IIA theory; more concretely, the F_0 flux is an integer in the quantum theory [28], which jumps by one unit as crossing the D8-brane.

Type IIB theory

The type IIB theory has R-states with the same chirality and thus the theory is chiral. Therefore, the anomaly cancellation is not trivial at all, although it does occur [45]. The massless fields of the theory are

$$\begin{gathered}(\mathbf{8_v} + \mathbf{8_+}) \otimes (\mathbf{8_v} + \mathbf{8_+}) \ = \\ (\mathbf{1} \ + \ \mathbf{28} \ + \ \mathbf{35_v})_{\mathrm{NS-NS}} + (\mathbf{1} \ + \ \mathbf{28} \ + \ \mathbf{35_+})_{\mathrm{R-R}} + (\mathbf{8_-} + \mathbf{56_+})_{\mathrm{NS-R}} + (\mathbf{8_-} + \mathbf{56_+})_{\mathrm{R-NS}} .\end{gathered} \tag{1.35}$$

While sharing the same NSNS sector with the IIA theory, the RR sector has the zero, two- and 4-forms C_0, C_2 and C_4 respectively. Again, we shall introduce their corresponding Hodge duals for convenience, which are an 8-form C_8 and a 6-form C_6. Finally it is also

as a 2-form can be integrated on a 2-dimensional worldvolume. On the other hand, an NS5-brane is magnetically charged with respect to B_2, given that the EM dual of a 2-form is a 6-form $\star dB_2 = dB_6$, that can be integrated on the 6-dimensional worldvolume of the NS5-brane.

[12]This discovery triggered the so-called *second superstring revolution*, leading to both the holographic and M-theory dualities that are going to appear in different parts of this thesis.

[13]Gravitational waves carry momentum charge in a certain direction, whereas the less familiar KK-monopoles are purely gravitational solutions that only appear in KK compactifications [63].

possible to introduce a 10-form C_{10}. This 10-form does not have any propagating dof as its field strength would be a 11-form in a 10D space-time, and so no space-time kinetic term can be generated.

This type IIB theory has $\mathcal{N}=2$ chiral SUSY and is effectively described by IIB SUGRA, whose bosonic part is described by [35, 36, 37, 38]

$$\begin{aligned}\mathcal{S}_{IIB} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{|g|}\Bigg\{ e^{-2\phi}\left(\mathcal{R}-4(\partial\phi)^2-\frac{1}{2\cdot 3!}H^2\right)-\frac{1}{2}\left(G_1\right)^2 \\ -\frac{1}{2\cdot 3!}\left(G_3-C_0H\right)-\frac{1}{2\cdot 5!}\left(G_5\right)^2\Bigg\} -\frac{1}{\kappa^2}\int d^{10}x\, C_4\wedge G_3\wedge H.\end{aligned} \tag{1.36}$$

In this case, the field strengths are given by $H = dB$, $G_1 = dC_0$, $G_3 = dC_2$, $G_5 = dC_4 - H\wedge C_2$. Additionally we have the self duality condition $G_5 = \star G_5$. In this theory we also find NS-1 and NS-5, as well as Dp-branes for $p=-1,1,3,5,7$ [14] deduced from the RR field content of the theory.

1.2.2 Type I string theory

Modding out the type IIB theory by a particular $\mathbb{Z}_2$ discrete symmetry (known in general as performing an *orientifold* projection) one obtains another new theory with half of the initial SUSYs, the type I superstring theory, containing open and closed non-oriented strings. Open strings appear as fundamental objects of the theory, while closed strings appear once interactions are considered. Type IIB theory has this worldsheet parity symmetry Ω acting in the following way in the open and closed string modes

$$\begin{aligned} &closed: \quad \Omega\, X(\tau,\sigma)\,\Omega^{-1} = X(\tau, 2\pi-\sigma) \quad \Rightarrow \quad \alpha_m^{\mu(+)} \leftrightarrow \alpha_m^{\mu(-)}, \\ &open: \quad \Omega\, X(\tau,\sigma)\,\Omega^{-1} = X(\tau, \pi-\sigma) \quad \Rightarrow \quad \alpha_n^{\mu} \rightarrow \pm(-1)^n\alpha_n^{\mu}\,. \end{aligned} \tag{1.37}$$

By effect of the first transformation left- and right-movers become interchanged[15], while end points of the open strings are exchanged by the second one. Projecting the type IIB spectrum by this Ω transformation changes the sign of C_0, B_2 and C_4 and leaves the rest of the massless spectrum invariant, in this way the spectrum of the type I string theory is

[14] The D(-1)-brane is an instanton-like solution, whereas the D9-brane is charged under a 10-dimensional RR-form C_{10}. That C_{10} is non-dynamical in 10 dimensions, as it not possible to define an 11-dimensional field-strength.

[15] This $\mathbb{Z}_2$ discrete symmetry is not a symmetry of IIA theory, as left- and right- movers have opposite chirality in that case.

obtained

$$\frac{(\mathbf{8_v}+\mathbf{8_+})\otimes(\mathbf{8_v}+\mathbf{8_+})}{\Omega} = (\mathbf{1}+\mathbf{35_v})_{\mathrm{NS-NS}} + (\mathbf{28})_{\mathrm{R-R}} + (\mathbf{8_-}+\mathbf{56_+})_{\mathrm{R-NS}}. \tag{1.38}$$

This leads to the appearance of a graviton, a scalar and an antisymmetric tensor, as well as some fermionic states, which can all be combined into an $\mathcal{N}=1$ 10-dimensional SUGRA multiplet. However type I SUGRA in 10-dimensional Minkowski space is inconsistent due to gravitational anomalies. The only way to avoid this is by coupling it to an $\mathcal{N}=1$ super-Yang-Mills (SYM) 10-dimensional gauge theory with an $SO(32)$ or $E_8\times E_8$ gauge group. For the type I superstring only the former case is possible, and the corresponding gauge symmetry comes from certain non-dynamical dof related to open string ends; these are the *Chan-Paton* factors[16]. According to this, the low energy type I effective action is given by $\mathcal{N}=1$ D=10 SUGRA coupled to $\mathcal{N}=1$ SYM with gauge group $SO(32)$. Its bosonic part reads [40, 41, 42, 43]

$$\mathcal{S}_I = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{|g|}\left\{e^{-2\phi}\left(\mathcal{R}-4(\partial\phi)^2\right)-\frac{1}{2\cdot 3!}\left(G_3\right)^2-\frac{\alpha'}{4}e^{-\phi}\mathrm{tr}\left(F^2\right)\right\}. \tag{1.39}$$

The gauge fields F^a_b transform in the adjoint of $SO(32)$ and are given by

$$F_{\mu\nu}(b) = \partial_\mu b_\nu - \partial_\nu b_\mu + \sqrt{\frac{2}{\alpha'}}\left[b_\mu, b_\nu\right], \tag{1.40}$$

whereas the field strength G_3 introduces C_2 and the YM fields

$$G^{(3)}_{\mu\nu\rho} = \partial_\mu C^{(2)}_{\nu\rho} - \frac{1}{2}\mathrm{tr}\left(b_\mu F_{\nu\rho}(b) - \frac{1}{3}\sqrt{\frac{2}{\alpha'}}\, b_\mu\left[b_\nu, b_\rho\right]\right) + \text{cyclic permutations}. \tag{1.41}$$

No charged $NSNS$ solitons appear (and therefore no fundamental strings), but Dp-branes with $p=1,5,9$ are found in this theory, as their associated RR charges are not affected by (1.37). Associated to the previous orientifold projection Ω there is a so-called *orientifold* O9-plane. O-planes are non-dynamical, mirror-like extended objects defined as fixed points of an special orientifold projection[17]. Indeed, type I anomalies cancel out by considering a vacua filled by an O9-plane with -16 units of D9-brane charge, cancelled by the addition of 16 space-time-filling D9-branes and resulting in $SO(32)$ gauge symmetry [39]. This configuration preserves one of the two type IIB SUSYs, and its total energy density cancels.

[16] Chan-Paton factors work as a kind of label that, when considering multiple coincident D-branes, indicates on which of the D-branes an open string ends. They are going to be introduced in the next chapter, where D-branes are studied in more detail

[17] Those objects, as the Dp-branes, are also sources of electric and magnetic RR forms, and appear in superstring theories with different tensions, charges and dimensionality. Depending on the kind of O-plane, it can have a positive or negative tension, given by $T_{Op}=\pm 2^{p-5}T_{Dp}$.

1.2.3 Heterotic strings

Finally, the possibility of mixing both bosonic and superstring modes in a consistent way was not overlooked, and it is indeed possible to compactify 16 dimensions of the bosonic string states in order to match them with the 10-dimensional superstring ones. Those heterotic strings do not have any open string sectors, just closed ones, because locking left and right sectors is no longer a consistent condition. One bosonic mode can not be reflected in the boundary if there are only fermionic modes in the other direction and vice versa. Without entering in much detail, there only exist two consistent ways of constructing those theories, which result in two different theories with gauge groups $SO(32)$ and $E_8 \times E_8$, depending on whether periodic or anti-periodic boundary conditions are taken in the fermionic sector. The right-moving sector contains the usual closed string spectrum, while the left-moving one has vector fields in the adjoint of either of those gauge groups

$$(\mathbf{8_v} + \mathbf{8_+})_R \otimes (\mathbf{8_v})_L \;= (\mathbf{1} + \mathbf{28} + \mathbf{35_v})_B + (\mathbf{8_-} + \mathbf{56_+})_F . \tag{1.42}$$

These states assemble an $\mathcal{N} = 1$ SUGRA multiplet. On top of this there are vector bosons in the adjoint representation of the gauge group, therefore the low energy effective action involves $\mathcal{N} = 1$ SUGRA and 10-dimensional $\mathcal{N} = 1$ SYM with gauge group $SO(32)$ or $E_8 \times E_8$. Its bosonic part is then given by [40, 41, 42, 43]

$$\mathcal{S}_{Het} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{|g|}\, e^{-2\phi} \left\{ \mathcal{R} - 4(\partial\phi)^2 - \frac{1}{2 \cdot 3!} H^2 - \frac{\alpha'}{4} \mathrm{tr}\left(F^2\right) \right\}, \tag{1.43}$$

where the YM field strength F is

$$F_{\mu\nu}(V) = \partial_\mu V_\nu - \partial_\nu V_\mu + \sqrt{\frac{2}{\alpha'}}\, [V_\mu, V_\nu] , \tag{1.44}$$

and the 3-form field strength H reads

$$H^{(3)}_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} - \frac{1}{2} \mathrm{tr} \left(V_\mu F_{\nu\rho}(V) - \frac{1}{3}\sqrt{\frac{2}{\alpha'}}\, V_\mu\, [V_\nu, V_\rho] \right) + \text{cyclic permutations.} \tag{1.45}$$

These theories have no RR fields, and so no D-branes appear on it. On the other hand NSNS strings and 5-branes do show up. Should also be remarked that the YM $SO(32)$ coupling to the $\mathcal{N} = 1$ SUGRA is different in type I and $SO(32)$ heterotic theories. Nevertheless a connection between both theories does exist, concretely in terms of a duality relation [39].

Indeed, the five superstring theories turn out to be all interrelated as we are going to see in the next section. It is remarkable the fact that all these theories are non-anomalous, and therefore quantum-consistent, which is not the case with certain SUGRAs. In this way, string theory does not only provide a wide framework for them, but a consistent one.

Theory	Strings	Chirality	SUSY	NSNS	RR
Type I	open & closed, non-orientable	chiral	$\mathcal{N}=1$	$g,\ \phi$	C_2
Type IIA	open & closed, orientable	non-chiral	$\mathcal{N}=2$	$g,\ \phi,\ B$	C_1, C_3
Type IIB	open & closed, orientable	chiral	$\mathcal{N}=2$	$g,\ \phi,\ B$	C_0, C_2, C_4
Heterotic SO(32) / $\mathbf{E_8 \times E_8}$	closed, orientable	chiral	$\mathcal{N}=1$	$g,\ \phi,\ B$	–

Table 1.1: 10-dimensional superstring theories, with: type of strings, chirality, SUSYs preserved and NSNS and RR massless field content.

Theory	Dp-branes	NS q-branes
Type I	$p = 1, 5, 9$	–
Type IIA	$p = 0, 2, 4, 6, 8$	$q = 1, 5$
Type IIB	$p = -1, 1, 3, 5, 7$	$q = 1, 5$
Heterotic SO(32) / $\mathbf{E_8 \times E_8}$	–	$q = 1, 5$

Table 1.2: Stable Dp-brane and NS q-brane content of the different 10-dimensional superstring theories. As was previously pointed out, GW and KK monopole solutions are also present in all these theories.

1.3 String dualities and M-theory

By duality relation we understand a set of rules relating two equivalent descriptions of a given physical system. It results that relations of this type relate sectors of the same string theory, or even sectors of different theories. It was a great surprise to discover that the five different consistent superstring theories are in fact related among them. What is more, they can be seen as different limits of a unique non-perturbative 11-dimensional theory. First of all, let us present two types of string dualities that play a crucial role in this understanding of the superstring theories as a whole, the *target space duality* or *T-duality*, and the *strong-weak coupling duality* or *S-duality*.

1.3.1 T-duality

Target space duality may occur when at least one of the dimensions is toroidally compactified[18]. Let us consider the simpler case of the bosonic string. For a closed bosonic string with an spatially compactified dimension k

$$x^k \ \equiv \ x^k + 2\pi R \quad \Rightarrow \quad X^k(\tau, \sigma + 2\pi) \ \equiv \ X^k(\tau, \sigma) + 2\pi R w, \quad w \in \mathbb{Z}, \tag{1.46}$$

where R is the radius of the dimension and w the winding of the string around it (its sign encodes the direction). Due to periodicity $p^k = m/R$, with $m \in \mathbb{Z}$, so that it is possible to describe the winding and momentum modes by w and m respectively. In this case, all fields are expanded as in (1.11) except for the k direction, for which

$$X^k_\pm(\tau \pm \sigma) = \frac{1}{2}x^k + \alpha' p^k_\pm(\tau \pm \sigma) + oscillations, \tag{1.47}$$

with

$$p^k_\pm = \left(\frac{m}{R} \pm \frac{wR}{\alpha'}\right). \tag{1.48}$$

We notice some kind of invariance under the following transformation, called T-duality

$$R \ \leftrightarrow \ \frac{\alpha'}{R}, \qquad w \leftrightarrow m. \tag{1.49}$$

Indeed, it is possible to check that right-moving oscillators change sign under (1.49)

$$\alpha_n^{k(+)} \to \alpha_n^{k(+)}, \qquad \alpha_n^{k(-)} \to -\alpha_n^{k(-)}, \tag{1.50}$$

[18] An Abelian isometry in the target space must exist so that this duality can be generalized to other spaces, through the so-called Buscher procedure ([15, 16]).

and so it can also be seen as a space-time parity transformation acting only on the right-moving dof

$$X^k = X^k_L + X^k_R \quad \rightarrow \quad X'^k = X^k_L - X^k_R. \tag{1.51}$$

Indeed, imposing the on-shell condition $m^2 = -p^2$ for the ($(D-1)$-dimensional) mass of the states, it is possible to see that the mass spectrum is given by

$$M^2 = \frac{m^2}{R^2} + \frac{R^2 w^2}{(\alpha')^2} + \frac{2}{\alpha'}(N_L + N_R - 2), \tag{1.52}$$

which is invariant under transformation (1.49) [17, 18]. This is the basis of T-duality, it relates a theory compactified on a radius R with the same or a different theory compactified on a radius α'/R [19], with an exchange of momentum and winding modes. Indeed physical quantities such as the energy-momentum tensor and correlation functions are also left invariant by this transformation, from what follows that T-duality is not only a symmetry of the spectrum, but a symmetry of the theory.

Whereas a T-duality transformation leaves the closed string invariant, a change in the open strings boundary conditions does occur. Let us come back to the open bosonic string expansion (1.16) with Neumann-Neumann boundary conditions and take the k-direction to be compact $p^k = m/R$. By applying the transformation rules (1.51) the expansion remains the same except for the k-th direction, which becomes

$$X'^k = y^k + 2\alpha' \frac{m}{R}\sigma + i\sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} \alpha^k_n e^{-in\tau} \sin n\sigma. \tag{1.53}$$

y^k is just the magnitude of a constant vector pointing in the k-th direction, and can be introduced in the mode expansion without modifying the $X^\mu_L + X^\mu_R$ sum. This k-th component carries winding but no momentum, and satisfies Dirichlet boundary conditions. The endpoints of the string will then be fixed and located at

$$X^k(\sigma = 0) = y^k, \qquad X^k(\sigma = \pi) = y^k + 2\pi m \frac{\alpha'}{R}, \tag{1.54}$$

where $\frac{\alpha'}{R}$ plays the role of the dual radius[20]. This also works the other way around and so, surprisingly, by T-dualizing a compact direction of the open string Neumann and Dirichlet boundary conditions become exchanged. As Dirichlet conditions are provided by D-branes, this T-duality transformation makes these hyperplanes to naturally appear in the theory.

[19] There is a minimum length below which there is no new physics, the fixed point of the transformation: $R_0 = \sqrt{\alpha'} = l_s$.

[20] The initial open string had no winding modes, as with Neumann-Neumann boundary conditions in all directions the configuration is topologically trivial. Nevertheless the winding modes are now topologically stable, since the endpoints of the open string are fixed in the k direction by the Dirichlet condition.

The transformation is such that a D-brane not extended in the original circle will wrap the dual one, while momentum modes of the strings in the original dimension are transformed into winding modes in the dual version. This transformation can in general be realized over n circular directions, resulting in a T-dual description in which the open strings have fixed extrema in n directions. Therefore, as a general rule, T-duality changes the dimensionality of a Dp-brane by $p \to p \pm 1$ when moving from a compact dimension to the dual circle (a D-brane wrapping a circle, does not wrap the dual one, and vice-versa). As a consequence, we should now understand the original case of open strings with Neumann-Neumann boundary conditions in every direction ($n = 0$) as having them ending on a space-time-filling D-brane. All this occurs both in bosonic and superstring theories. On the other hand, we have that when T-dualizing a compact coordinate of radius R on which an orientifold projection Ω is defined as in (1.37), two fixed 9-dimensional hypersurfaces appear in the 10-dimensional case: two O8 planes. In general, n T-dualities can in this way generate 2^n non-dynamical O$(9-n)$ planes extended in $10-n$ dimensions[21].

It is remarkable that a T-duality transformation indeed maps the type IIA theory into the type IIB and vice versa [47, 48], given that (1.51) changes the chirality of the right-moving fermions in (1.34) and (1.36). Because of this, type IIA theory with one dimension compactified on a certain circle is equivalent to type IIB theory compactified on a circle with inverse radius. This is valid at both perturbative and non-perturbative levels. At low energies these compactified theories can be written in terms of 9-dimensional fields, and in 9 dimensions there is only one $\mathcal{N} = 2$ SUGRA (in Minkowski space). Hence it is obvious that both type II theories should be mapped to the same 9-dimensional theory, which allows to write down a set of transformation rules between their massless fields [46]. The explicit rules can be found in [33]. These rules can be successfully used to interrelate the worldvolume effective actions for D-branes in type IIA and type IIB theories (their bosonic part is going to be introduced in section 1.4.2).

Finally, we would like to mention that T-duality also appears in the type I theory[22], and links the two heterotic string theories once their gauge groups are both broken down to the subgroup $SO(16)\times SO(16)$ [50, 51, 52]. Let us now introduce the S-duality transformation.

[21] For an extensive review on orientifolds, see for example [44].

[22] T-duality acting on the type I superstring theory bring us to the so-called *type I' theory*. A review in the subject can be found in [49].

1.3.2 S-duality

Another important duality is the so-called strong-weak coupling duality or S-duality. Classical solitonic solutions of field theory can sometimes fulfill a very interesting property, namely they can saturate a bound for their mass, called the *BPS bound* (in honor to Bogomol'nyi, Prasad and Sommerfield) [53]. This relates the mass and charge for the soliton[23] so that when moving from weak to strong coupling states are preserved, transforming in such a way that the perturbative spectrum become heavy while the solitonic solution becomes light. The existence of an exact symmetry in the theory (the S-duality) was then conjectured to be responsible for that behavior, interchanging strong and weak coupling limits and solitons with perturbative states [54]. This symmetry also appears in string theory, in the context of supersymmetry, where the concept of BPS state is generalized to states preserving a certain amount of SUSY, as will be detailed in chapter 2. S-dualities sometimes relate the weak coupling limit of a string theory with its own strong coupling limit, or even with the strong coupling limit of a different one. This occurs by means of an inversion of the coupling constant g_s to $1/g_s$, in a similar way to the radius transformation of T-duality. In certain situations one can determine properties of those BPS states not depending on the coupling, and then, extrapolate them to a different coupling regime by an S-duality transformation. This makes BPS states, in combination to S-duality, an incredible tool for exploring otherwise inaccessible strong coupling regimes[24].

More concretely, S-duality is present in type IIB superstring theory as part of a bigger symmetry. Type IIB SUGRA has a global $SL(2,\mathbb{R})$ invariance [37, 38]. However, this symmetry of the low energy effective action is not shared by the full type IIB theory. Instead, it is conjectured that quantum effects break it down to a discrete $SL(2,\mathbb{Z})$ subgroup. One way of seeing this is by considering strings charged either under B_2 (fundamental strings) or under C_2 (called *D-strings*). Their charges are usually denoted by $(1,0)$ and $(0,1)$ respectively, as each one has just one unit of charge. These strings transform as a doublet of $SL(2,\mathbb{R})$ because so do the 2-forms under which they are charged[25]. In order to ensure that these charges are integers, as required by *Dirac quantization condition*[26], the global sym-

[23] Details about this are given in section 1.4.1.

[24] By using this kind of duality it is possible, for example, to take a system in its strong coupling regime and, once being transformed, carry out perturbative computations on the weakly coupled system. In this way we can finally return to the original system and get some kind of additional information.

[25] In general they transform into (p,q) strings, carrying both types of charge

[26] In quantum-mechanical systems with magnetic monopoles, the wave function of an electrically charged particle is uniquely defined only if its charge e satisfies the Dirac quantization condition [55] $e \cdot g \in 2\pi n$, $n \in \mathbb{Z}$, being g the magnetic charge. This result can be generalized to p-branes with charge μ_p by requiring the wave function of an electric p-brane to be consistently defined in the field of the magnetic (D-p-4)-brane $\mu_p\ \mu_{D-p-4} = 2\pi$ (single p-branes turn out to satisfy the relation for $n = 1$. Polchinski showed this for D-branes in [28]).

metry must be restricted to $SL(2,\mathbb{Z})$. This symmetry group is generated by the following transformations

$$\mathcal{S} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \mathcal{T} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \qquad \mathcal{R} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{1.55}$$

$\mathcal{T}$ corresponds to a shift in the RR scalar $C_0 \to C_0 + 1$, while $\mathcal{R}$ leaves the type IIB SUGRA complex field $\tau = C_0 + ie^{-\phi}$ invariant. The S-duality subgroup is given by the $\mathcal{S}$ transformation

$$C_0 \to \frac{-C_0}{(C_0)^2 + e^{-2\phi}}\,, \qquad e^{-\phi} \to \frac{e^{-\phi}}{(C_0)^2 + e^{-2\phi}}. \tag{1.56}$$

Taking $C_0 = 0$ and recalling that $g_s = e^{-\phi_0}$ this transformation produces the inversion of the coupling constant of the theory

$$g_s \to \frac{1}{g_s}\,, \tag{1.57}$$

as was previously advanced. The complete set of S-duality transformation rules for the massless forms of type IIB theory is given in [33].

Heterotic $SO(32)$ and type I string theories are also related by S-duality. As we said, both theories have low energy descriptions given by 10-dimensional $\mathcal{N} = 1$ SUGRA coupled to a YM $SO(32)$ theory, although with different dilaton couplings. Nevertheless both actions, (1.39) and (1.43), become equivalent under the following identifications

$$\phi = -\phi\,, \qquad g_{\mu\nu} = e^{-\phi} g_{\mu\nu}\,, \qquad B = C_2\,, \qquad A^a = V^a\,. \tag{1.58}$$

The inversion of the dilaton coupling suggests the existence of a strong-weak coupling relation between both theories [56]. This leads to the conjecture that both theories, not just their low energy limits, are actually dual to one another, being just two descriptions of two different regions of the same quantum theory. A possible non-perturbative check of this duality consists in showing that certain objects of one of the theories map into objects of the other theory under the conditions indicated. This indeed can be shown for the heterotic string, a perturbative object which becomes mapped to a D1-brane of the type I theory, a non-perturbative soliton. On the other hand, a type I D5-brane is also mapped to a NS5-brane of the heterotic superstring theory.

1.3.3 An 11-dimensional theory

As we have previously mentioned, there exists an 11-dimensional theory that can be accessed from those interrelated 10-dimensional superstring theories. More concretely, it can be obtained as a strong coupling limit of the type IIA and heterotic $E_8 \times E_8$ theories ([57],[56]). This 11-dimensional theory is known as *M-theory* and its elementary objects are no longer strings. Stable BPS SUGRA solutions do exist, certain non-perturbative objects called *M-branes* (from *membrane*[27]), but the fact that there is not a perturbative fundamental object eliminates any possible perturbative approach, with awful consequences for the study of the theory. What is more, M-theory does not have any tunable coupling that could give rise to a perturbative regime either, being the only parameter the Planck length in eleven dimensions l_p.

Following a top-down approach from M-theory compactifications it is possible to obtain either the IIA theory or the heterotic $E_8 \times E_8$ theory, depending on whether we compactify the 11-th dimension on a circle or on an interval respectively. Indeed, these compactifications give rise to the corresponding SUGRA theories in the low energy limit. In the first case Type IIA 10-dimensional SUGRA can be obtained as a limit of the 11-dimensional one [58], and it can be verified that their number of dof do indeed match. The bosonic content of the 11-dimensional SUGRA consist of the metric $\hat{g}$ and a 3-form gauge potential $\hat{C}_3$, combined in the following way in the 11D action [59]

$$\mathcal{S}_{11D} = \frac{1}{2\hat{\kappa}^2} \int d^{11}x \left\{ \sqrt{|\hat{g}|} \left(\hat{\mathcal{R}} - \frac{1}{2 \cdot 4!} (\hat{G}_4)^2 \right) - \frac{1}{6} \hat{G}_4 \wedge \hat{G}_4 \wedge \hat{C}_3 \right\} . \tag{1.59}$$

The 4-form $\hat{G}_4$ is the field strength of the 3-form potential $\hat{G}_4 = d\hat{C}_3$. It is possible to see that considering the maximum amount of SUSYs in 11 dimensions leads to the appearance of different brane solitonic solutions in M-theory. These are a KK monopole, a GW (M0-brane), an M2-brane (also called *membrane*), its EM dual, the M5-brane, and a domain-wall M9-brane[28]. Each of these carries a certain (SUSY central) charge, being therefore stable. In the same way D-branes are charged under RR forms of the 10-dimensional superstring, M2 and M5-branes are charged under the 11-dimensional $\hat{C}_3$ form and its $\hat{C}_6$ dual.

11-dimensional fields can be expressed in terms of 10-dimensional ones after a Kaluza-Klein reduction. In order to do that, let us split the space-time coordinates as $x^{\hat{\mu}} = (x^\mu, y)$ and consider the isometry generated by the Killing vector in the compactified dimension $\hat{k} = \partial_y$. The reduction rules for the reduction of the 11-dimensional metric and 3-form

[27]Although this terminology is usually reserved for the 3-dimensional M2-brane.

[28]The M9-brane in M-theory plays the same role as the D8-brane in type IIA (although there does not seem to exist a massive extension for uncompactified M-theory). For this reason the M9 seems to be the M-theoretic origin of massive type IIA string theory.

obtained in this way are [33] [29]

$$
\begin{aligned}
\hat{g}_{\mu\nu} &= e^{-\frac{2}{3}\phi} g_{\mu\nu} + e^{\frac{4}{3}\phi} C^{(1)}_\mu C^{(1)}_\nu , & \hat{C}^{(3)}_{\mu\nu\rho} &= C^{(3)}_{\mu\nu\rho} , \\
\hat{g}_{\mu y} &= e^{\frac{4}{3}\phi} C^{(1)}_\mu , & \hat{C}^{(3)}_{\mu\nu y} &= B_{\mu\nu} , \\
\hat{g}_{yy} &= e^{\frac{4}{3}\phi} .
\end{aligned}
\tag{1.60}
$$

From here we can see that the string coupling constant in the type IIA theory is related to the asymptotic value of $\hat{g}_{yy}$, which can be identified with the compactification radius R as well, as

$$
|\hat{k}|^2 = |\hat{g}_{yy}| = \left(\frac{R}{l_p}\right)^2 \qquad \Rightarrow \qquad g_s = \left(\frac{R}{l_p}\right)^{3/2} . \tag{1.61}
$$

Now, in order to relate units in eleven dimensions with the string units, that is to rewrite R and l_p in terms of l_s, we impose that the 10-dimensional Newton constant has to coincide with the 11-dimensional one

$$
G_{10} = \frac{G_{11}}{2\pi R} \qquad \Rightarrow \qquad l_p = g_s^{1/3} l_s \, , \qquad R = g_s l_s . \tag{1.62}
$$

Here it was used that the Newton constant in an arbitrary number of dimensions D is given in terms of the Plank length by

$$
16\pi G_D = 2\kappa_D^2 = (2\pi)^{D-3} l_p^{D-2} . \tag{1.63}
$$

From (1.62) we can see that for $g_s >> 1$, which corresponds to the IIA theory at strong coupling, the radius of the 11-th dimension becomes very large (in both string and eleven dimensional units). Therefore the compactified dimension opens up, and a 11-dimensional approach should then be used. In this way M-theory can be seen as the strong coupling limit of IIA superstring theory, not being possible to access that additional dimension while being in perturbation theory.

On the other hand, the strong coupling limit of the heterotic $E_8 \times E_8$ superstring theory is also given by M-theory. Starting again from an 11-dimensional point of view, it is possible to perform the dimensional compactification in an interval. This compactification leads to an 11-dimensional space-time which is a slab with two parallel 10-dimensional faces that break half of the SUSYs, having one set of E_8 gauge fields in each face whereas gravitational fields reside in the bulk. These E_8 gauge fields appear in order to cancel gravitational anomalies [61]. The role played before by the compactification radius is now played by the length of the interval L, in such a way that

$$
g_{het}^{2/3} = \frac{L}{l_p} . \tag{1.64}
$$

[29] We are using a mostly plus Minkowski metric, resulting in some sign differences with respect to the reference [33].

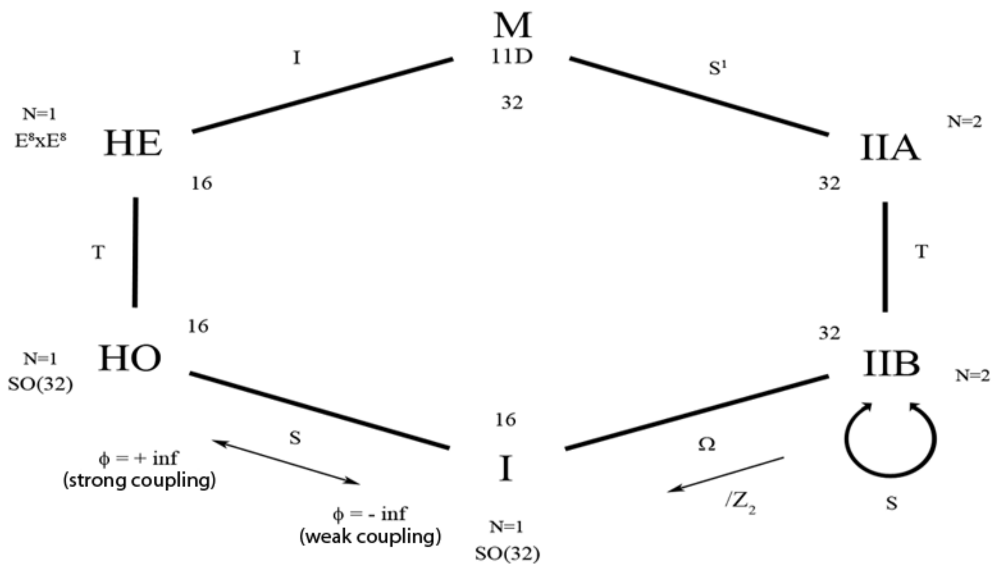

Figure 1.4: Scheme of the different superstring theories and their interrelations.

To conclude, fig. 1.4 shows the interrelations between the vacua of the different superstring theories. The numbers 16 and 32 indicate the amount of supercharges preserved by the vacuum of each theory, $\mathcal{N} = 1$ or $\mathcal{N} = 2$, being the number of space-time SUSYs preserved. For example in D=10 Majorana-Weyl spinors have 16 real components, and so those supercharges can be grouped together in one or two of these spinors depending on the case (we will study how strings and branes are related to the SUSY algebra in the following chapter). T-dual and S-dual relations are also shown.

1.4 More on D-branes

We have shown how by acting with T-duality on the open strings, certain p-dimensional dynamical hyperplanes called Dp-branes appear in different string theories, providing consistent boundary conditions for these strings. D-branes were indeed identified with certain solitonic solutions already found in SUGRA theories. In this section we elaborate a bit more on what is known about these objects[30], as some of their properties are going to be relevant later on.

[30] See [24] for a detailed review on the subject.

1.4.1 D-branes as BPS states

In supersymmetric theories BPS states are short multiplets of a representation of the supersymmetry algebra, preserving a certain amount of SUSY and having the property that their (invariant) masses are equal to some central charge, being therefore stable and protected from quantum radiative corrections. Indeed the sole presence of the soliton requires the extension of the original SUSY algebra, generated by Q_α and $Q^\dagger_\beta$ and describing non-singular and topologically trivial field configurations, to include the central charges Z as a boundary term[31]. These central charges are conserved quantities that commute with the other generators of the SUSY algebra.

BPS states do appear in string theory, well as point-like soliton SUGRA solutions, or as extended objects, the BPS p-branes. In general, the commutation algebra of the SUSY charges has the form

$$\{Q_\alpha, Q_\beta\} = (\Gamma^\mu C)_{\alpha\beta} P_\mu + \frac{1}{p!}(\Gamma^{\mu_1\cdots\mu_p} C)_{\alpha\beta} Z_{\mu_1\ldots\mu_p}. \tag{1.65}$$

It is found that the central charges are given by antisymmetric tensors proportional to the charges carried by the p-brane solitons. Therefore, the BPS condition relate these charges with the tensions[32] of these branes, that can be regarded as extended soliton solutions of the effective field theory. A p-form central charge in D dimensions is originated by a p-brane with charge density

$$\mathcal{Q}_p = \frac{1}{V_S}\int_{S^{D-p-2}} \star F^{(p+2)}, \tag{1.66}$$

where the integral is taken over a $(D-p-2)$ sphere of volume V_S transverse to the p-brane. For a p-brane extended in the directions $i_1, ..., i_p$ [33], and after identifying the p-brane charge density with the magnitude of the p-form central charge per unit volume of the brane $\mathcal{Q}_p = |Z_p|/V_{D_p}$, the central charge associated is given by

$$Z^{i_1\cdots i_p} = \mathcal{Q}_p \int dX^{i_1} \wedge \cdots \wedge dX^{i_p}. \tag{1.67}$$

Polchinski [28] related type II p-brane SUGRA solutions to D-branes, which are able to describe non-perturbative $\frac{1}{2}$BPS states carrying non-trivial RR charges. D-branes break only half of the $\mathcal{N}=2$ SUSY of the theory when their tensions saturate the BPS bound

[31] An explanation can be found in [62].

[32] The tension of a brane and not its mass is the relevant quantity to work with, as branes can in general be extended in non-compact dimensions, causing its mass to diverge.

[33] Only the spatial components $Z_{i_1\cdots i_p}$ of the $p-form$ central charge are associated to the $p-brane$. $Z_{0i_1\cdots i_{p-1}}$ components turn out to be associated to a $(D-p)$-brane after dualizing them [63, 62].

[28], in which case some remarkable properties shows up. A non-renormalization theorem protects the (semi-classical) spectrum of masses and charges of BPS states from quantum radiative corrections to all order in perturbation theory, as long as the corresponding SUSYs are preserved. These are the special states that we referred to in the advance given at introducing S-duality, and play a privileged role in order to study supersymmetric theories beyond its low energy limit since their properties cannot depend on any continuous parameter of the theory (for example, the string coupling g_s).Other interesting property of these BPS states is that the force between such states is zero.

1.4.2 Bosonic effective actions

As we already mentioned, D-branes are hyperplanes with certain dynamics associated. Indeed D-branes can be deformed and move[34] as both open and closed strings are able to transfer momentum by interacting with them[35]. We can ask ourselves if it would be possible to write down certain actions for describing the low energy dynamics of these objects by taking into account the strings attached. The answer is yes, as we are going to see. At weak coupling, D-branes can be described by certain low energy (semiclassical) worldvolume effective actions where the massless modes of the strings attached are restricted to the worldvolume of the D-brane. In this way low energy Dp-brane dynamics are described by a $(p+1)$-dimensional effective field theory of massless fields (scalars, spinors, and vectors). In the action of the whole configuration is in general possible to distinguish a contribution coming from the bulk of the strings, and other contribution associated to their extrema, where the D-branes are

$$S_{\text{eff}} = S_{\text{bulk}} + S_{\text{D-brane}}. \tag{1.68}$$

We are going to only work with bosonic actions in the following. Although fermionic excitations could be taken into account in these formulas, they sometimes increase the complexity in former calculations so much, that certain results had not even yet been proven when those superpartners are present. On the other hand, it should be understood that we are working in type II string theory unless the contrary is indicated, as well as the D-branes here considered are going to be BPS objects. Non-BPS configurations are introduced in

[34] Although it is usually difficult to talk about movement if one considers an infinite extended brane.

[35] Closed strings can also be emitted and absorbed by D-branes, and so they feel gravitational interaction. One can see this by considering an open string with both ends attached to a D-brane. If both ends join, the resultant closed string can leave the brane worldvolume and be absorbed by a different D-brane by the inverse process. This scattering can be computed in terms of open strings by means of the so-called *worldsheet duality transformation*, which interchanges σ and τ string coordinates, turning the closed string exchange (a cylinder between branes) into an open string 1-loop vacuum diagram, a Casimir effect between D-branes.

chapter 2.

As a first step, an approximate description of a p-brane can be given by generalizing the classical dynamics for the trajectory of a point-like particle, in the same way the Nambu-Goto action (1.3) was already a generalization to a bidimensional surface. An action minimizing a $(p+1)$-dimensional hypersurface associated to a p-brane evolving in a D-dimensional space-time is given by

$$\mathcal{S}_p = T_p \int d^{p+1}\xi \left(\sqrt{-detG_{ab}}\right), \tag{1.69}$$

where $G_{ab}(\sigma) = g_{\mu\nu}(X)\, \partial_a X^\mu \partial_b X^\nu$ ($\mu, \nu = 0, ..., D-1$; $a, b = 0, ..., p$) is again the *pull-back* from the D-dimensional metric to its worldvolume, parametrized by the coordinates ξ^a. Nevertheless, by using a so simple action we are not taking into account any of the aforementioned dynamical dof of Dp-branes.

In order to properly describe a Dp-brane at low energies compared to the string scale, we can construct effective actions by just taking into account the massless modes of the open strings ending on it, integrating the massive modes. In this manner, in superstring theories with Dp-branes an effective $(p+1)$-dimensional theory of massless fields with scalars, vectors, and spinors is obtained for them.

Dirac-Born-Infeld action

Let us first just take into account the coupling of the NSNS fields to the Dp-brane, assuming a 10-dimensional space-time and $\mathcal{N} = 2$ SUSY, i.e., type IIA/IIB. This results in a $U(1)$ gauge supersymmetric theory with a vector field A_a, associated to the endpoints of open strings, $9-p$ real scalars Φ^i ($i = p+1, ..., 9$), which describe transverse excitations of the D-brane, and the corresponding fermionic superpartners, a pair of real Majorana-Weyl spinors Θ^{Am} ($A = 1, 2$; $m = 1, ..., 32$) that, as was previously said, we are going to systematically skip. More concretely, only $9-p$ dof of the original $X^\mu(\xi)$ functions describing the embedding of the brane in the ambient space do propagate, these are the Φ^i fields. On the other hand, only 16 of the 32 fermionic dof propagate, from where only 8 turn out to be independent. The vector gauge field A_a has its origin in the open strings attached to the Dp-brane, and it provides the rest of bosonic physical dof required by SUSY. This is because gauge invariance requires two of its $p+1$ components to be non-dynamical, and so it results in a total of $(9-p) + (p-1) = 8$ bosonic dof.

The bosonic action describing this interaction between string boundaries and D-branes was proposed by Leigh [64] who, following some previous works, introduced a non-linear $\sigma-$model in order to describe such a theory in arbitrary massless backgrounds. After one-loop renormalization, the $\beta-$functions obtained lead to certain eom for the background fields equivalent to the ones obtained by the so-called Dirac-Born-Infeld (DBI) (or just Born-Infeld) action in $p+1$ dimensions[36]. The DBI action for a Dp-brane is[37]

$$\mathcal{S}_{DBI} = -\mathcal{T}_{\mathrm{D}p} \int d^{p+1}\xi \left(e^{-\phi} \sqrt{-det\,(P[G+B_2]_{ab} + 2\pi\alpha' F_{ab})} \right), \tag{1.70}$$

where $T_{\mathrm{D}p}$ is the tension of the Dp-brane, $F = dA$ is the field strength of the Born-Infeld vector A associated to open strings, ϕ the dilaton field and B_2 the NS 2-form. Usually the dilaton field is taken to be constant and its contribution is extracted from the integral. In this way it can be absorbed in an effective tension

$$T_{\mathrm{D}p} = \frac{1}{g_s (2\pi)^p (\alpha')^{(p+1)/2}}. \tag{1.71}$$

The $1/g_s$ dependence of $T_{\mathrm{D}p}$ provides a non-perturbative character to D-branes, becoming very heavy at weak coupling[38]. This dependence can be deduced from the Maxwell term $-\frac{1}{g^2}\int F^2$ coming from the expansion of (1.70). The gauge coupling in $p+1$ dimensions g is proportional to the open string coupling constant g_{open}, since the gauge field comes from the open string sector, and this coupling is related to the closed string one g_s by $g_s = g^2_{open}$. On the other hand, the numerical factors in (1.71) are computed in [28].

Let us now make use of the so-called *static gauge*, on which worldvolume coordinates of the D-brane are identified with $p+1$ of the space-time coordinates $\xi^a \equiv x^a$, by using the diffeomorphism invariance of the action. In this way, and by relabeling the $9-p$ coordinates remaining as $x^i \equiv 2\pi\alpha'\Phi^i$, (1.70) takes the more convenient form

$$\mathcal{S}_{DBI} = -T_p \int d^{p+1}\xi \left(\sqrt{-det\,(g_{ab} + (2\pi\alpha')^2 \partial_a\Phi^i\partial_b\Phi^i + \mathcal{F}_{ab})} \right) \tag{1.72}$$

where $\mathcal{F} = B_2 + 2\pi\alpha' F$ is a gauge-invariant combination of B_2 and F in the worldvolume[39]. Note that, after an expansion of (1.72) in powers of $\mathcal{F}$ and for constant dilaton, the first term corresponds to the generic p-brane action 1.69.

[36]The original Born-Infeld Lagrangian $\mathcal{L} = \sqrt{\eta + \chi F}$ is an example of non-linear electrodynamics. It cures the divergence of the electrostatic field of an stationary point-like charge at its origin by smoothing the field at that region, providing the usual Coulomb field outside.

[37]The supersymmetric extension of the DBI action was constructed in [65, 66, 67, 68, 69].

[38]Therefore, in this limit, they can be treated as rigid objects, or when necessary, considered just as small deformations when interacting with strings. It should be pointed out that other solitonic solutions in string theories have tensions that grow faster when approaching $g_s = 0$. For example, the NS5-brane tension is proportional to $1/g_s^2$. Due to this, D-branes have a wider weak coupling regime in which it is possible to neglect the gravitational backreaction of the background geometry. This gives the D-branes even more relevance, as they can be useful for probing string geometry. We will come back to this in chapter 4.

[39]The concrete gauge transformations of the world volume fields in D-brane actions can be read in [33].

Chern-Simmons action

RR string excitations are taken into account in the *Chern-Simons* (CS) or *Wess-Zumino* term, which has the following form[40] [70, 71, 29]

$$\mathcal{S}_{CS} = T_p \int d^{p+1}\xi \left(\sum_{n=0}^{10} C_n \, e^{\mathcal{F}} \right)_{p+1} . \tag{1.73}$$

The parenthesis in this action indicates that only terms with $(p+1)$-form total degree must be considered, so that in general a certain electric coupling to the $p+1$ RR form exists, in addition to some multipole couplings. Note that when considering a background with vanishing B-field, the expression has the form of an α' expansion. Let us for instance think of a D3-brane of the type IIB theory. In that case the CS action is in general given by

$$\mathcal{S}_{CS_{D3}} = T_4 \int d^4\xi \bigg(C_4 \, + \, C_2 \wedge \mathcal{F} \, + \, \frac{1}{2} \, C_0 \wedge \mathcal{F} \wedge \mathcal{F} \bigg), \tag{1.74}$$

consistently with the massless RR field content of the theory. Notice that a single D-brane always couples to RR forms with equal or less degree than its dimensionality. However, when multiple coincident D-branes are considered higher dimensional couplings can be developed in certain backgrounds ([75]). This effect is closely connected to a non-Abelian character exhibited by stacks of coincident D-branes, as we will see in section 1.4.3.

Additionally, when massive IIA SUGRA is considered, couplings to the F_0 Romans mass do also appear in the actions [26, 28, 27, 29] [41]. More concretely, the Bianchi identities for the RR gauge potentials C_{2k-1} can be violated by adding certain sources ρ

$$ddC_{2k-1} = \rho, \tag{1.75}$$

providing an extra contribution to the CS actions of D$(8-2k)$-branes of massive IIA.

Anti-D-Branes

It is possible to define *anti-Dp-branes* ($\bar{D}p$) as objects identical to the corresponding Dp-branes, but with inverted orientation in the worldvolume, such that they can carry opposite charge with respecto to the RR fields[42]. In terms of the effective action, this produces a

[40] An extension of (1.73) considering couplings to a general space-time curvature can be found at [73, 74]

[41] The M-theoretical interpretation of this F_0 Romans mass is not yet fully complete, although some advances as [60, 72] have been done.

[42] As well as a D0 particle exists, its antiparticle exits too, but cannot be defined as a D0-brane with opposite orientation.

change in the relative sign between the DBI and CS parts of the original action for a D-brane, so that they continue being $\frac{1}{2}$BPS objects but preserving the other one-half of the supersymmetries. This implies that coincident interacting Dp-anti-Dp ($(Dp, \bar{Dp})$) configurations break all SUSYs in the theory, eliminating their initial BPS character. This subject is explained in chapter 2.

1.4.3 Multiple coincident D-branes and Chan-Paton factors

The previously mentioned non-Abelian character of an N D-brane stack is reflected in the promotion of the worldvolume $U(1)^N$ theory of N separated Dp-branes, to a non-commutative $U(N)$ gauge theory. This promotion is driven by the fact that the initial transverse scalar fields Φ_i, which described the position of a single brane in the ambient space, become matrices when the D-branes are superimposed. In some cases the D-branes constituting the ground state of the system cannot be treated as individual objects any longer, forming what is known as a *fuzzy manifold*.

When considering multiple coincident D-branes, the open strings have freedom to end on any of the different D-branes. That freedom can be described by "labeling" the string endpoints. Additional indices i and j account for the fact that the string stretches from a D-brane j to a D-brane i in the following way

$$|\text{open string state}\rangle \otimes |ij\rangle. \tag{1.76}$$

If the state is a linear combination of eigenstates of the center-of-mass momentum k, it can be described by

$$|k; a\rangle = \sum_{i,j=1}^{N} |k; ij\rangle \, \lambda^a_{ij} \,, \tag{1.77}$$

being λ^a_{ij} Hermitian $N \times N$ matrices. These matrices form a representation of a $U(N)$ gauge group when the normalization $\text{Tr}\lambda^a\lambda^b = \delta^{ab}$ is chosen, and are called Chan-Paton factors. The index i transforms in the fundamental representation of the group, whereas j transforms in the antifundamental representation. In this way, Chan-Paton factors λ^a_{ij} belong to the adjoint representation of $U(N)$. It is remarkable that such simple non-dynamical dof lead, from the space-time point of view, to a non-Abelian gauge symmetry. This can be seen for example by considering a 3-point vertex (fig. 1.5), which contains an extra factor $\text{Tr}\lambda^1\lambda^2\lambda^3$. The sum coming from the internal boundary gives a factor of N that guarantees scattering amplitudes to be $U(N)$ invariant.

Notice that, since Chan-Paton dof are non-dynamical, they do not show up in the Hamiltonian, not affecting the quantization process of open strings.

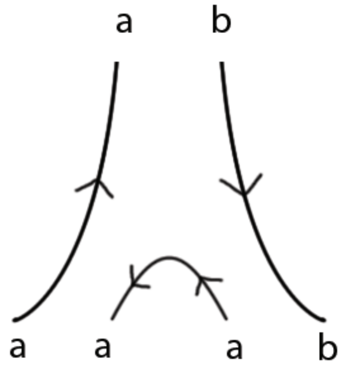

Figure 1.5: Chan-Paton factors are non-dynamical, and so the label of an endpoint propagates unchanged along the endpoint worldline. On the other hand one should sum over all possible index values for internal boundaries.

$$\begin{aligned} IIA/S^1 &\leftrightarrow IIB/(S^1)' \\ R &\leftrightarrow R' = \alpha'/R \\ Dir./New. &\leftrightarrow New./Dir. \\ p - brane &\leftrightarrow (p \pm 1) brane \\ 2\pi\alpha'(i\partial_a + A_a) &\leftrightarrow \Phi^a \end{aligned}$$

Figure 1.6: Certain p-branes in type II theories are mapped one into each other under T-duality. D-branes can be transformed in other D-branes with lower or higher dimensionality, this can be used in order to generalize a result obtained for a concrete system of D-branes to a generic dimension.

1.4.4 Branes and string dualities, a resume

After having studied D-branes in some more detail and having introduced their corresponding semiclassical effective actions, let us come back to the T- and S- duality transformations previously introduced in section 1.3 (fig. 1.6 summarizes some T-duality relations among branes in type IIA and IIB theories.) By using these rules (and by showing how the different dualities act on branes it is possible to establish a whole net of duality relations among them. A summary of these relations can be found in fig. 1.7. Two different options do always appear when relating M-theory and IIA branes, depending on whether we compactify on a longitudinal or transverse direction to the brane.

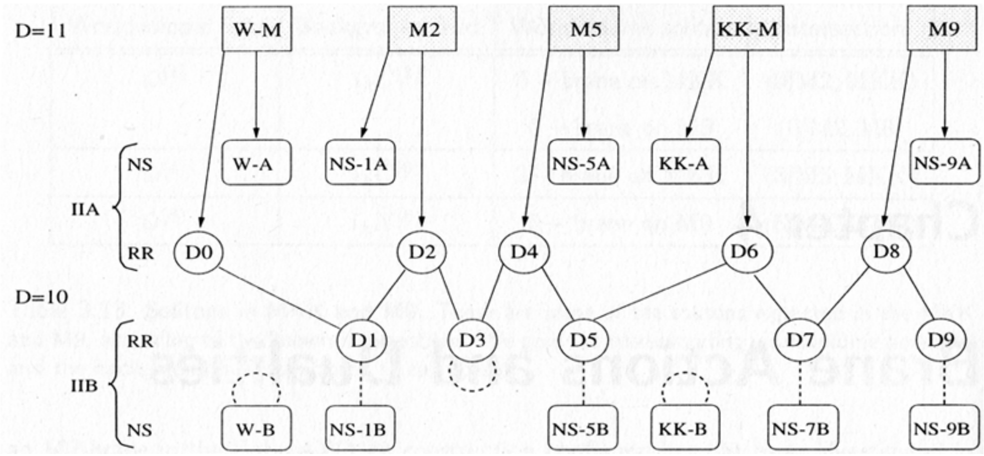

Figure 1.7: Different relations between branes, GW and KK monopoles of D=10 IIA and IIB theories, as well as from D=11. Arrows represent dimensional reduction relations, normal lines T-dualities, and dashed ones S-duality relations.

1.5 The AdS/CFT correspondence

In [20] Maldacena presented his celebrated conjecture. In its strongest form, that AdS/CFT correspondence is an holographic strong-weak coupling equivalence between a string theory realization in an d-dimensional Anti-de Sitter background (AdS_d) and a conformal field theory (CFT) living on its $(d-1)$-dimensional frontier. This was a very important result by itself, not only due to its continuously emerging new applications, but because the correspondence could be revealing something really deep about gravity, as a theory of gravitation could somehow be encoded in a lower dimensional local field theory.

In [140] 't Hooft considered the rank N of the gauge group of a $U(N)$ YM theory as a free parameter, together with the coupling g_{YM}. He realized that taking a large N limit ($N >> 1$) and holding the *'t Hooft coupling* $\lambda = g_{YM}^2 N$ fixed, the diagrams appearing in the perturbative expansion of the theory came multiplied by N to a certain power, depending only on the genus of the surface they span[43]. The similarity with the sum over worldsheet topologies of the string theory perturbative expansion was the first indication towards the so called AdS/CFT correspondence.

On the other hand, the AdS/CFT correspondence is a manifestation of 't Hooft holographic principle [155, 156], according to which all the information of a given volume can be contained on its boundary. This idea was based in the renowned black hole thermody-

[43]When N is taken to infinity this 't Hooft limit is also called *planar limit*, as in that limit non planar diagrams (with higher-genus contributions) are suppressed, providing a great simplification.

namics studies elaborated by Bekenstein and Hawking [157], by which the entropy S (and therefore the number of dof) of a black hole is proportional to the area A of its horizon $S = A/4G$ (with G the Newton's constant). In local field theories one would expect an entropy proportional to the volume of the horizon, instead to its area. This picture could be consistent if gravity in d dimensions is equivalent, in some way, to a local field theory in $d-1$ dimensions.

Building on these considerations and on results from Witten [141] and Klebanov [142] on coincident D-brane stacks, Maldacena conjectured the existence of a duality relation between certain CFTs and supergravity [20] [44]. Indeed, the strongest interpretation of that initial proposal is an exact equivalence between the whole type IIB string theory compactified in an $AdS_5 \times S^5$ space and $\mathcal{N} = 4$ SYM theory on $\mathbb{R} \times S^3$ with $U(N)$ gauge group. This proposal should hold provided the following identification is made

$$4\pi g_s = g_{YM}^2 = \frac{\lambda}{N}, \qquad \frac{L^2}{\alpha'} = \sqrt{\lambda}, \tag{1.78}$$

where $\alpha' = l_s^2$ and L is the common radius of AdS_5 and the S^5. Note that the Maldacena conjecture is a strong-weak coupling duality. As a good understanding of non-perturbative type IIB string theory is still pending, it could be that a more appropriate perspective is to look at the $\mathcal{N} = 4$ SYM theory as the definition of that IIB string theory in the $AdS_5 \times S^5$ background.

Although the strongest version of the conjecture is supposed to hold for any value of N, it is much easier to study it in the planar limit, where from (1.78) the string coupling constant vanishes. This is the regime of the weak formulation of the AdS/CFT correspondence, in which (in the AdS_5/CFT_4 case) the planar limit of the $\mathcal{N} = 4$ SYM theory ($N \to \infty$, $\lambda = g_{YM}^2 N$) is associated to non-interacting strings in the $AdS_5 \times S^5$ background, and where most efforts have been focusing. Since the initial breakthrough, this version of the duality has been very well tested by now. Nevertheless, providing a rigorous proof (or disproof) of its strongest form has not yet been possible (although lots of partial tests and hints do appear), mainly due to the aforementioned problems of testing any strong-weak coupling duality and our current ignorance of the strong coupling regimes of both theories. Proofs in both intermediate coupling regions could be the best way to face the problem, but this continues to be a really tough task. Additionally, at strong t' Hooft coupling ($\lambda >> 1$) and large N type IIB string theory on $AdS_5 \times S^5$ is reduced to type IIB SUGRA, resulting in that free type IIB in $AdS_5 \times S^5$ should be dual to planar $\mathcal{N} = 4$ SYM on $\mathbb{R} \times S^3$.

On the other hand, its strong-weak coupling nature would make the correspondence an incredible potential tool in order to explore the darker areas of one side of the duality from

[44] For a review on this topic, the classic [143] can be read.

the well known part of the other. Additional dualities between different theories (including M-theory) have been proposed since the original conjecture[45], even advances hinting for certain non-AdS/non-CFT dualities have also been posed. In this way, in the last years, the AdS/CFT correspondence has opened an incredibly wide window towards arid areas as highly coupled QFT systems, usually unreachable by analytical methods[46].

Before giving more explicit details about the AdS_5/CFT_4 duality and the novel ABJM proposal, let us first briefly describe how branes can create a gravitational background, and comment on the $AdS_m \times S^n$ case. After this, we will discuss the role played by particle-like brane solutions, which are going to be directly related to the research work presented in chapter 5.

1.5.1 Branes as gravitational sources

It is possible to look for solutions of the previously introduced bosonic type IIA (1.34) and type IIB (1.36) SUGRA actions containing only the metric $g_{\mu\nu}$, the dilaton ϕ, and just one of the antisymmetric tensors, say the $(p+1)$-form potential[47]. Imposing[48] space-time Poincaré invariance in $p+1$ directions, rotational invariance in the remaining $D-p-1$ directions, a Minkowski asymptotical limit in the radial direction $r \to \infty$, and the preservation of half of the SUSYs, it is possible to obtain (in the string frame) a rather simple metric[49] [151]

$$ds^2 = H(r)^{-1/2}\eta_{ab}dx^a dx^b + H(r)^{1/2}\delta_{ij}dy^i dy^j, \tag{1.79}$$

where $H(r) = 1 + \left(\frac{L}{r}\right)^{7-p}$, and L is a certain characteristic length scale proportional to $\sqrt{\alpha'} = l_s$. There exists a unique p-brane gravitational solution with a given tension T_p and "electric" charge Q_p (w.r.t. the antisymmetric tensor field chosen). For $T_p > Q_p$ the solution appears to have a horizon in the transverse directions, and so it is usually called *black p-brane*. In the BPS case, the condition $T_p = Q_p$ is satisfied and the solution is called *extremal black p-brane*, a denomination clearly inherited from classical black hole physics.

[45]For instance, the duality between certain 1+1 dimensional relatively well-understood CFT and type IIB string theory compactified on $AdS_3 \times S^3 \times T^4$ [20]. That CFT was the one that featured in the first counting of black hole microstates done by Strominger and Vafa [148].

[46]Numerical approaches such as lattice are the only way to attempt to face the problem.

[47]It is possible to check [149] that the fields kept do not act as sources for those not taken into account (see [150] for a review on BPS branes as SUGRA solutions).

[48]Apart from an "electric" type ansatz for the antisymmetric potential, and a certain ansatz for the dilaton field.

[49]We should remark that, in this way, it can also be seen how the generalization of Dirac's quantization condition for a p-brane and its EM dual $(6-p)$ brane arises.

This equality indicates the existence of an exact cancellation between the attractive influence of the NSNS fields, due to the brane tension T_p, and the repulsive (Coulomb-like) force of the RR potentials due to the charge Q_p. This precise cancellation of forces allows us to pile up any number of coincident BPS p-branes.

We should note that three different regions appear in (1.79) depending on the radial distance to the solution r. An *asymptotic region* $r >> L$, in which the metric becomes flat, an *intermediate region* $r < L$, in which the deviation from Minkowski space-time is fast, and a *throat region* $r \to 0$, where the metric has an apparent singularity (which is considered to be a horizon, and would correspond to the interior of the black-brane). We should remark that the region of validity of (1.79) is $g_s N > 1$, where the characteristic length scale is greater than the string scale (as $g_s N < 1$ would mean $L < \sqrt{\alpha'}$, and in that case the characteristic length scale would be less than the string scale, where we do not expect to trust this solution).

$AdS_m \times S^n$ spaces from string/M theory

Maldacena [20] conjectured that "the full quantum M/string theory on AdS space, plus suitable boundary conditions is dual to the corresponding brane theory". At that point, he did not specify the boundary conditions in AdS, instead, he made his conjecture based on an $AdS\times$(spheres) description at large N where it was possible to isolate some local processes from that boundary conditions question, remarking that the supersymmetries of both theories do indeed agree as was demonstrated by previous works: Although the superconformal group has twice the amount of SUSYs of the corresponding super-Poincaré group [152, 153], a SUSY enhancement near the horizon of extremal black holes was observed [154] by showing that the near throat geometry reduces to those $AdS\times$(spheres) geometries. In the case of the $AdS_5 \times S^5$ background, its SUSY group was known to be the same as the superconformal group in 3+1 dimensions [152].

More concretely, in string theory and M-theory it is possible to generate $AdS_m \times S^n$ backgrounds in the near horizon limit of certain brane stacks. For maximal supersymmetry the possible values for the (m, n) pair are: $(5, 5)$, by considering a stack of type IIB D3-branes, and $(4, 7)$ or $(7, 4)$ in M-theory, by considering an stack of M2 or M5 branes respectively. The general metric for those spaces can be written as

$$ds^2 = ds^2_{AdS_m} + ds^2_{S^n}, \tag{1.80}$$

with

$$ds^2_{AdS_m} = -\left(1 + \frac{r^2}{\tilde{L}^2}\right) dt^2 + \frac{dr^2}{\left(1 + \frac{r^2}{\tilde{L}^2}\right)} + r^2 d\Omega^2_{m-2}, \tag{1.81}$$

and

$$ds^2_{S^n} = L^2\left(d\theta^2 + cos^2\theta\, d\phi^2 + sin^2\theta d\Omega^2_{n-2}\right). \tag{1.82}$$

Here $d\Omega_{m-2}$ and $d\Omega_{n-2}$ are the metrics of a unit S^{m-2} and S^{n-2} respectively. The unit metric of a S^d can be written as

$$d\Omega_d = d\alpha_1^2 + sin^2\alpha_1\left(d\alpha_2^2 + sin^2\alpha_2(\cdots + sin^2\alpha_{d-1}d\alpha_d\right). \tag{1.83}$$

Given that the branes source the background, certain p-form fluxes exist with potentials given by

$$C_{t\tilde{\alpha}_1\cdots\tilde{\alpha}_{m-2}} = -\frac{r^{m-1}}{\tilde{L}}\sqrt{|\tilde{g}_\alpha|}, \tag{1.84}$$

in the AdS_m part (tilded quantities are related to AdS), and

$$C_{\phi\alpha_1\cdots\alpha_{n-2}} = \beta_n L^{n-1} sin^{n-1}\theta\sqrt{|g_\alpha|}, \tag{1.85}$$

in the S^m. $g_{\tilde{\alpha}}$ and g_α are respectively the unit metric over the spheres parametrized by $\tilde{\alpha}^i$ and α^i. The coefficient β_n of the magnetic potential is responsible for the potential sign, being $\beta_4 = 1$, $\beta_5 = 1$, $\beta_7 = -1$ its possible values. On the other hand, L and $\tilde{L}$ are respectively the S^n and AdS_m radius, which are related by $L = \frac{n-3}{2}\tilde{L}$, and depend on the number of branes that source the background. In the $AdS_5 \times S^5$ case, we have that $L^4 = 4\pi g_s(\alpha')^2 N$, where N stands for the number of D3-branes in the stack, and so both the radius of the AdS and the sphere are controlled by the total number of D3-branes that source the background.

In addition to the global parametrization (1.81) of AdS, a suitable local coordinate system called *Poincaré coordinates* $(u, t, \vec{x})$ can be used. In these coordinates, the AdS_m space-time metric takes the following form[50] [143]

$$ds^2 = R^2\left(\frac{du^2}{u^2} + u^2(-dt^2 + d\vec{x}^2)\right), \tag{1.86}$$

from where it can be more easily seen that AdS spaces have two quite special limits, directly related to the coordinate u. This u coordinate sets the scale of the Minkowski part of the metric (1.86). One possible limit, $u \to 0$, completely contracts the Minkowski space to a point, while the other one, in which we take the u coordinate to $u \to \infty$, and so blowing up the size of the Minkowski subspace, takes us to the so-called *boundary of AdS.* It is in this AdS boundary where the stack of branes that source the background is located and

[50]In this form of the AdS metric, the subgroups $ISO(1, m-2)$ and $SO(1,1)$ of the $SO(2, m-1)$ isometry are manifest. $ISO(1, m-2)$ is the Poincaré transformation on $(t, \vec{x})$ and $SO(1,1)$ ($(u, t, \vec{x}) \to (c^{-1}u, ct, c\vec{x})$) is identified with the dilatation transformation D in the conformal symmetry group of $\mathbb{R}^{1,m-2}$ by the AdS/CFT correspondence.

so the corresponding CFT.

Type IIB SUGRA has only two maximally SUSY vacua: flat Minkowski space, and the $AdS_5 \times S^5$ background just presented [37]. On the other hand, in order to consider a probe D3-brane in the AdS background instead of being part of the source stack, which is extensively used for diverse purposes in the context of the correspondence,we refer to the initial works of Metsaev and Tseytlin [146, 144]. On the other hand, actions corresponding to the M2-brane and the M5-brane in an $AdS_4 \times S^7$ and $AdS_7 \times S^4$ background were derived in [145].

1.5.2 The AdS_5/CFT_4 duality

Before giving more details on the AdS_5/CFT_4 correspondence, let us just sketch some relevant information about the 4-dimensional $\mathcal{N} = 4$ SYM CFT with $U(N)$ gauge group.

4D $\mathcal{N} = 4$ SYM

The $U(N)$ $\mathcal{N} = 4$ SYM theory has the maximum number of supercharges in 4 dimensions, and its Lagrangian is completely determined by SUSY:

$$\mathcal{L} \sim \frac{N}{\lambda} \mathrm{Tr} \left(\frac{1}{4} \mathcal{F}^2 + \frac{1}{2} (\mathcal{D}_\mu \Phi_m)^2 - \frac{1}{4} [\Phi_m, \Phi_n]^2 + \cdots \right) . \tag{1.87}$$

The field content of the theory is given by an $U(N)$ gauge field $\mathcal{A}_\mu$, 4 fermions Ψ^a_α, and 6 scalars Φ_m. The theory has an (exact) global superconformal symmetry $PSU(2,2|4)$, and three unrenormalized (due to the conformal invariance) couplings λ, N, θ_{top}. The superconformal group is composed by the conformal symmetry group in 4D $SO(4,2)$ [51], and a global $SU(4) \cong SO(6)$ *R-symmetry*[52] which rotates the scalars and fermions of the theory.

The theory has a lot of interesting simplifying properties in its planar limit, many of those under continuous development at the moment, although we are not going to detail them.

[51] $SO(4,2)$ includes Poincaré invariance, scale transformations, and an special conformal transformation which includes the inversion symmetry $x^\mu \to \frac{x^\mu}{x^2}$.

[52] Generically, an R-symmetry is just a symmetry that commutes with SUSY (i.e., transforms the different supercharges into each other). In extended SUSYs, as in this case, the R-symmetry group is non-Abelian.

The correspondence

The subgroup $SO(4,2)$ of the global $PSU(2,2|4)$ symmetry of 4-dimensional $\mathcal{N}=4$ SYM is (locally) the isometry group of the AdS_5 space-time. On the other hand, $SO(6)$ happens to be the isometry group of S^5.

We have already seen that the near horizon limit of N D3-branes has an $AdS_5 \times S^5$ geometry, which is a valid description for great values of $g_s N$ ($R >> l_s$). Note that this is a closed string description, as the graviton particle in string theory is a closed string state. On the other hand, we have also seen that the low energy worldvolume dynamics of a system of D-branes generates a $U(N)$ gauge symmetry for the theory of open strings attached, which in this case, is going to have $\mathcal{N}=4$ SUSY. This later point of view is valid in the perturbative regime, for low values of $g_s N$. In the gravitational description, people observed that by getting closer to a black brane horizon of an N D-brane stack, the large gravitational field produces a redshift in the brane excitations seen from the far transversal region, in such a way that only the highly energetic modes do get to there, and they do it as low energy modes. In this way, for large N, the strongly coupled CFT in the boundary is associated to low energy excitations in the bulk of AdS space (and so the AdS/CFT correspondence appears as a consequence of the duality between open and closed strings). Indications of this came from calculations of low energy graviton absorption cross sections, in which gravity and SYM calculations do agree[53].Additionally, in using this picture it is important to keep in mind that the AdS boundary is at spatial infinity, and although it is possible for a light ray to go and come back in a finite time, massive particles can never get to there.

Maldacena, in its initial proposal [20], did not yet provide a map between AdS and CFT quantities. That map came with [19, 28], and made the correspondence much more explicit. For illustration, let us just consider the bosonic dof here. Considering a free scalar field ϕ with mass m moving in AdS, its Klein-Gordon equation has two linearly independent solutions behaving as

$$e^{-\Delta r}, \qquad e^{\Delta - 4}r, \tag{1.88}$$

in the $r \to \infty$ limit, and having that $\Delta(\Delta-4) = m^2$. The coordinate r is defined in terms of the u Poincaré coordinate of AdS that we used in (1.86) by $u = e^r$. If we now consider a SUGRA solution with the following condition for the fields at the boundary

$$\phi(r, x^\mu) \sim \phi_i^0(r, x^\mu) e^{(\Delta-4)r}, \tag{1.89}$$

[53]Inspired in calculations of D1-D5 systems.

then the AdS/CFT map is given by

$$\exp\left(-\Gamma_{\text{sugra}}(\phi_i)\right) = \left\langle \exp\left(\int d^4x \phi_i^0 \mathcal{O}_i\right)\right\rangle . \tag{1.90}$$

The LHS of the equality represents the SUGRA action evaluated on the classical solution given by ϕ_i, whereas the RHS is a generating function of correlation functions in the SYM theory. Indeed, at the LHS of (1.90) we should be using the full string theory partition function subject to the relevant boundary conditions (this SUGRA version is only its saddle-point approximation). We should note that there should be a YM operator $\mathcal{O}_i$ for every ϕ_i in AdS. On the other hand, it is possible to show that this operator needs to have conformal dimension δ_i. We have restricted to the scalar fields ϕ_i, but the full AdS/CFT correspondence should involve all the AdS dof.

In this way, under the correspondence, the gauge-invariant local operators of the conformal theory are associated to certain states in the string theory side, and their conformal scaling dimensions Δ are associated to the energy of the stringy states. The dilatation operator corresponds to a non-compact generator of the global symmetry group, that can get quantum corrections and depends on the coupling constant. The rest of the charges are discrete and correspond to angular momenta of certain string states moving in non-trivial cycles of the geometry.

More concretely, in order to avoid the problem of testing a strong-weak coupling duality, the early tests were realized working with the *chiral primary operators*[54] Tr Z^{J_1}, Tr Y^{J_2}, Tr X^{J_3} [173, 21], being Z, Y, X complex linear combinations of the scalar fields of the SYM theory. Each of these linear combinations has a single unit of R-charge, J_1, J_2 and J_3 respectively. The importance of these operators is that they are BPS, and because of that, their scaling dimensions are protected by supersymmetry, so that they do not receive quantum corrections and are independent of the value of the 't Hooft coupling λ [55]. Due to this, it is possible to compare them directly to classical string energies in the $\lambda >> 1$ limit. Their dual string states turned out to be point-like particles rotating along a great circle of the S^5 from $AdS_5 \times S^5$ with angular momentum J_1, J_2 and J_3 respectively. Berenstein, Maldacena and Nastase [158] proposed to study operators with a very large R-charge $J = J_1$ with the idea of turning the perturbative expansion of the gauge theoretical conformal dimension Δ in a power series with corrections scaling as higher powers of $1/J$ (this was called BMN scaling). This spectrum could be maintained well-behaved even

[54] Chiral primary operators are those with minimal conformal weight. We refer to [147] for an introduction to CFTs with applications to string theory .

[55] Massive string modes correspond to operators in long multiplets with dimensions diverging for large λ, and therefore, the stringy nature of the dual picture is hard to guess due to the decoupling of the non-chiral operators, which constitute the large majority of possible gauge invariant operators.

at strong coupling and compared to perturbative string energies. Following that, it was possible to reproduce at one loop and leading order in J the quantum spectrum of certain gravity states. Something very remarkable was that, following very general arguments in the CFT side, families of quiral primaries expected to have an infinite number of elements, are instead truncated and finite. By relating (via R symmetry of the CFT) the charge of these operators with the angular momenta of the aforementioned stringy states, one deduces that their angular momenta have to be bounded from above. This is the so-called *string exclusion principle* [204], and is understood in terms of *giant graviton states* corresponding to certain D-branes moving inside an S^r part of the geometry, stable as their tensions equal their charges, and seeing as point-like objects from outside the compact space in which they move. These states turns out to have their momenta associated to their radius, and being the S^r a compact space, the hole picture makes sense.

Quarks and duality

A *Wilson loop* is a is a quantity defined by the trace of the path-ordered exponential of a gauge field transported along a closed loop in space-time. In a Yang-Mills theory

$$W(\mathcal{C}) = \frac{1}{N}\mathrm{Tr}\mathcal{P}e^{i\oint_c A}, \tag{1.91}$$

where Tr stands for the trace taken over the fundamental representation and $\mathcal{P}$ is the path-ordered operator. We can view the Wilson loop as the phase factor associated to the propagation of a very massive quark in the fundamental representation of the gauge group. Its vev gives the quark-antiquark ($q\bar{q}$) potential, making this quantity a valuable tool for example, to study whether a theory is confining or not. $\mathcal{N}=4$ SYM has no dynamical quarks, although it is still possible to introduce static quarks and compute the appropriate Wilson loop to obtain the $q\bar{q}$ potential.

The problem of computing static Wilson-loops in $\mathcal{N}=4$ SYM in the 't Hooft limit is mapped, under the AdS/CFT duality, to the problem of minimizing the classical action for a string that extends into the radial direction and connects the quark and antiquark at the AdS_5 boundary (fig. 1.8) [161]

$$W(\mathcal{C}) \sim e^{-S}. \tag{1.92}$$

Taking this into account a Wilson loop can be computed both at weak 't Hooft coupling, directly in the field theory [160], or at strong 't Hooft coupling in the gravity side of the correspondence [161, 162]. An agreement is found as in both regimes the $q\bar{q}$ potential turns out to be proportional to $1/d$, being d the separation between quark and antiquark. This particular coulombian shape exhibits no confinement, which is related to the conformal

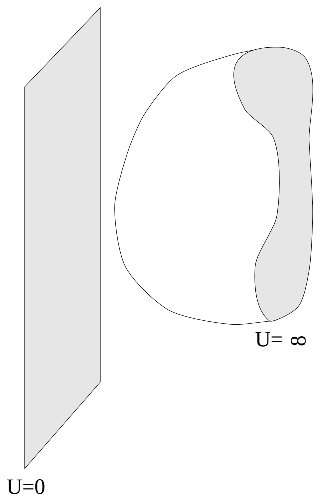

Figure 1.8: REHACER FIGURA The string worldsheet ends in the contour $\mathcal{C}$ on the AdS boundary.

invariance of the theory[56].

It was already shown in [161, 162] that fundamental strings ending on the D3-brane and going all the way to the AdS boundary are seen as external quarks or antiquarks in the dual theory, depending on the strings orientation. We can consider a U-shaped string with both ends attached to the boundary. That would be seen as a $q\bar{q}$ pair on the field theory side. It is remarkable how, after so many advances in string theory, the fundamental string (in AdS) ends up identified with the QCD string of YM theory, which motivated the initial development of the theory as an effective way for describing strong interaction.

On the other hand we can ask ourselves how to study more realistic CFTs, i.e. with confinement and no SUSY, similar to QCD. There are different ways of breaking SUSY, one example is to consider a certain orbifolding that produces the SUSY breaking, as occurs with the conifold, at which tip SUSY is broken[57].

[56]Confinement breaks the conformal invariance of a theory by the introduction of an energy scale, below which, quarks form bound states. In QCD that scale is $\lambda_{QCD} \sim 200MeV$, obtaining a conformal theory in its chiral limit.

[57]Additionally in this case, by smothering the shape of the conifold we can obtain a linear potential for the $q\bar{q}$ pair ending on the AdS boundary, and henceforth, confinement [177].

Towards a theory of coincident supermembranes

In the last three years important progress have taken place in understanding the world volume theory of coincident supermembranes of M-theory in the context of the AdS_4/CFT_3 correspondence. These interacting superconformal CFTs are expected to be Chern-Simons gauge theories coupled to massless matter [176]. These highly supersymmetric three dimensional CFTs are of interest in the description of some conformal fixed points in condensed matter systems. Their high degree of SUSY makes them more solvable, and can be used as toy models for certain studies.

The theories initially studied had $\mathcal{N} \leq 3$ supersymmetry, thus not meeting the required amount SUSY for describing theory of M2-branes (the M2-branes exhibit an $\mathcal{N} = 8$ SUSY). Nevertheless a significant advance in that direction occur. Bagger and Lambert [178] (and independently [179]) constructed a theory with $\mathcal{N} = 8$ supersymmetry that was conjectured to be related to a specific theory of M2-branes; however, it only described correctly two M2-brane systems. More details about this and the more general ABJM proposal are given in chapter 4.

Summary

We are going to present two independent research works in the area of string theory, each one presented in a separate part of this thesis (parts I, II). The following chapters are organized so that each part contains one chapter (chapters 2, 4) briefly introducing relevant advances related to the correspondent research, that appears in the very next chapter (chapters 3,5). The researches are presented merely as an adaptation of the original articles. Conclusions are given separately at the end of each part of the thesis. A summary of the following chapters is presented below.

Part I

Chapter 2:

In this chapter we introduce certain unstable non-BPS brane configurations, the non-BPS Dp-branes and the $(Dp, \bar{D}p)$ systems. Their instability is reflected in the presence of tachyonic modes in their string spectra. These tachyonic modes can decay ('condense') giving rise to a new configuration, which can in turn be stable or not. The boundary state formalism is introduced, as well as an effective potential approach, in order to tackle the problem.

Chapter 3:

This chapter is an adaptation of [22]. In this work, made in collaboration with my supervisor Y. Lozano, we present a worldvolume effective action suitable for the study of the confined phase of a $(Dp, \bar{D}p)$ system at weak coupling. We identify the mechanism by which the fundamental string arises from this action when the Dp and the $\bar{D}p$ annihilate. We also construct an explicit dual action, more suitable for the study of the strong coupling regime. Our dual description indicates that the condensing tachyonic objects originate from open D$(p-2)$-branes stretched between the brane and the antibrane.

Part II

Chapter 4:

In this fourth chapter we present the most relevant results from the particle-like branes appearing in relation to the AdS/CFT conjecture. These configurations are made of branes living in the bulk of AdS and a certain number strings stretched all the way to the boundary, where they are seen as external quarks. We review the stability study of the baryon vertex in the $AdS_5 \times S^5$ background and how this configuration was generalized by introducing a magnetic flux. We also comment on the Di-baryon configurations that appear in the same context. Finally, we explain the basis of the ABJM theory, an AdS/CFT proposal realized over an AdS_4 space and related to a three dimensional supersymmetric CS matter theory. A review of the particle-like branes appearing in this theory let us lay the ground for the research presented in chapter 5.

Chapter 5:

Here we present the research work [23] in which we study the effect of adding lower dimensional brane charges, generalizing the particle-like brane configurations of $AdS_4 \times \mathbb{P}^3$ introduced in the previous chapter. We show that these configurations require additional fundamental strings in order to cancel certain worldvolume tadpoles appearing. A dynamical study reveals that the charges must lie inside some interval in order to find well defined configurations, and for the baryon vertex and the di-baryon, the number of fundamental strings must also lie inside an allowed interval. As our configurations are sensitive to the flat B-field recently suggested in the literature, we make some comments on its possible role. We also discuss how these configurations are modified in the presence of a non-zero Romans mass.

Part I

Tachyon condensation, confinement and non-perturbative phenomena in $(Dp, \bar{D}p)$ systems

Chapter 2

Non-BPS D-branes, $(Dp, \bar{D}p)$ systems and tachyon condensation

In the previous chapter we have shown what the BPS Dp-branes are and how they appear in certain string theories. We are now going to consider non-BPS configurations, which stability is no longer assured by SUSY. Directly related to this stability is the process known as *tachyon condensation*, which can take place in a system containing tachyons in their string spectra. In this process a tachyonic mode which is initially in a relative maximum or a saddle point of a certain potential rolls down towards a stable classical minimum, in a way analogous to the *spontaneous symmetry breaking* (SSB) mechanism in QFT. The process is known as the “condensation” of the tachyon, by which the unstable initial configuration can decay into a different one, that can in turn be stable or not. Is in this context in which the research work presented on chapter 3 was done. In order to study the tachyon condensation phenomena, we will introduce the boundary state formalism and the effective potential approach for the tachyonic modes.

Non-BPS configurations and unstable systems are also interesting for various reasons in string theory, beyond the simple search for stability. For example $(Dp, \bar{D}p)$ systems, apart from having been widely used in the literature in the study of string theory in time dependent backgrounds, they have also been used more recently in the study of chiral symmetry breaking in holographic models of QCD [100, 101, 102, 103].It should be emphasized that states which are not BPS can be stable due to charge conservation (a pedagogical review on this topic can be seen in [82]). In most cases the description of these states in the strong coupling limit remains unknown, although in some cases string dualities do enable us to identify the strongly coupled states and calculate their masses [81]. On the other hand, there are configurations unstable due to the presence of tachyonic modes in the system, for example, the bosonic string due to the existence of the closed string tachyon. In a similar fashion, tachyonic open strings ending on unstable non-BPS D-brane systems do also appear in superstring theories. More concretely, non-BPS $D\bar{D}$ systems have been widely used in the literature in the study of string theory in time dependent backgrounds[1].

As we had previously mentioned, an anti-Dp-brane is a 1/2 BPS object preserving the half of SUSYs broken by the corresponding 1/2 BPS Dp-brane. On top of that, they have opposite charges, and therefore when considering a pair of coincident Dp and $\bar{D}p$ the total charge of the configuration exactly vanishes. In this way, when interactions are taken into account (so that each one cannot be taken as an independent object anymore), SUSY becomes completely broken and with zero total charge nothing prevents the branes to annihilate each other. At low energies, this instability is realized by the presence of tachyonic modes in the open strings connecting brane and antibrane. One can start to separate the $(Dp, \bar{D}p)$ pair until the system gets close to be decoupled, which is usually called *BPS limit*. In that limit, interaction between branes practically vanishes and so it is possible to neglect its effect in their separate effective actions, and hence recovering SUSY. If that distance is not too large in comparison to the string scale, open massive strings will stretch between the two[2].

On the other hand, we had observed that in type IIA (IIB) there are only odd (even) RR forms. As suitable RR fields are needed in order to form the CS term of the SUSY action for a Dp-brane, Dp-branes with odd (even) p in type IIA (IIB) are not BPS nor stable, as they do not preserve any RR charge; these non-BPS Dp-branes are said to have *wrong* p. The instability of those systems is again reflected in the presence of tachyonic modes in the open strings attached, in this case, strings with both ends ending in the unstable Dp.

The existence of tachyons in a physical system is a reflection that the configuration is settled in a relative maximum or saddle point of the potential. In that case, one has to search for a stable solution at some relative minimum in order to be able to make a perturbative treatment, as happens with the Higgs mechanism in the SM. For the tachyons appearing in the low energy spectrum of the bosonic string, a fully satisfactory interpretation has not been found yet. The vacuum itself seems to be unstable, and it is not even known whether a minimum of the corresponding tachyonic potential exists. The corresponding open string tachyons have been understood in the previous terms, however the problem of the closed string tachyon appearing after quantization of the bosonic string remains unsolved.

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[1] For an extensive review on the subject see [90], whereas a shorter and very clear introduction can be found in [91].

[2] If the distance separating the branes becomes too large in string units, a perturbative treatment would not be possible. On the other hand we can also imagine the opposite case, with brane and antibrane getting closer to each other and recovering the massless open strings with tachyonic modes in their spectra. Indeed, the open strings themselves tend to approximate the branes to each other as they tend to minimize its area.

2.0.3 Motivation

In the late 90's, a new framework was developed in which D-branes can be understood as topological solitons in the worldvolume of unstable brane configurations of higher dimensionality [89, 92]. It is remarkable that, through a hierarchy of embeddings (the so-called *brane descent relations*), a complete classification of D-brane charges in terms of K-theory groups [92, 93] was possible in this way. As we are going to see, $(Dp, \bar{Dp})$ systems can decay into a SUSY D$(p-2)$ stable brane, or into a non-SUSY D$(p-1)$-brane. Depending on the theory considered, this D$(p-1)$-brane can be stable of not. In Type II theories this brane is going to be unstable, able to decay into a SUSY D$(p-2)$-brane. In turn, in type I the D$(p-1)$-brane resulting from the $(Dp, \bar{Dp})$ system annihilation can be stable, as happens with the D0-brane coming from an unstable $(D1, \bar{D}1)$ pair. This has been used to provide a test, not based on SUSY, of the duality between type I and the heterotic $SO(32)$ superstring theories. We will show this more explicitly below, but we should mention that there are other similar results in testing non-perturbative dualities in string theory beyond the BPS limit, related to orbifold compactifications, for example in testing the S-duality symmetry of type IIB theory [81].

Usually, one looks at the BPS states in order to test a non-perturbative duality relation. As we have discussed, they are stable and protected by quantum radiative corrections. In this way their properties can be studied perturbatively at weak coupling and be safely extrapolated to strong coupling, and reinterpreted in terms of non-perturbative configurations of the dual theory. However, non-perturbative tests based on BPS states are really an evidence for a duality conjecture, or just unavoidable results dictated by supersymmetry? In order to clarify this, any non-perturbative tests of the string dualities beyond the BPS limit are key. Nevertheless, stable non-BPS states do receive quantum corrections, and so identifying the strong coupling states is a much more complicated task.

An interesting work in testing the S-duality between the heterotic $SO(32)$ and the type I theories at a non-BPS limit was carried out by Sen in [86, 87]. Certain massive perturbative states, stable but not BPS, exist in the heterotic $SO(32)$ theory. They are stable because they are the lightest states carrying the quantum numbers of the spinorial representation. However, the dual of these states remained unknown until it was found out that a $(D1, \bar{D}1)$ pair carries the same quantum numbers of the spinor representation of $SO(32)$. Although this $(D1, \bar{D}1)$ system is unstable, the condensation of its tachyonic mode gives rise to a non-BPS D0-brane, which in type I is stable. This is the S-dual of the states in the spinorial.

As non-BPS D-branes are non-supersymmetric objects, another topic in which the tachyonic condensation may be useful is in getting results about non-SUSY gauge theories via the AdS/CFT correspondence (which is going to be introduced in chapter 4). For example, Drukker, Gross and Itzhaki [83] constructed unstable classical solutions of $\mathcal{N}=4$ SYM and found their dual unstable states in type IIB in AdS_5 (an unstable D0-brane located at the AdS center). This kind of studies are non-SUSY tests of the AdS/CFT duality, however, some problems exist related to the SUGRA solutions.

2.0.4 $(Dp, \bar{Dp})$ systems

As we have mentioned $(Dp, \bar{Dp})$ systems are unstable due to the presence of a tachyonic mode in the open strings connecting brane and antibrane. In order to see this, let us make use of the *boundary state formalism* [84]. In this formalism, a stable SUSY Dp-brane is represented by a boundary state

$$|Dp\rangle = \frac{1}{\sqrt{2}} \left(|Dp\rangle_{NSNS} + |Dp\rangle_{RR} \right) , \tag{2.1}$$

representing a source for both NSNS and RR closed strings states emitted by a Dp-brane. By taking the GSO projection into account, the NSNS sector has the form

$$|Dp\rangle_{NSNS} = \frac{1}{2} \left(|Dp; +\rangle_{NSNS} - |Dp; -\rangle_{NSNS} \right) , \tag{2.2}$$

where $|Dp; \pm\rangle_{NSNS}$ represent the two possible implementations of the fermionic boundary conditions appropriate for a Dp-brane

$$(\psi^a \mp i\tilde{\psi}^a)\Big|_{\tau=0} = 0 \; ; \qquad (\psi^i \pm i\tilde{\psi}^i)\Big|_{\tau=0} = 0 \; . \tag{2.3}$$

Similarly, for the RR sector we have

$$|Dp\rangle_{RR} = \frac{1}{2} \left(|Dp; +\rangle_{RR} - |Dp; -\rangle_{RR} \right) . \tag{2.4}$$

Projecting with $(-1)^{F_L}$

We are now going to see that, acting with the $(-1)^{F_L}$ proyection (where F_L stands for the left-moving space-time fermion number operator), we can relate different BPS and non-BPS D-brane systems. The boundary state formalism can be used to compute the spectrum of the open strings that begin and end on a Dp-brane. The closed string exchange between two Dp-branes (2-point function tree level cylinder amplitude) can be rewritten as a trace over open string states[3] (annulus amplitude). The NS sector is then GSO projected with[4]

[3] A modular transformation $\tau_c \to \tau_a = 1/\tau_c$ maps the length τ_c of the cylinder described by the closed strings into the modular parameter of the annulus τ_a spanned by the open strings.

[4] The modular transformation maps the contribution coming from the exchange of RR states into the contribution of the NS $(-1)^F$ sector of the open strings. The sum of the NSNS and RR sector of the closed string is then equivalent to the NS sector of the open string with the usual GSO projection.

$\frac{1+(-1)^F}{2}$. Therefore, the NS sector ground state, which by convention is taken $(-1)^F$-odd, is projected out and there is no tachyon, resulting in a SUSY spectrum for the open strings in that case.

Now, if the closed string exchange takes place between a Dp and a $\bar{D}p$, we have the same sign for the NSNS sector closed string exchange and opposite sign for the RR one (it is proportional to the product of the RR charges of the two branes), which implies a GSO projection operator for the NS sector[5] $\frac{1-(-1)^F}{2}$. Therefore, the NS sector ground state, which is tachyonic, is not projected out in this case, and both the $D\bar{D}$ and $\bar{D}D$ open strings contain tachyonic excitations. An open string state in this system is characterized by a Chan-Paton (CP) factor, which we take to be

$$\begin{aligned} &\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \text{ for a } (pp) \text{ string,} \qquad \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \text{ for a } (\bar{p}\bar{p}) \text{ string,} \\ &\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \text{ for a } (p\bar{p}) \text{ string,} \qquad \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \text{ for a } (\bar{p}p) \text{ string.} \end{aligned} \tag{2.5}$$

In this way the tachyonic states are

$$\begin{aligned} |t\rangle &= |k^a\rangle_{NS} \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} , \\ |\bar{t}\rangle &= |k^a\rangle_{NS} \otimes \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} , \end{aligned} \tag{2.6}$$

being $|k^a\rangle_{NS}$ the NS ground state (which is a complex field as it comes from two sectors), or

$$\begin{aligned} |T\rangle &= |t\rangle + |\bar{t}\rangle = |k^a\rangle_{NS} \otimes \sigma^1 , \\ |\bar{T}\rangle &= |t\rangle - |\bar{t}\rangle = |k^a\rangle_{NS} \otimes i\sigma^2 . \end{aligned} \tag{2.7}$$

being σ^i the Pauli matrices.

Now, let us look at the action of $(-1)^{F_L}$. Suppose that we start, for definiteness, in type IIA. It is well known that orbifolding type IIA by $(-1)^{F_L}$ gives type IIB [85]. Since $(-1)^{F_L}$ reverses the sign of the left-moving space-time fermions, it reverses the sign of all RR fields, and so maps a Dp into a $\bar{D}p$ and vice versa. Therefore the $(Dp, \bar{D}p)$ system is invariant under $(-1)^{F_L}$. If we project out by $(-1)^{F_L}$ ($(-1)^{F_L} = \sigma^1$ over the CP factors, as

[5] One can say that the requirement that a cylinder diagram can be reinterpreted as an annulus diagram through a modular transformation fixes the type of GSO projection that one has to perform on the various open strings.

it exchanges brane and antibrane), we have

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \xrightarrow{(-1)^{F_L}} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; \quad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \xrightarrow{(-1)^{F_L}} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix};$$
$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \xrightarrow{(-1)^{F_L}} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \quad \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \xrightarrow{(-1)^{F_L}} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}. \tag{2.8}$$

Due to this, the only states that are invariant under $(-1)^{F_L}$ are those with CP factors commuting with σ^1, i.e. $\mathbb{1}$ and σ^1. States with CP factor $\mathbb{1}$ come from pp and $\bar{p}\bar{p}$ open strings, with the right GSO projection. The states with CP factor σ^1 come from $p\bar{p}$ and $\bar{p}p$ open strings, with the wrong GSO projection.

In that way, schematically the two contributions to the NS sector are

$$\frac{1}{2}(NS + NS(-1)^F) + \frac{1}{2}(NS - NS(-1)^F) = NS. \tag{2.9}$$

The resulting configuration is non-supersymmetric. It defines a non-BPS type IIB Dp-brane[6]. From the point of view of the closed strings, the fact that the contribution of $NS(-1)^F$ is missing in (2.9) means that there is no RR sector for the closed strings. Therefore the non-BPS Dp-brane does not carry RR charge. There are other interesting properties that can be easily deduced from here. For example the open strings ending on this non-BPS Dp can carry CP factors $\mathbb{1}$ or σ^1, and those with σ_1 contain a tachyonic excitation as well, which in this case is real. On the other hand, the tension for the non-BPS Dp ($|Dp\rangle = |Dp\rangle_{NSNS}$) can be calculated as compared with the tension for a BPS Dp (2.1), obtaining a $\sqrt{2}$ factor of difference

$$T_{\cancel{BPS}\,Dp} = \frac{T_{Dp}}{\sqrt{2}} = \frac{1}{g_s\sqrt{2}(2\pi)^p(\alpha')^{(p+1)/2}}. \tag{2.10}$$

Let us now mod out the non-BPS Dp-brane by $(-1)^{F_L}$ again, as since it does not carry RR charge, it is invariant under that projection. In this case the effect of $(-1)^{F_L}$ is not clear, because it does not exchange brane with antibrane. The CP factors of both type of strings survive the projection, however

$$\frac{1}{2}(NS + NS(-1)^F) + \frac{1}{2}(NS - NS(-1)^F) = NS. \tag{2.11}$$

The states represented in the first of the two terms in the LHS of (2.11), coming from open strings with CP$=\mathbb{1}$, do survive the projection, as one can check by looking at 2-point functions. $< B_{\mu\nu} A_\rho >$ does not vanish, and so $B_{\mu\nu}$ is even iff A_μ is even, and this

[6] In order to see that it describes a single object and not a pair, one must check that the additional dof appearing when separating the two branes reside in the sector with CP factor σ_3, and therefore it is projected out by $(-1)^{F_L}$.

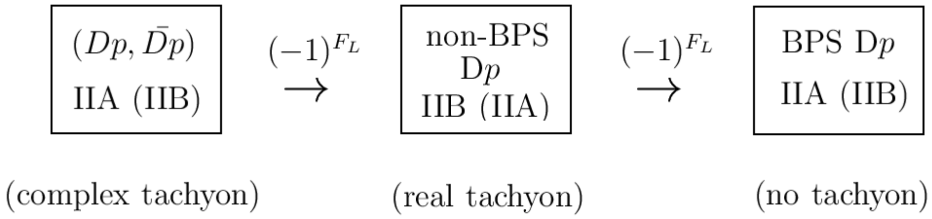


Figure 2.1: Subsequent actions of the $(-1)^{F_L}$ projection in a $(Dp, \bar{D}p)$ type II system remove the tachyon instability.

field comes from the open strings with CP= $\mathbb{1}$ (this can be checked in full generality). Regarding the other states, we can look at the 2 point functions $< C_{p+1}T >$, which is also non-vanishing, and therefore C_{p+1} is odd iff T is odd, and this field comes from the open strings with CP= σ_1. As states with CP= σ_1 are odd under $(-1)^{F_L}$, these modes are projected out, and we find, finally, the states associated to a BPS Dp-brane

$$\frac{1}{2}(NS + NS(-1)^F). \tag{2.12}$$

Hence, summarizing, we have that subsequent actions of $(-1)^{F_L}$ remove the tachyonic instability. A first non-BPS tachyonic Dp-brane appears in the other type II theory after projecting the $(Dp, \bar{D}p)$ system by $(-1)^{F_L}$. In this way, there are non-BPS Dp-branes with p odd (even) in IIA (IIB). These non-BPS Dp-branes are such that after another $(-1)^{F_L}$ projection, a BPS Dp-brane appears in the initial type II theory (fig. 2.1).

Let us now see that there are other connections between these objects, that appear after tachyon condensation, as can be seen in fig. 2.2. The analysis of the action of $(-1)^{F_L}$ in $(Dp, \bar{D}p)$ systems is useful in order to define non-BPS Dp-branes, however, it is more interesting to study how non-BPS D$(p-1)$'s are obtained from $(Dp, \bar{D}p)$ systems in the same theory, and D$(p-2)$'s from non-BPS D$(p-1)$'s, via tachyon condensation, since this is giving rise to the brane descent relations (the vertical arrows in fig. 2.2).

Implications of duality

At strong coupling the instability of these $(Dp, \bar{D}p)$ systems should be manifested through the existence of tachyonic modes in open D$(p-2)$-branes stretched between the Dp and the anti-Dp or with both ends on a non-BPS Dp-brane. This is indeed an implication of the S-duality symmetry of type IIB theory. This can be seen, for example, taking a $(D3, \bar{D}3)$ pair with an F1 connecting the branes and applying an S-duality transformation

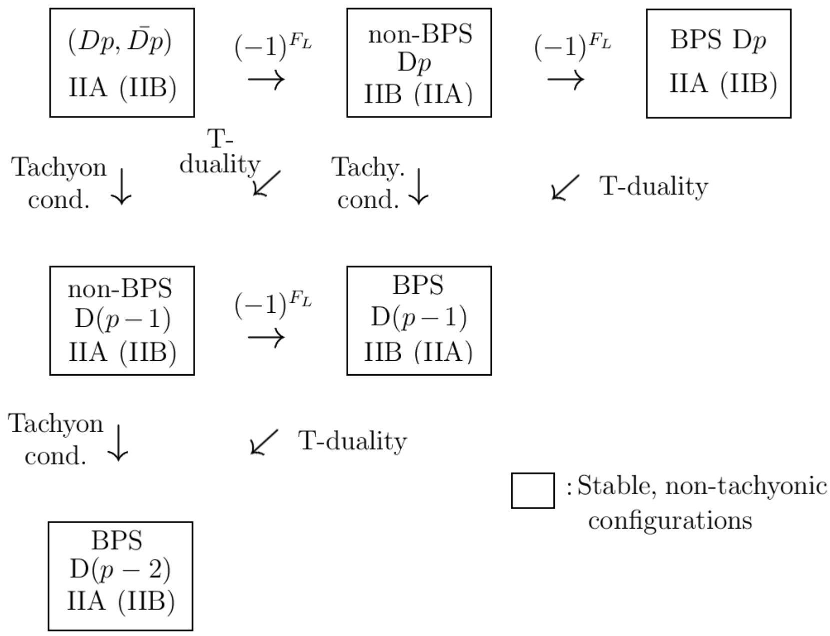


Figure 2.2: Net of different BPS and non-BPS configurations in type IIA and type IIB theories related via T-duality, $(-1)^{F_L}$ projection, and tachyon condensation.

to it. This transformation acting on a $D3$ has no effect, given that under S-duality type IIB fields can be transformed in the following way [33]

$$g \;\rightarrow\; \frac{1}{g}, \qquad B_2 \;\rightarrow\; C_2, \qquad C_2 \;\rightarrow\; -B_2, \qquad C_4 \;\rightarrow\; C_4, \tag{2.13}$$

(i.e. a D3 transforms as an $SL(2,\mathbb{Z})$ singlet) but the S-dual of a fundamental string is a D1-brane. The result will be a $(D3, \bar{D}3)$ system with a tachyonic D1-brane stretched between both, as it is showed schematically in fig. 2.3. A T-duality generalization implies that the $(Dp, \bar{D}p)$ system should contain tachyonic open D$(p-2)$-branes stretched between them.

Tachyon condensation

In order to know if a tachyonic instability is an incurable problem of the unstable system considered, or on the contrary, we are just expanding the tachyonic field around a false vacuum, one should compute the tachyon potential. In this section, we are going to comment on some useful results from string field theory (SFT) that allow for the construction of effective actions for the previous unstable systems.

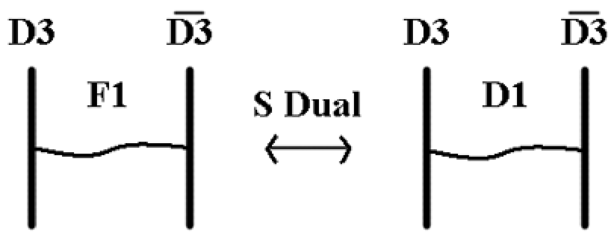


Figure 2.3: The $(D3, \bar{D}3)$ system is self-dual under S-duality whereas the F1 is transformed into a D1 and vice versa.

As it is explained in [90], some properties of the tachyonic fields can be deduced from the analysis of the S-matrix at tree level, such that the real tachyon field T in the open strings attached to non-BPS branes has $m^2 = -1/2$ $(\alpha' = 1)$ and it is $\mathbb{Z}_2$ invariant $(T \to -T)$. On the other hand, the two real tachyons of the $(Dp, \bar{D}p)$ system (one going from the brane to the antibrane and the other going from the antibrane to the brane) that can be combined into a single complex (tachyonic) field, also have $m^2 = -1/2$, in this case respecting a $U(1)$ invariance $(T \to e^{i\alpha}T)$. SFT results are important for studying the tachyon condensation process[7], although the tachyon mass-squared is of the order of the string tension and so there is no systematic way for computing the effective potential $V(T)$ (usually calculated in perturbation theory as an expansion in α'). Such potential would be created by the branes to which the endpoints of those open strings belong.

Sen demonstrated ([88]) that the tachyon condensation process in a $(Dp, \bar{D}p)$ system can occur as the total energy of the pair, which is twice its tension, is exactly cancelled by the negative value of the potential, giving a zero-energy configuration and resulting in the annihilation of the pair. An equivalent situation appears with a non-BPS brane, as can be seen in fig. 2.4. It is also inferred that the non-BPS (real) tachyon potential should have two minima, whereas the (complex) brane-antibrane tachyon potential should have a continuous ensemble of vacua parametrized by $e^{i\alpha}$. Moreover, SFT results reflect that, in order to describe correctly these systems, the contribution of the tachyonic potential to the effective action must be of the following form when all the massless fields are set to zero (assuming a space-time independent field configuration)

$$\mathcal{S}_{eff} = -\int d^{p+1}\xi \; V(T). \tag{2.14}$$

Obviously, another constraint will be that $V''(0) < 0$ in order to be tachyonic. Additionally the additive constant in the potential must be chosen such that $V(0) = 0$.

[7]As the problem rests on the fact that the physics is necessarily “off-shell”, this is outside the domain of the first quantized theory and needs a SFT treatment

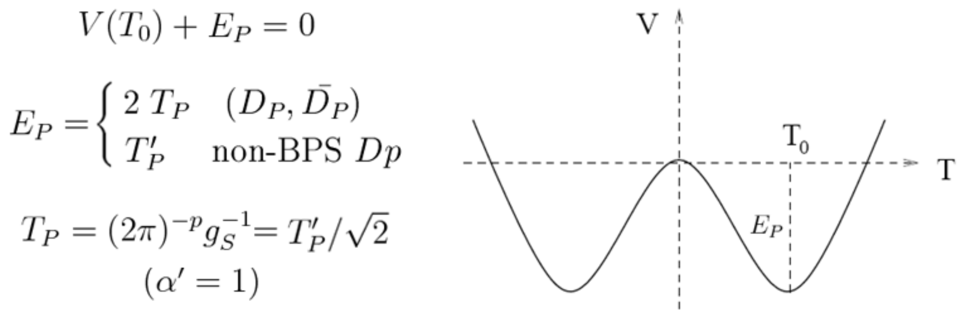


Figure 2.4: Tachyonic potential on an unstable non-BPS D-brane in superstring theories. Revolving this diagram about the vertical axis the tachyon potential on a brane-antibrane system is obtained. Here T_P is the tension of a BPS Dp-brane, whereas T'_P is the corresponding one for the non-BPS case.

A DBI action for the $(Dp, \bar{D}p)$ system

Despite not being able to construct a worldvolume effective action for the $(Dp, \bar{D}p)$ system from first principles, this action has been calculated in the literature (including the complex tachyonic field) by performing disk amplitudes with an open string tachyon inserted at the boundary and a RR boson in the interior. In this way, some symmetries and consistency conditions derived from SFT results have to be kept in mind. Let us now briefly elaborate on the DBI part of the $(Dp, \bar{D}p)$ action, and afterwards, present similar results for the non-BPS Dp-brane. Finally, a CS part of this kind of actions for a $(Dp, \bar{D}p)$ system is presented in section 2.0.4.

Symmetries that must be taken into account in order to derive the DBI part of a $(Dp, \bar{D}p)$ worldvolume effective action are ([90]):

- The usual Poincaré invariance in the corresponding dimensions.

- The gauge symmetry

$$T \to e^{2i\alpha(x)}T, \qquad A_\mu^{(1)} \to A_\mu^{(1)} + \partial_\mu \alpha(x), \qquad A_\mu^{(2)} \to A_\mu^{(2)} - \partial_\mu \alpha(x). \tag{2.15}$$

- The symmetry at interchanging brane and antibrane produced by $(-1)^{F_L}$, imposing the conditions

$$T = real, \qquad A_\mu^{(1)} = A_\mu^{(2)} \equiv A_\mu, \qquad \Phi^I_{(1)} = \Phi^I_{(2)} \equiv \Phi^I. \tag{2.16}$$

On top of this, the consistency condition of recovering the separate brane and antibrane actions in the BPS limit must be added.

Notwithstanding it is important to stress that the later conditions do not fix uniquely the form of the action. Imposing them Sen [98] proposed the following effective DBI action for a brane-antibrane system

$$\mathcal{S}_{DBI} = -\int d^{p+1}\xi \; V\big(T, \Phi^I_{(1)} - \Phi^I_{(2)}\big)\left(\sqrt{-det\boldsymbol{A}_{(1)}} + \sqrt{-det\boldsymbol{A}_{(2)}}\right), \tag{2.17}$$

where

$$\boldsymbol{A}_{(i)\mu\nu} = \eta_{\mu\nu} + F^{(i)}_{\mu\nu} + \partial_\mu\Phi^I_{(i)}\partial_\nu\Phi^I_{(i)} + \frac{1}{2}(D_\mu T)^*(D_\nu T) + \frac{1}{2}(D_\nu T)^*(D_\mu T) \tag{2.18}$$

and

$$F^{(i)}_{\mu\nu} = \partial_\mu A^{(i)}_\nu - \partial_\nu A^{(i)}_\mu, \qquad D_\mu T = \left(\partial_\mu - iA^{(1)}_\mu + iA^{(2)}_\mu\right)T, \tag{2.19}$$

with $V(T)$ depending only on $|T|$ and $\sum_I(\Phi^I_{(1)} - \Phi^I_{(2)})^2$, and such that for small T:

$$V(T, \Phi^I_{(1)} - \Phi^I_{(2)}) = T_p\left[1 + \frac{1}{2}\left\{\sum_I\left(\frac{\Phi^I_{(1)} - \Phi^I_{(2)}}{2\pi}\right)^2 - \frac{1}{2}\right\}|T|^2 + \mathcal{O}(|T|^4)\right] \tag{2.20}$$

It must be kept in mind that this is an effective action, providing a good description of the system for T large and small second and higher derivatives.

A DBI action for non-BPS D-branes

As we have already mentioned, Dp-branes with p even/odd in the type IIB/IIA theories are unstable, in a way similar to the brane-antibrane systems. Defining the corresponding potential for the tachyonic modes and searching for the classical minimum, the original Dp-brane will decay in a BPS stable D$(p-1)$-brane, with the "correct" p depending on the theory.

The construction of effective actions describing non-BPS branes was carried out in [94] and [95]. An effective action describing the dynamics of the tachyonic field is given by:

$$\mathcal{S} = -\int d^{p+1}\xi V(T)\sqrt{-det\boldsymbol{A}}, \tag{2.21}$$

7 with

$$\boldsymbol{A}_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu T\partial_\nu T + \partial_\mu\Phi^I\partial_\nu\Phi^I + F_{\mu\nu}. \tag{2.22}$$

In this case certain results imply that $V(T)$ must be symmetric under $T \to -T$, with a maximum located in $T = 0$ producing the instability. On the other hand, its minimum is at $T = \pm\infty$, where the tachyon potential vanishes.

One must always keep in mind that this effective action describes a non-BPS brane as long as the tachyon field T is large and we are able to consider the second and higher derivatives of its potential $V(T)$ small.

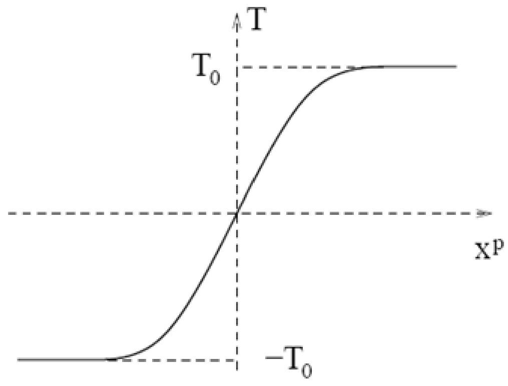


Figure 2.5: Kink solution from of non-BPS Dp-brane.

Non-trivial time independent solutions

Some non-trivial static solutions can be obtained from the previous actions. For example, a D$(p-1)$-brane can appear as a kink solution both in the non-BPS Dp-brane case and in the $(Dp, \bar{D}p)$ one. In the first one, the kink solution connects two opposite vacua in the form showed in fig. 2.5, where the energy density is localized around a $(p-1)$-dimensional plane at the center. That is a BPS D$(p-1)$-brane, and it is indeed possible to check that the energy per unit volume in that plane corresponds to this type of object. In a similar fashion a BPS D$(p-1)$-brane can emerge from a $(Dp, \bar{D}p)$ system for $Im(T) = 0$ and $Re(T)$ playing the role of the (real) tachyon of the previous case.

Other different types of solutions of these actions can be written down. For example a $(Dp, \bar{D}p)$ system can also give rise to a BPS D$(p-2)$-brane as a vortex solution. Writing the complex tachyonic field in cylindrical coordinates, as

$$T = T_0 f(\rho) e^{i\theta}, \qquad f(\infty) = 1 \qquad f(0) = 0, \tag{2.23}$$

with

$$\rho = \sqrt{(\xi^{p-1})^2 + (\xi^p)^2}\ , \qquad \theta = tan^{-1}(\xi^p/\xi^{p-1}), \tag{2.24}$$

the energy density represents a codimension-two soliton localized in the $(p-2)$-dimensional region around $\rho = 0$.

Another possible solution is a BPS D$(p-3)$-brane, obtained as a t'Hooft-Polyakov monopole solution from two non-BPS D-branes. Finally, it has also been shown that a BPS D$(p-4)$-brane can arise from a system of two $(Dp, \bar{D}p)$ pairs.

The Chern-Simons effective action for the $(Dp, \bar{D}p)$ system

Partial results for the CS effective action for the $(Dp, \bar{D}p)$ pair have been obtained in [96] The result is the following

$$S_{CS} = \int_{\mathbb{R}^{1,p}} C \wedge \mathrm{STr}\, e^{\mathcal{F}}, \tag{2.25}$$

where the supertrace (STr) is defined such that the leading term is

$$S_{CS} = \int_{\mathbb{R}^{1,p}} C_{p-1} \wedge \mathrm{Tr} \left(\mathcal{F}^{(1)} - \mathcal{F}^{(2)} - \frac{1}{2}\mathcal{F}^{(1)}, t\bar{t} + \frac{1}{2}\mathcal{F}^{(2)}, \bar{t}t + Dt \wedge \bar{D}t \right) + \cdots . \tag{2.26}$$

Now, by looking at the first part of the supertrace expansion one can read

$$\int_{\mathbb{R}^{1,p}} C_{p-1} \wedge \mathrm{Tr} \left(\mathcal{F}^{(1)} - \mathcal{F}^{(2)} \right) = \int_{\mathbb{R}^{1,p}} C_{p-1} \wedge \mathrm{Tr} \left(dA^{(1)} - dA^{(2)} \right) , \tag{2.27}$$

which by integrating over the flux on the transverse $\mathbb{R}^2$ plane, results in the appearance of a D$(p-2)$-brane charge

$$\int_{\mathbb{R}^{1,p-2}} C_{p-1}. \tag{2.28}$$

In this way we can see the soliton is indistinguishable from the D$(p-2)$-brane, being simply a different representation of the same object. This result can be proved as well by using CFT techniques. In this case the difficulty is that we start without any RR charge, but end up with a non-zero RR charge, and so we do not expect to find a marginal deformation that continuously interpolates between the two configurations. However, Majumder and Sen [97] showed that there is a marginal deformation that converts the boundary CFT of the $(Dp, \bar{D}p)$ system to that of a $(D(p-2), \overline{D(p-2)})$ system. This deformation interpolates between the original $(Dp, \bar{D}p)$ and a vortex-antivortex pair on this system, and so establishes the equivalence between a vortex solution and a D$(p-2)$-brane.

Summarizing the two-step construction, we have that a BPS D$(p-2)$-brane can be obtained as a kink solution from a non-BPS D$(p-1)$-brane which, in turn, can also appear as a kink solution of a $(Dp, \bar{D}p)$ system. The vortex solution connects the first and last systems, since a BPS D$(p-2)$ brane can be directly obtained from a $(Dp, \bar{D}p)$ system in that way (this construction appears as a vertical sequence in fig. 2.2). This observation can be made more explicit by showing that the worldvolume theory on the vortex solution is given by the DBI action on a $D(p-2)$-brane [98, 104] (see also [105]), as we are going to see in chapter 3.

To conclude this section, we recall that the vortex configuration can be generalized to higher codimension solitons on the brane-antibrane pair [92], which is the base of the aforementioned K-theory classification. On the other hand, the condensation of the complex tachyon occurs as a Higgs mechanism in which the tachyon plays the role of the Higgs field, providing mass to the relative $U(1)$ vector field $A^{(1)} - A^{(2)}$, being removed from the low energy spectrum. The overall $U(1)$ vector field $A^{(1)} + A^{(2)}$, under which the tachyon is neutral, remains however unbroken, posing a puzzle [106, 107, 108]. We are going to discuss this in the next chapter, where a novel result clarifying this is presented.

Finally, we would like to mention that relevant results in the type I theory can be obtained via worldsheet parity reversal from type IIB. In this way the existence of D(-1), D0, D7 and D8 non-BPS D-branes has been predicted.

Chapter 3

Confinement and non-perturbative tachyons

As we have already mentioned at the end of the last chapter, the vortex solution coming from the $D\bar{D}$ tachyon condensation process is not completely understood, as in this process the relative $U(1)$ vector field is removed from the low energy spectrum whereas the overall $U(1)$ vector field, under which the tachyon is neutral, remains unbroken [106, 107, 108]. It was suggested in [108], based on the duality relation between the Type IIA superstring and M-theory, that the overall $U(1)$ is in the confined phase. The suggested mechanism for this confinement is a dual Higgs mechanism in which magnetically charged tachyonic states associated to open D$(p-2)$-branes stretched between the Dp and the $\bar{D}p$ condense. Evidence for such a situation comes from the M-theory description of a $(D4, \bar{D}4)$ system.

The superposition of a $D4$ and a $\bar{D}4$ is described in M-theory as an $(M5, \bar{M}5)$ pair wrapped in the eleventh direction. The open strings that connect the $D4$ and the $\bar{D}4$ are realized as open M2-branes wrapped in the eleventh direction and stretched between the $M5$ and the $\bar{M}5$. These M2-branes must contain as well a complex tachyonic excitation. Since the tachyon condensing charged object is in this case extended (a tachyonic worldvolume string) there are no ways to describe quantitatively this type of mechanism. However, duality with the Type IIA superstring implies that whatever this mechanism is the condensation of this tachyonic mode should be accompanied by a non-trivial magnetic flux, in this case of the relative antisymmetric tensor field in the worldvolume of the $(M5, \bar{M}5)$. This magnetic flux generates charge with respect to the 3-form potential of eleven dimensional supergravity, as inferred from the coupling in the $(M5, \bar{M}5)$ Chern-Simons action[1]

$$\int_{\mathbb{R}^{1,5}} \hat{C}_3 \wedge (d\hat{A}_2 - d\hat{A}'_2)\,. \tag{3.1}$$

[1]Here $\hat{C}_3$ stands for the 3-form of eleven dimensional supergravity and $\hat{A}_2$ and $\hat{A}'_2$ for the worldvolume 2-form fields on the $M5$ and the $\bar{M}5$. Note that $\hat{A}_2$ (self-dual) and $\hat{A}'_2$ (anti-self-dual) combine to give an unrestricted relative 2-form field [108].

An M2-brane would then emerge as the remaining topological soliton.

Let us suppose that one performs now the reduction from M-theory along a worldvolume direction of the $(M5, \bar{M}5)$ transverse to the stretched M2-branes [108]. In this case a $(D4, \bar{D}4)$ system is obtained in which tachyonic D2-branes are stretched between the $D4$ and the $\bar{D}4$. Again, if this tachyonic mode condenses in a vortex-like configuration, B_2-charge will be induced in the system, as the reduction from the previous coupling along a worldvolume direction transverse to the stretched M2-branes shows

$$\int_{\mathbb{R}^{1,4}} B_2 \wedge (dA_2 - dA'_2)\,, \tag{3.2}$$

where now A_2 and A'_2 are associated to open D2-branes ending on the $D4$ and the $\bar{D}4$. A fundamental string would then arise as the remaining topological soliton.

Note that in this case the Higgs mechanism is intrinsically non-perturbative, given that this description emerges after interchanging two compact directions in M-theory. Indeed, the coupling (3.2) shows that the worldvolume dynamics of the $(D4, \bar{D}4)$ system is governed by the 2-form gauge fields dual in the five dimensional worldvolume to the BI vector fields. These fields couple in the worldvolume with inverse coupling, and are therefore more adequate to describe the strong coupling regime of the system.

Therefore, qualitatively the duality between Type IIA and M-theory predicts the occurrence of both the perturbative and non-perturbative Higgs mechanisms for the $(D4, \bar{D}4)$ system. The same conclusion can be reached for arbitrary $(Dp, \bar{D}p)$ systems by T-duality arguments [108]. Applying T-duality to the coupling (3.2) along $(p-4)$ transverse directions[2] one gets

$$\int_{\mathbb{R}^{1,p}} B_2 \wedge (dA_{p-2} - dA'_{p-2})\,. \tag{3.3}$$

This coupling indicates that the fundamental string would arise as a topological soliton in a dual Higgs mechanism [109] in which magnetically charged tachyonic states associated to open $D(p-2)$-branes stretched between the Dp and the $\bar{D}p$ condensed[3]. In terms of the original variables this would translate into confinement of the overall $U(1)$, given that due to the opposite orientation of the $\bar{D}p$-brane the relative $(p-2)$-form field is dual in the $(p+1)$-dimensional worldvolume to the overall BI vector field. Therefore, its localized magnetic flux at strong coupling translates into a confined overall $U(1)$ electric flux at weak coupling.

The explicit action that describes the dual Higgs mechanism at strong coupling has not been constructed in the literature, although some qualitative arguments pointing at

[2] Or along a spatial direction of the stretched D2-brane if $p < 4$.

[3] When $p = 3$ this is exactly the S-dual picture of the creation of a $D1$-brane as a vortex in a $(D3, \bar{D}3)$ system [108].

particular couplings have been given [108, 110, 111]. In any case, as we have mentioned, this mechanism is intrinsically non-perturbative, and this makes this description highly heuristic.

A related crucial question which was first addressed in [112, 113, 110, 111] is the possibility of describing both the perturbative and the non-perturbative Higgs mechanisms simultaneously at weak coupling. Starting with Sen's action [112, 113] reference [111] studied the Hamiltonian classical dynamics of the $(Dp, \bar{Dp})$ system, and showed that it describes a massive relativistic string fluid. The possibility of describing the region of vanishing tachyonic potential in terms of the $(p-2)$-form fields dual to the BI vector fields was also addressed[4] and although the explicit dual action was not constructed it was argued that the dual Higgs mechanism proposed in [108] could be realized explicitly if this action was the one associated to an Abelian Higgs model for the relative $(p-2)$-form dual field. The fundamental string would then arise as a Nielsen-Olesen solution. In this construction, however, the $(p-3)$-form field playing the role of the Goldstone boson associated with the dual magnetic objects did not have a clear string theory origin.

One of the results that we will present in this study will be the construction of the explicit dual action describing the strongly coupled dynamics of the $(Dp, \bar{Dp})$ system in terms of the $(p-2)$-form dual potentials and a $(p-3)$-form Goldstone boson. The generalization of Sen's action to include tachyonic couplings in a $(Dp, \bar{Dp})$ system [114, 99, 115, 116, 117, 118, 119, 120, 98, 121, 122, 123, 124] describes, to second order in α', an Abelian Higgs model in which the Abelian field is the relative BI vector of the brane and the antibrane and the phase of the tachyon plays the role of the associated Goldstone boson. We will show however that the dual of this action does not describe an Abelian Higgs model for the relative $(p-2)$-form potential, contrary to the expectation in [111]. The explicit dual Abelian Higgs model will instead arise from a different generalization of Sen's action from which we will be able to describe the confining phase (for the overall $U(1)$) of the $(Dp, \bar{Dp})$ system at weak coupling.

The dualization of the four-dimensional Abelian Higgs model is known since long ago [125], motivated by the study of the confining phases of four dimensional Abelian gauge theories in the context of Mandelstam-'t Hooft duality [126]. The dual action constructed by Sugamoto describes the confining phase of four dimensional vector fields in terms of a massive 2-form field theory which is an extension of the model for massive relativistic hydrodynamics of Kalb and Ramond [127]. This field theory allows a quantized vortex solution similarly to the creation of the Nielsen-Olesen string in the Abelian Higgs model. The extension of Sugamoto's construction to arbitrary d-dimensional p-form Abelian Higgs models was carried out more recently in [128], with the aim at describing the confining

[4] This idea was also put forward in [110] in the 2+1 dimensional case.

phases of p-form field theories in a generalization of Mandelstam-'t Hooft duality. In this general case the dual action describing the confining phase is a massive $(p+1)$-form field theory.

In this study we will develop on the work of [128] and we will extend the construction in [125] to the $(p+1)$-dimensional Abelian Higgs model that describes the Higgs phase (for the relative $U(1)$) of a $(Dp, \bar{D}p)$ system. As we will see the massive Abelian field of the Abelian Higgs model can still be dualized in the standard way into a massless $(p-2)$-form field once the phase of the tachyon is dualized into a $(p-1)$-form. We will show that the dual action is of the type of the massive $(p-1)$-form field theories discussed in [128]. Furthermore, we will show that a $D(p-2)$-brane can emerge as a confined electric flux brane associated to the overall $(p-2)$-form dual field. The precise mechanism involved in this process is the Julia-Toulouse mechanism [129, 128], which as we will see is the exact contrary of the more familiar Higgs mechanism.

The construction of the dual action is therefore useful in order to identify the mechanism by which a $D(p-2)$-brane can emerge at strong coupling after the annihilation of a Dp and a $\bar{D}p$. However, it sheds no light on the issue of the unbroken overall $U(1)$, nor on the creation of the fundamental string, since it involves only the overall $(p-2)$-form potential, and this field is dual to the relative BI vector field. Indeed, inspired by Mandelstam-'t Hooft duality one expects that the dual action describes the creation of the $D(p-2)$-brane in dual variables, since it should provide an explicit realization of the duality between the Higgs phase (for the relative $U(1)$), described at weak coupling by Sen's action, and the confinement phase (for the overall $(p-2)$-form field) at strong coupling. The Higgs phase for the relative $(p-2)$-form gauge potential at strong coupling should instead be dual to the confining phase for the overall $U(1)$ at weak coupling.

In this study we will present a worldvolume effective action suitable to describe perturbatively the dynamics of the $(Dp, \bar{D}p)$ system in the confining phase for the overall $U(1)$. Developing on the work of [128] we will start in the phase in which the tachyon vanishes, the Coulomb phase, and show that the confining phase arises after the condensation of $(p-3)$-dimensional topological defects which are interpreted as the end-points of $D(p-2)$-branes. We will see that the fundamental string emerges at weak coupling as a confined electric flux string after a Julia-Toulouse mechanism in which a 2-form gauge field associated to the fluctuations of the topological defects eats the overall $U(1)$ vector field. We will also show, following [128] closely, that the confined phase for the original overall $U(1)$ vector field can be studied in the strong coupling regime as a generalized Higgs-Stückelberg phase for its dual $(p-2)$-form field. The explicit dual action is given by an Abelian Higgs model for the relative $(p-2)$-form potential. In this description the condensing tachyonic objects are identified as $(p-3)$-branes that originate from the end-points of open $D(p-2)$-

branes stretched between the Dp and the $\bar{D}p$. The fundamental string then emerges as a topological soliton after the condensation of this tachyonic mode through a dual Higgs mechanism [109]. Therefore, through this construction we can make explicit the mechanism suggested in [108] for realizing non-perturbatively the confinement of the overall $U(1)$.

As we have seen the $(Dp, \bar{D}p)$ system admits two types of topological defects: particles and $(p-3)$-branes. The first originate as the end-points of open strings and are therefore perturbative in origin. The second originate as the end-points of non-perturbative open $D(p-2)$-branes and can therefore only be described in terms of $D(p-2)$-brane degrees of freedom in the strong coupling regime. We have seen however that using Julia and Toulouse's idea we can incorporate these degrees of freedom in the perturbative action, and study the confining phase for the overall $U(1)$. If we combine the effective actions describing the Higgs phase for the relative $U(1)$ and the confining phase for the overall $U(1)$ we will be able to describe perturbatively the breaking of both gauge groups. We will see that from this action both the $D(p-2)$-brane and the fundamental string are realized as solitons in the common $(p+1)$-dimensional worldvolume. The $D(p-2)$-brane arises after a Higgs mechanism involving the relative $U(1)$, and the F1 after a Julia-Toulouse mechanism involving the overall $U(1)$.

The organization of this chapter is as follows. In section 2.1 we construct the dual of the Abelian Higgs model that describes the $(Dp, \bar{D}p)$ system at weak string coupling. We see that contrary to expectation in [110] it does not describe an Abelian Higgs model for the dual relative $(p-2)$-form potential. The worldvolume field content of the dual action consists on a $(p-1)$-form, dual to the phase of the tachyon, and two $(p-2)$-form fields dual to the BI vectors. We show that the $(p-1)$-form can become massive by eating the overall dual $(p-2)$-form potential through the Julia-Toulouse mechanism, and that a $D(p-2)$-brane arises as a confined electric flux brane in this process. Therefore the Higgs phase for the relative BI vector is mapped onto the confining phase for the overall $(p-2)$-form field, with a $D(p-2)$-brane arising either as a vortex solution after the Higgs mechanism at weak coupling or as a confined electric flux brane after the Julia-Toulouse mechanism at strong coupling. In section 2.2 we present our candidate action for describing the confining phase of the overall BI vector field at weak coupling. We show that from this action the fundamental string arises as a confined electric flux string after a Julia-Toulouse mechanism. In section 2.3 we construct the dual of this action and show that it realizes a generalized Higgs-Stückelberg phase for the relative $(p-2)$-form field. Therefore, the confining phase for the overall BI vector is mapped onto the Higgs phase for the relative $(p-2)$-form field, with a fundamental string arising either as a confined electric flux string after the Julia-Toulouse mechanism at weak coupling or as a generalized vortex solution after the Higgs mechanism at strong coupling. Section 2.4 is our Discussion section. Here we present the action from which we can describe simultaneously the Higgs phase for the relative $U(1)$ and the confinement phase for the overall $U(1)$ at weak string coupling.

3.1 The $(Dp, \bar{D}p)$ system in dual variables

The effective action describing a brane-antibrane pair has been extensively studied in the literature using different approaches [114, 99, 115, 116, 117, 118, 119, 120, 98, 121, 122, 123, 124]. Although the complete action has not been derived from first principles it is known to satisfy a set of consistency conditions [98]. It is invariant under gauge transformations of the tachyon phase and the relative BI vector: $\chi \to \chi + \alpha(x)$, $A^- \to A^- + d\alpha$, it reduces to the sum of the BI effective actions for the Dp and the $\bar{D}p$ for zero tachyon, and it gives rise to the action for a non-BPS Dp-brane [112, 130, 131, 132] when modded out by $(-1)^{F_L}$ [90]. In the context of our discussion in this study this action describes the Higgs phase for the relative BI vector field.

In this study we will work to second order in α', and take the RR potentials $C_{p-3}, C_{p-5}, \ldots$ to zero. We will also ignore the tachyonic couplings to the C_{p-1} RR-potential derived in [99, 116, 123]. Thus, our action represents a truncated version of the $(Dp, \bar{D}p)$ action that can be derived from the results in [114, 99, 115, 116, 117, 118, 119, 120, 98, 121, 122, 123, 124] [5]. We will see however that it contains the relevant couplings for describing the most important aspects of the dynamics of the $(Dp, \bar{D}p)$ system, both in the Higgs and in the confining phases[6].

Our starting point is the action:

$$
\begin{aligned}
S(\chi, A) = \int d^{p+1}x \Big\{ & e^{-\phi}\Big(\frac{1}{2}F^+ + B_2\Big) \wedge *\Big(\frac{1}{2}F^+ + B_2\Big) + \\
& +\frac{1}{4}e^{-\phi}F^- \wedge *F^- + |T|^2(d\chi - A^-) \wedge *(d\chi - A^-) + d|T| \wedge *d|T| - V(|T|) \\
& +C_{p-1} \wedge F^- \Big\}\,. \qquad (3.4)
\end{aligned}
$$

Here we have set $2\pi\alpha' = 1$, A^+ and A^- are the overall and relative BI vector fields: $A^+ = A + A'$, $A^- = A - A'$, and the complex tachyon is parametrized as $T = |T|e^{i\chi}$. $V(|T|)$ is the tachyon potential [113], whose precise form will be irrelevant for our analysis. Finally, the background fields B_2 and C_{p-1} are implicitly pulled-back into the $(p+1)$-dimensional worldvolume of the $(Dp, \bar{D}p)$.

[5] Note that in comparing with the boundary string field theory results [133] there is the usual discrepancy by $2\log 2$ in the kinetic term of the tachyon [116, 117, 120].

[6] Once it is extended as we do in next section in order to incorporate the non-perturbative degrees of freedom associated to the $(p-3)$-brane topological defects.

The coupling $\int C_{p-1} \wedge F^-$ is the one that we discussed at the end of the last chapter. It shows that when the tachyon condenses in a vortex-like configuration a $D(p-2)$-brane is generated as a topological soliton [90], since the associated localized F^- magnetic flux generates C_{p-1} charge. In this process the relative $U(1)$ vector field eats the scalar field χ, gets a mass and is removed from the low energy spectrum. The overall $U(1)$ vector field, under which the tachyon is neutral, remains unbroken, but it is believed to be confined [108, 112, 113, 110].

In this section we construct the dual of the action (3.4), and show that it describes the confining phase for the $(p-2)$-form potential dual to the relative BI vector field, thus providing an explicit realization of Mandelstam-'t Hooft duality for the Abelian Higgs model associated to the $(Dp, \bar{D}p)$ system. We also discuss the mechanism by which the $D(p-2)$-brane arises as a confined electric flux brane.

3.1.1 The duality construction

Let us focus on the worldvolume dependence of the action (3.4) on A^+, A^- and the phase of the tachyon. Note that since A^- is massive it cannot be dualized in the standard way. We can however use the standard procedure to dualize the phase of the tachyon and A^+. These fields are dualized, respectively, into a $(p-1)$-form, W_{p-1}, and a $(p-2)$-form, that we denote by A^-_{p-2} given that due to the opposite orientation of the antibrane the relative and overall gauge potentials should be interchanged under duality. The intermediate dual action that is obtained after these two dualizations are carried out is such that A^- becomes massless[7] and can therefore be dualized in the standard way into a $(p-2)$-form, which we denote as A^+_{p-2} [8].

The final dual action reads:

$$S(W_{p-1}, A_{p-2}) = \int d^{p+1}x \Big\{ e^{\phi} \Big(\frac{1}{2} F^+_{p-1} + W_{p-1} + C_{p-1} \Big) \wedge * \Big(\frac{1}{2} F^+_{p-1} + W_{p-1} + C_{p-1} \Big) + \frac{1}{4} e^{\phi} F^-_{p-1} \wedge * F^-_{p-1} + \frac{1}{4|T|^2} dW_{p-1} \wedge * dW_{p-1} + d|T| \wedge * d|T| - V(|T|) - B_2 \wedge F^-_{p-1} \Big\} \quad (3.5)$$

[7] Up to a total derivative term.

[8] Alternatively, one can use a generalization of the intermediate action presented in [125], from which it is possible to dualize a massive Abelian 1-form field.

with the explicit duality rules being given by:

$$\frac{1}{2}F^{+} + B_2 = \frac{1}{2}e^{\phi} * F^{-}_{p-1} \tag{3.6}$$

$$\frac{1}{2}F^{-} = e^{\phi} * \left(\frac{1}{2}F^{+}_{p-1} + W_{p-1} + C_{p-1}\right) \tag{3.7}$$

$$d\chi - A^{-} = \frac{1}{2|T|^2}(-1)^{p-1} * dW_{p-1}\,. \tag{3.8}$$

Here we see that the relative and overall gauge potentials are interchanged, as expected due to the opposite orientation of the antibrane. Note that for $p = 3$ our notation is ambiguous. When analyzing this particular case we will use A^+ and A^- to denote the BI vector fields and $\tilde{A}^+$, $\tilde{A}^-$ to denote the dual vector fields associated to open D-strings ending on the branes.

The action (3.5) is an extension of the actions proposed in [128] for describing the confining phases of field theories of compact antisymmetric tensors. After we discuss these actions in some detail in the next section it will become clear that (3.5) describes the confining phase for the overall $(p-2)$-form dual potential. This phase arises after the condensation of zero-dimensional topological defects which originate from the end-points of open strings stretched between the brane and the antibrane. The interpretation of the low energy mode W_{p-1} is as describing the fluctuations of these defects, and is such that away from the defects $W_{p-1} = dA^{+}_{p-2}$.

Note that the gauge invariance $\chi \to \chi + \alpha(x)$, $A^- \to A^- + d\alpha$ of the original action has been mapped under the duality transformation into $W_{p-1} \to W_{p-1} + d\Lambda_{p-2}$, $A^{+}_{p-2} \to A^{+}_{p-2} - 2\Lambda_{p-2}$. This symmetry can be gauge fixed by absorbing F^{+}_{p-1} into W_{p-1}, which becomes then massive. The overall A^{+}_{p-2} gauge potential is then removed from the low energy spectrum, through a mechanism that is the exact contrary of the Higgs mechanism. This is the Julia-Toulouse mechanism mentioned in the introduction. Thus, the Julia-Toulouse mechanism is identified as the mechanism responsible for the removal of the relative $U(1)$at strong coupling. However it clearly sheds no light on the removal of A^+.

When comparing the action (3.5) to the actions describing the confining phases of antisymmetric field theories presented in [128] one sees that the modulus of the tachyon plays the role of the density of condensing topological defects. In a way one can think of $|T|$ as an indicator of how unstable the system is. Since the instability in the confining phase is originated by the presence of the topological defects it is reasonable to expect a relation between both quantities. In the confining models of Quevedo and Trugenberger a consistency requirement is that the antisymmetric field theory in the Coulomb phase is recovered for zero density of topological defects. This is indeed satisfied by our action (3.5) for vanishing tachyon, since the $|T| \to 0$ limit forces the condition that W_{p-1} must be exact and can therefore be absorbed through a redefinition of A^+. The action is then reduced to the action describing the $(Dp, \bar{D}p)$ system in the Coulomb phase, i.e. to (3.4) for zero tachyon.

Finally, following the analysis in [125] we can see that a $D(p-2)$-brane arises as a confined electric flux brane after the Julia-Toulouse mechanism. In order to see this explicitly we need however to recall some basic facts on the construction of [125], so we will postpone this discussion till the end of next section.

In the next section we present our candidate action for describing the confining phase for the overall $U(1)$ at weak coupling. We show that the fundamental string arises from this action as a confined electric flux string. By direct generalization of this analysis we also show that the $D(p-2)$-brane arises as a confined electric flux brane from the action (3.5) derived in this section.

3.2 Confinement at weak string coupling

In this section we present our candidate action for describing the dynamics of the $(Dp, \bar{D}p)$ system in the confining phase. We use the results in [128], where an action describing the confined phase of field theories of compact antisymmetric tensors of arbitrary rank was derived. We start by summarizing the qualitative points that are relevant for our construction, to later concretize these ideas to the $(Dp, \bar{D}p)$ system. The reader is referred to [128] for a more detailed discussion.

Quevedo and Trugenberger made explicit in the framework of antisymmetric field theories an old idea in solid-state physics due to Julia and Toulouse [129]. These authors argued that for a compact tensor field of rank $(h-1)$ in $(p+1)$-dimensions a confined phase might arise after the condensation of $(p-h-1)$-dimensional topological defects[9]. The fluctuations of the continuous distribution of topological defects generate a new low-energy mode in the theory which can be described by a new h-form, W_h, such that away from the defects $W_h = dA_{h-1}$, where A_{h-1} is the original tensor field. The main idea is to extend the h-form in the topological invariant term[10]

$$\int_{S_h} \omega_h \tag{3.9}$$

to the whole $\mathbb{R}^{p+1}$ space-time. In this way the $(p-h)$-form $J_{p-h} = *(d\omega_h)$, which is zero

[9]The mechanism by which these defects originate is irrelevant for the nature of the confining phase.

[10]S_h is an h-dimensional sphere surrounding the defect on an $(h+1)$-dimensional hyperplane perpendicular to it, and ω_h is an h-form which is exact outside S_h.

outside the defect, picks up delta-like singularities at the locations of the topological defects and can describe the conserved fluctuations of their continuous distributions. Note that due to $J_{p-h} = *(d\omega_h)$ the new degrees of freedom are associated only with the gauge-invariant part of ω_h.

The effective action describing the confining phase of the antisymmetric tensor field then depends on a gauge-invariant combination of the antisymmetric tensor field, A_{h-1}, and the extended h-form, W_h. This combination is such that when the density of topological defects vanishes the original action describing the antisymmetric tensor field theory in the Coulomb phase is recovered.

As discussed in [128], the finite condensate phase is a natural generalization of the confinement phase for a vector gauge field. For compact QED in four dimensions the induced static potential between a particle and an antiparticle is linear at large distances, identifying the monopole condensate phase as a confinement phase. This computation can be generalized to arbitrary $(h-1)$-forms in d dimensions. In this case the leading term in the induced action is the h-dimensional hypervolume enclosed by the $(h-1)$-dimensional closed hypersurface to which the $(h-1)$-form couples. For a more detailed discussion on the confining properties of these actions see [128, 134].

Given that the worldvolume theory of a $(Dp, \bar{D}p)$ system is a vector field theory, the results in [128] for $h = 2$ can be applied to this case, with some obvious modifications coming from the couplings to the background gauge potentials associated to the closed strings. In this case the Coulomb phase is the phase with zero tachyon, and it is therefore described[11] by the Lagrangian:

$$L(A) = e^{-\phi}\Big(\frac{1}{2}F^+ + B_2\Big) \wedge *\Big(\frac{1}{2}F^+ + B_2\Big) + \frac{1}{4}e^{-\phi}F^- \wedge *F^- + C_{p-1} \wedge F^- \,. \tag{3.10}$$

Developping now on the ideas in [128] for the $(Dp, \bar{D}p)$ system we have that the topological defects whose condensation will give rise to the confining phase are $(p-3)$-branes, which originate in this case from the end-points of $D(p-2)$-branes stretched between the Dp and the $\bar{D}p$. The new mode associated to the fluctuations of the defects is described by a 2-form, W_2, which will couple in the action through a gauge invariant combination with the overall $U(1)$ vector field[12]. The action should depend as well on the density of topological defects, such that when this density vanishes the original action in the Coulomb phase, given by (3.10), is recovered. In the actions constructed in [128] the density of topological defects entered as a parameter which was interpreted as a new scale in the theory. We

[11]To second order in α' and for $C_{p-3} = C_{p-5} = \cdots = 0$.

[12]One could in principle expect that W_2 coupled to either combination of the $U(1)$ vector fields, but we will see that consistency with S- and T- dualities implies that it must couple only to the overall vector field. This will allow ultimately to explain the puzzle of the unbroken overall $U(1)$ through confinement.

will see however that in the $(Dp, \bar{D}p)$ case duality implies that the density of topological defects must be a dynamical quantity, because it is related to the modulus of the tachyonic excitation of the open $D(p-2)$-branes in the dual Higgs phase. We will denote this field by $|\tilde{T}|$ and, moreover, we will use the duality with the Higgs phase to include in the action its kinetic and potential terms.

The action that we propose for describing the confining phase of the $(Dp, \bar{D}p)$ system is then given by:

$$\begin{aligned} S(W_2, A) &= \int d^{p+1}x \Big\{ e^{-\phi}\Big(\frac{1}{2}F^+ + W_2 + B_2\Big) \wedge * \Big(\frac{1}{2}F^+ + W_2 + B_2\Big) + \frac{1}{4}e^{-\phi}F^- \wedge *F^- + \\ &\quad + \frac{1}{4|\tilde{T}|^2} dW_2 \wedge *dW_2 + d|\tilde{T}| \wedge *d|\tilde{T}| - V(|\tilde{T}|) + C_{p-1} \wedge F^- \Big\}. \end{aligned} \tag{3.11}$$

This action has been constructed under four requirements. One requirement is gauge invariance, both under gauge transformations of the BI vector fields and under $W_2 \to W_2 + d\Lambda_1$, which ensures that only the gauge-invariant part of W_2 describes a new physical degree of freedom. This transformation must be supplemented by $A^+ \to A^+ - 2\Lambda_1$, a symmetry that has to be gauge fixed. The second is relativistic invariance. The third requirement is that the original action describing the Coulomb phase must be recovered when $|\tilde{T}| \to 0$. Indeed, when $|\tilde{T}| \to 0$ we must have that $dW_2 = 0$, so that $W_2 = d\psi_1$ for some 1-form ψ_1. This form can then be absorbed by A^+, and the original action (3.10) is recovered. These requirements were the ones imposed in [128]. The $(Dp, \bar{D}p)$ system, being a string theory object, must also satisfy consistency with the duality symmetries of string theory. The implications of this requirement will become more clear when we show the duality between this action and the action describing the Higgs phase for the dual $(p-2)$-form gauge field. It implies in particular that W_2 must couple only to the overall $U(1)$ vector field.

Now, in (3.11) F^+ can be absorbed by W_2, fixing the gauge symmetry

$$\begin{aligned} W_2 &\to W_2 + d\Lambda_1 \\ A^+ &\to A^+ - 2\Lambda_1\,, \end{aligned} \tag{3.12}$$

and the action can then be entirely formulated in terms of W_2 and the relative vector field:

$$\begin{aligned} S(W_2, A^-) &= \int d^{p+1}x \Big\{ e^{-\phi}\Big(W_2 + B_2\Big) \wedge * \Big(W_2 + B_2\Big) + \frac{1}{4}e^{-\phi}F^- \wedge *F^- + \\ &\quad + \frac{1}{4|\tilde{T}|^2} dW_2 \wedge *dW_2 + d|\tilde{T}| \wedge *d|\tilde{T}| - V(|\tilde{T}|) + C_{p-1} \wedge F^- \Big\}. \end{aligned} \tag{3.13}$$

In this process the original gauge field A^+ has been eaten by the new gauge field W_2, and has therefore been removed from the low energy spectrum. This solves the puzzle of the unbroken overall $U(1)$ at weak string coupling through the Julia-Toulouse mechanism, which, as we have seen, is the exact opposite of the Higgs mechanism. Let us now see how the fundamental string arises from this action.

Consider first the $p = 3$ case, which can be directly compared to the results in [125]. In this case the action (3.13) is a generalization of the action proposed in [125] to describe the confining phase of a four dimensional Abelian gauge theory. We recall from the introduction that this action was constructed as the dual of the four dimensional Abelian Higgs model, and that it allows a quantized electric vortex solution similar to the Nielsen-Olesen string. We see below that in our case this solution is identified as a fundamental string.

The construction of the vortex solution in [125] considers a non-vanishing 2-form vorticity source[13] along the x^3 axis:

$$V_e^3 = n\delta(x^1)\delta(x^2)\,, \qquad V_e^i = 0 \text{ for } i = 1, 2\,, \qquad \vec{V}_b = 0 \tag{3.14}$$

where the subindices e and b refer to the electric and magnetic components, and looks for a static and axially symmetric solution with the following assumptions:

$$\partial_0 e^3 = \partial_0 |\tilde{T}| = 0\,, \qquad e^3 = e^3(r)\,, \qquad |\tilde{T}| = |\tilde{T}|(r)\,, \tag{3.15}$$

$$e^i = 0 \text{ for } i = 1, 2\,, \qquad \vec{b} = 0 \tag{3.16}$$

where $\vec{e}$ and $\vec{b}$ refer to the electric and magnetic components of W_2, and $r = \sqrt{(x^1)^2 + (x^2)^2}$. The solution that is found represents a static circulation of flow around the x^3 axis, and satisfies the quantization condition

$$\int_{D_\infty} e^3 ds = 2\pi n\,, \tag{3.17}$$

where D_∞ is a large domain in the (x^1, x^2) plane including the origin. This solution corresponds to the Nielsen-Olesen string in the original Higgs model. As expected, the magnetic flux quantization condition has been mapped under duality onto an electric flux quantization condition, given by (3.17). The reader is referred to [125] for a more detailed discussion. For arbitrary p it is easy to find a similar, generalized, electric vortex solution with the same properties.

Let us now see that the confined electric flux string solution corresponds in the $(Dp, \bar{D}p)$ case to the fundamental string. In this case we have an additional coupling

$$\int B_2 \wedge *W_2 \tag{3.18}$$

[13] In the construction in [125] the vorticity source is created by the phase component of the Higgs scalar of the original Abelian Higgs model. In our case it is created by the phase component of the tachyon field associated to open D-strings connecting the $D3$ and the $\bar{D}3$. This will become clear after the analysis in the next section.

in the effective action (3.13), which shows that the quantized electric flux generates B_2-charge in the system. Charge conservation then implies that the remaining topological soliton is the fundamental string.

As mentioned in the previous section, the $D(p-2)$-brane arises from the strongly coupled confining action (3.5) derived in that section in a very similar way. In this case the vorticity source is a $(p-1)$-form which is created by the phase of the tachyon field in the original action (3.4). Note that in all the duality transformations that we have discussed in this study we have ignored total derivative terms. Had we kept these terms in the dualization of the action (3.4) we would have found a coupling $\int dW_{p-1} \wedge d\chi$ in the dual action. This coupling can be rewritten in terms of a vorticity source, $V_{p-1} = *dd\chi$, as $\int W_{p-1} \wedge *V_{p-1}$, giving then the generalization to arbitrary dimensions of the vorticity coupling in [125]. Let us suppose that we fix now $\chi = n\theta$, where θ is the azimuthal angle in the (x^{p-1}, x^p) plane. For $n \neq 0$ θ is not well defined on the worldvolume of a $(p-2)$-brane, and therefore the vorticity source is non-vanishing in this worldvolume. Taking then $V_{p-1}^{012\ldots p-2} = n\delta(x^{p-1}, x^p)$ and zero otherwise, we can look for a static and axially symmetric solution with the assumptions

$$\partial_0 W_{p-1}^{012\ldots p-2} = \partial_0 |T| = 0\,, \qquad W_{p-1}^{012\ldots p-2} = W_{p-1}^{012\ldots p-2}(r)\,, \qquad |T| = |T|(r)\,, \tag{3.19}$$

where $r = \sqrt{(x^{p-1})^2 + (x^p)^2}$ and all other components of W_{p-1} are taken to vanish. In this case the solution that is found represents a static circulation of flow around the $(p-2)$-brane, and satisfies the quantization condition

$$\int_{D_\infty} W_{p-1}^{01\ldots p-2} ds = 2\pi n\,. \tag{3.20}$$

The coupling

$$\int C_{p-1} \wedge *W_{p-1} \tag{3.21}$$

in the dual effective action (3.5) then implies that this confined electric flux brane corresponds to the $D(p-2)$-brane, since it shows that the quantized electric flux (3.20) generates C_{p-1}-charge in the system. Therefore, the $D(p-2)$-brane arises either as a magnetic vortex solution after the Higgs mechanism at weak coupling or as confined electric flux brane after the Julia-Toulouse mechanism at strong coupling.

In the next section we show that the action (3.11) can be made exactly equivalent to an action describing the Higgs phase for the dual relative $(p-2)$-form potential. We also show that, as expected, the fundamental string arises from this strongly coupled action as a generalization of the Nielsen-Olesen magnetic vortex solution.

3.3 Confinement at strong string coupling: The dual Higgs mechanism

Let us consider the action (3.11) describing the confining phase for the overall $U(1)$ at weak string coupling. Inspired by Mandelstam-'t Hooft duality we expect that the dual of this action describes the Higgs phase for the $(p-2)$-form field dual to the overall BI vector. The dualization of the BI vector fields in (3.11) takes place in the standard way, given that they only couple through their derivatives. In turn, the 2-form W_2 is massive, but it can still be dualized in the standard way from the intermediate dual action that is obtained after dualizing the BI vector fields, in which it only couples through its derivatives. Let us call the dual of this form, a $(p-3)$-form, χ_{p-3}. The final dual action reads:

$$S(\chi_{p-3}, A_{p-2}) = \int d^{p+1}x \Big\{ e^{\phi}\Big(\frac{1}{2}F^{+}_{p-1} + C_{p-1}\Big) \wedge * \Big(\frac{1}{2}F^{+}_{p-1} + C_{p-1}\Big) + \frac{1}{4}e^{\phi}F^{-}_{p-1} \wedge *F^{-}_{p-1}$$
$$+|\tilde{T}|^2\Big(d\chi_{p-3} - A^{-}_{p-2}\Big) \wedge *\Big(d\chi_{p-3} - A^{-}_{p-2}\Big) + d|\tilde{T}| \wedge *d|\tilde{T}| - V(|\tilde{T}|) - B_2 \wedge F^{-}_{p-1}\Big\} \quad (3.22)$$

and the explicit duality relations are given by

$$\frac{1}{2}F^{-} = e^{\phi} * \Big(\frac{1}{2}F^{+}_{p-1} + C_{p-1}\Big) \quad (3.23)$$
$$\frac{1}{2}F^{+} + W_2 + B_2 = \frac{1}{2}e^{\phi} * F^{-}_{p-1} \quad (3.24)$$
$$\frac{1}{2}dW_2 = |\tilde{T}|^2(-1)^{p-1} * \Big(d\chi_{p-3} - A^{-}_{p-2}\Big)\,. \quad (3.25)$$

Notice that once again the overall and the relative gauge fields are interchanged.

The action (3.22) describes an Abelian Higgs model for the relative $(p-2)$-form field, with the dual $(p-3)$-form χ_{p-3} playing the role of the associated Goldstone boson. The effective mass term reads

$$|\tilde{T}|^2\Big(d\chi_{p-3} - A^{-}_{p-2}\Big)^2 \quad (3.26)$$

and it is gauge-invariant under $\chi_{p-3} \to \chi_{p-3} + \Lambda_{p-3}$, $A^{-}_{p-2} \to A^{-}_{p-2} + d\Lambda_{p-3}$. That a coupling of this sort could drive the dual Higgs mechanism was suggested in [108, 110, 111] (see also [109]) although it could not be explicitly derived from the action describing the Higgs phase at weak coupling, i.e. from Sen's action. In this study we have seen that consistently with Mandelstam-'t Hooft duality the dual Abelian Higgs model arises from the action describing the confining phase at weak coupling. In the dual action (3.22) the dual Goldstone boson χ_{p-3} is associated to the fluctuations of the $(p-3)$-dimensional topological defects that originate from the end-points of the $D(p-2)$-branes stretched between the Dp and the $\bar{D}p$. This is consistent with the fact that this field is the worldvolume dual of the field W_2, which was accounting for these fluctuations in the confining action (3.11). Moreover, we can identify for $p = 3$ the condensing Higgs scalar as the modulus of the tachyonic mode associated to open D-strings stretched between the $D3$ and the $\bar{D}3$. Indeed when $p = 3$ the action (3.22) reads[14]:

$$L(\chi, A) = \int d^{p+1}x \Big\{ e^{\phi}(\frac{1}{2}\tilde{F}^{+} + C_2) \wedge *(\frac{1}{2}\tilde{F}^{+} + C_2) + \frac{1}{4}e^{\phi}\tilde{F}^{-} \wedge *\tilde{F}^{-}$$
$$+|\tilde{T}|^2(d\tilde{\chi} - \tilde{A}^{-}) \wedge *(d\tilde{\chi} - \tilde{A}^{-}) + d|\tilde{T}| \wedge *d|\tilde{T}| - V(|\tilde{T}|) - B_2 \wedge \tilde{F}^{-} \Big\}\,, \quad (3.27)$$

i.e. it is the S-dual of the original action (3.4) describing the perturbative Higgs phase of the $(D3, \bar{D}3)$ system. This is an important consistency check for the actions that we have constructed, although strictly speaking S-duality invariance would only be expected for zero tachyon, i.e. when the system becomes BPS and the worldvolume field content is not expected to change at strong coupling. Note that in this duality relation the modulus of the perturbative tachyon is mapped into $|\tilde{T}|$, which can then be interpreted as the modulus of the tachyonic excitation associated to the open D-strings. Since $\tilde{\chi}$ has also an interpretation as the phase of the dual tachyon we can think of $\tilde{T} = |\tilde{T}|e^{i\tilde{\chi}}$ as the complex tachyonic mode associated to the D-strings stretched between the $D3$ and the $\bar{D}3$. For $p \neq 3$ $|\tilde{T}|$ plays formally the role of the modulus of a tachyonic excitation. However, since the tachyonic condensing charged object is in this case a $(p-3)$-brane the phase of the tachyon is replaced by a $(p-3)$-form[15]. It would be interesting to clarify the precise way in which these fields arise as open $D(p-2)$-brane modes.

Finally, let us discuss the way the fundamental string arises from the action (3.22) when the Dp and the $\bar{D}p$ annihilate. If the brane and the antibrane annihilate through a generalized Higgs-Stückelberg mechanism in which A^{-}_{p-2} gets a mass by eating the Goldstone boson χ_{p-3}, we have that, if the Goldstone boson acquires a non-trivial winding number:

$$\int_{\mathbb{R}^{p-1}} F^{-}_{p-1} = \oint_{S^{p-2}} A^{-}_{p-2} = \oint_{S^{p-2}} d\chi_{p-3} = 2\pi n\,, \quad (3.28)$$

B_2-charge is induced in the configuration through the coupling in (3.22)

$$\int_{\mathbb{R}^{p,1}} B_2 \wedge F^{-}_{p-1}\,. \quad (3.29)$$

Charge conservation therefore implies that after the annihilation a fundamental string is left as a topological soliton. Since in this process the relative $(p-2)$-form field is removed

[14]Here we have used tildes to denote the dual fields, as mentioned in section 4.2.

[15]Reference [108] suggests a more concrete relation between the field χ_1 for $p = 2$ and the phase of the dual tachyon, by imagining the relevant string field defined over a loop space as $e^{i\oint \chi_1}$. Imposing single-valuedness in the loop space would then imply $\oint_{\Sigma} d\chi_1 = n$.

from the low energy spectrum, and this field is dual to the original overall $U(1)$, this solves the puzzle of the unbroken $U(1)$, through the mechanism suggested in [108] which is intrinsically non-perturbative.

3.4 Discussion

As we have seen, a $(Dp, \bar{D}p)$ system admits two types of topological defects: particles and $(p-3)$-branes. The first are perturbative in origin, while the second are non-perturbative. The combined electric and magnetic Higgs mechanisms introduce mass gaps to both $U(1)$ vector potentials, being the only remnants $D(p-2)$-branes and fundamental strings, realized as solitons on the common $(p+1)$-dimensional worldvolume. The system is described perturbatively in terms of Sen's action, which incorporates the tachyonic degrees of freedom associated to the perturbative point-like defects. However, in order to incorporate the non-perturbative degrees of freedom associated to the $(p-3)$-dimensional topological defects one has to restrict to the strong coupling regime of the theory, where the degrees of freedom associated to these defects become perturbative. Even in this case, as we have seen, it is not obvious to account for the right fields describing the tachyonic excitations. We have seen in this study that it is also possible to incorporate the non-perturbative degrees of freedom associated to the extended topological defects in the weak coupling regime, using Julia and Toulouse's idea. Essentially one introduces a new form which describes the fluctuations of these defects and imposes a set of consistency conditions based on gauge invariance and duality. In section 4.3 we have presented the weakly coupled action that is formulated in terms of this new form and the $U(1)$ vector fields associated to the open strings. In fact, one can combine this action with Sen's action in order to incorporate the degrees of freedom associated to both the zero dimensional and extended topological defects, with the explicit combined action being given by:

$$\begin{aligned} S(\chi, W_2, A) \;=\; & \int d^{p+1}x \Big\{ e^{-\phi}\Big(\frac{1}{2}F^+ + W_2 + B_2\Big) \wedge * \Big(\frac{1}{2}F^+ + W_2 + B_2\Big) + \frac{1}{4}e^{-\phi}F^- \wedge *F^- + \\ & + |T|^2(d\chi - A^-) \wedge *(d\chi - A^-) + d|T| \wedge *d|T| + \frac{1}{4|\tilde{T}|^2} dW_2 \wedge *dW_2 + \\ & + d|\tilde{T}| \wedge *d|\tilde{T}| - V(|T|) - V(|\tilde{T}|) + C_{p-1} \wedge F^- \Big\} . \end{aligned} \tag{3.30}$$

This action describes both the perturbative and the non-perturbative Higgs mechanisms simultaneously at weak coupling, and it admits both a magnetic vortex solution, which by charge conservation is identified with the $D(p-2)$-brane, and an electric vortex solution, identified as the fundamental string.

Finally, we would like to comment on an alternative mechanism for realizing perturbatively the breaking of the overall $U(1)$ that was proposed in [105, 135, 136] (see also [137, 138]). In this proposal the fundamental string emerges as a classical solution to Sen's action, with confinement being realized through the dielectric effect of [139], with the tachyon potential playing the role of the dielectric constant. This mechanism is distinct to the one that we have proposed in this study. In particular it does not seem to have a simple relation with the dual Higgs mechanism of [108, 110][16].

[16]See however [135], where it is argued that it corresponds to strongly coupled open strings.

Part II

Charged particle-like branes in AdS/CFT

Chapter 4

Particle-like branes and holography

4.1 Particle-like branes in AdS_5 spaces

The AdS/CFT correspondence relates the large N limit of the gauge theory not only to supergravity, but to string theory. If we do not impose $\lambda \to \infty$, then the decoupling of the non-chiral operators does not longer occur, and the complicated resulting spectrum of the gauge theory would be related to type IIB string theory in a certain background[1]. Nevertheless, even taking the large 't Hooft coupling limit, the stringy dual nature of the correspondence can sometimes be observed. One of the first insights was provided by Witten [163], who mapped branes wrapped on S^5, RP^5 and subspaces thereof, to 4-dimensional gauge theories. In this section we are going to present certain configurations in which strings stretch all the way from the AdS space boundary to different branes in the bulk. These branes wrap certain non-trivial circles of the background, and are seen as point-like particles from the AdS perspective.

An example is the baryon vertex in $AdS_5 \times S^5$ [163] (a baryon vertex is a static finite energy configuration of N external quarks). We are going to first elaborate on this configuration, which will lay the foundations for the research presented in the next chapter.

4.1.1 The baryon vertex in $AdS_5 \times S^5$

The first remarkable fact when considering a baryon-like configuration in the $\mathcal{N} = 4$ SYM theory is that its quarks are non-dynamical, as we mentioned in the last section, and therefore there will not be any kind of baryonic particle. Nevertheless we are still able to consider a baryonic vertex, i.e. a gauge-invariant coupling of N external charges whose color wave functions are contracted with the antisymmetric tensor of SU(N). Its gravity dual can be realized in the $AdS_5 \times S^5$ compactification of the type IIB theory, as N strings

[1] See [159] for example.

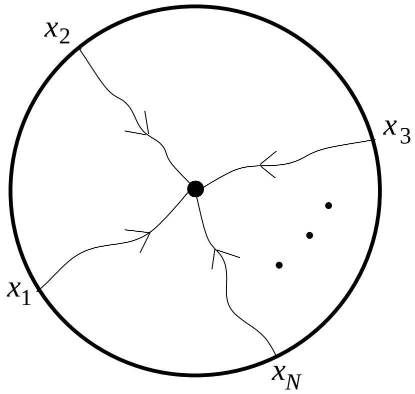


Figure 4.1: The strings connect points in the boundary of AdS to a point in the interior, where the baryon vertex is.

oriented in the same way and connecting points in the boundary of the AdS to some point in the interior, where a probe 5-brane is. That 5-brane wrapps the S^5 of the geometry and is static in a fixed point in AdS ([163]). Although we will consider the strings as fundamental ones, and so we will build the baryon vertex from a D5-brane, it is also possible to consider (p,q)-strings of the same type ending on a (q,p)-5-brane wrapping the 5-sphere as well.

The reason for considering a D5-brane comes from the fact that the D5 couples to the self-dual 5-form of the geometry (see section 1.5.1) through A_μ, the $U(1)$ gauge field living on its worldvolume. From the CS part of the D5-brane action the BI field strength F couples to the R-R 4-form of the background

$$-T_5 \int_{\mathbb{R}\times S^5} P[C_4] \wedge F, \tag{4.1}$$

and by integrating by parts we get to

$$T_5 \int_{\mathbb{R}\times S^5} P[G_5] \wedge A. \tag{4.2}$$

By $\int_{S^5} \frac{G_5}{2\pi} = N$, N units of $U(1)$ charge are produced in that way. Now, as the total charge must vanish in a compact space we must naturally take N fundamental strings ending on the D5-brane, all with the same orientation, in such a way that they will contribute with N units of opposite charge and cancel the previous contribution. Depending on the orientation of the D5 and the fundamental strings attached to it we will have either a baryon or an anti-baryon vertex configuration.

In this kind of cases, strings ending at the AdS boundary at infinity are regarded as ending on a D3-brane close to that boundary [161, 162]. At time zero that D3-brane can be considered as an static D3-brane whose world-volume is $S^3 \times R$, with S^3 a large 3-sphere near infinity and R a point in S^5. On the other hand, the worldvolume of the vertex D5-brane has the form $S^5 \times Q$ at time zero, being Q a point at AdS. Based on previous arguments in [163] it is argued that the ground state of strings stretching between those D-branes is fermionic and non-degenerate[2], making the configuration antisymmetric under permutation of the N fundamental strings as a baryon vertex has to be.

Stability

In order to study the stability of the system in SUGRA, energetic considerations are necessary. In [163] it is already discussed that for both electric and magnetic external charges, the energy of the 5-brane and the N strings attached to it are of the same order of magnitude in the 't Hooft limit ($g_s \to 0, N \to \infty, g_s N$ fixed). In the former case the vertex is realized by a D-brane with tension $\sim 1/g_s \sim N$, while in the later it is realized by a NS5-brane, whose tension has an extra $1/g_s$ factor so as the D-strings ending on it. A careful study was taken in [165] (see also [166]). They considered the combined actions of the wrapped D5-brane and the Nambu-Goto action of the N fundamental strings attached (as a function of the location of the baryon vertex in the bulk, that we will call u_0) in the gauge $x = \sigma$ and $t = \tau$

$$S_{total} = S_{D_5} + NS_{1F} = \frac{1}{(2\pi)^5(\alpha')^3 e^{\phi}} \int dx^6 \sqrt{h} + \frac{1}{2\pi} \int dtdx \sqrt{(u')^2 + u^4/L^4}. \tag{4.3}$$

Here h is the induced metric on the D5. The strings are taken to end symmetrically at the vertex (fig. 4.2) ensuring the net force to vanish in directions longitudinal to the AdS boundary. This additionally makes possible to ignore D-brane deformations due to strings tension as well as the electric field created by them on the brane worldvolume. Meanwhile, stability in the u direction will require a zero total net force on the vertex.

The variation of (4.3) ($u \to u + \delta u$) contains a volume and a surface term, that yield to the following equations of motion

$$\frac{u^4}{\sqrt{(u')^2 + \frac{u^4}{L^4}}} = \text{const.}, \qquad \frac{u_0'}{\sqrt{(u_0')^2 + \frac{u_0^4}{L^4}}} = \frac{1}{4}, \tag{4.4}$$

the first one for the bulk of the strings and the second one for its boundary, where the D5-brane contributes. There we have parametrized the fundamental strings worldvolume

[2]This can be either seen from the point of view of being two "linked" D-branes ([164]) in the $R^4 \times S^5$ nine-manifold, or from the local point of view of strings stretching between two transverse D-branes.

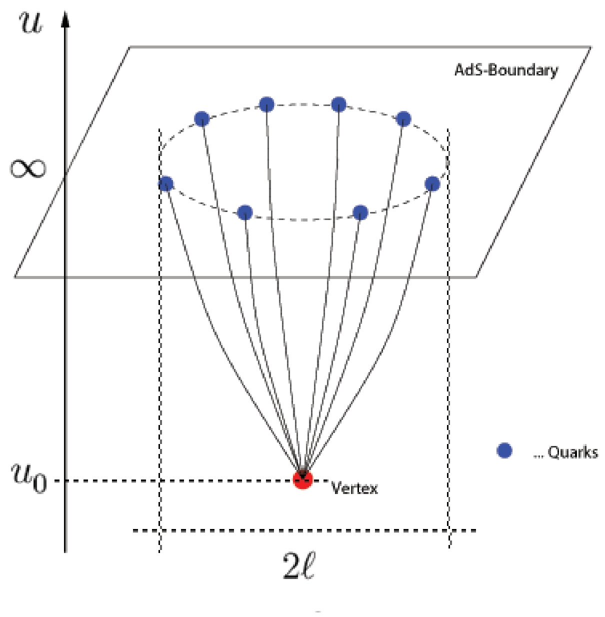


Figure 4.2: String endpoints in the boundary are by construction uniformly distributed around the vertex axis in a circumference of radius ℓ. The D5-brane is located at a u_0 position in the u direction of AdS.

by x,t and the position in AdS by $u = u(x)$, being $u'_0 = u'(u_0)$ and $L^4 = 4\pi g_s l_s^4 N$ the AdS radius. Combining both equations of motion one finds that

$$\frac{u^4}{\sqrt{u_x^2 + u^4/L^4}} = \sqrt{\frac{15}{16}} u_0^2 L^2, \tag{4.5}$$

At this point in time we can get an expression for the baryon size ℓ in terms of the position of the brane. We just have to integrate the previous equation

$$\frac{du}{dx} = f(u) \Rightarrow \int_{\rho_0}^{\infty} \frac{du}{f(u)} = \int_0^{\ell} dx \quad \overset{(4.5)}{\rightarrow} \quad \ell = \frac{L^2}{u_0} \int_1^{\infty} \frac{dy}{y^2 \sqrt{(\beta^2 y^4 - 1)}} \tag{4.6}$$

where $y = u/u_0$ and $\beta = \sqrt{16/15}$.
The energy of a single string is obtained by subtracting the (divergent) energy of its configuration with the D5-brane located at $u = 0$ [3]

$$E = \frac{1}{2\pi} u_0 \left(\int_1^{\infty} dy \frac{\beta y^2}{\sqrt{\beta^2 y^4 - 1}} - 1 \right) - \frac{u_0}{2\pi}. \tag{4.7}$$

[3]The D5-brane energy vanishes there, so just the fundamental strings contribute.

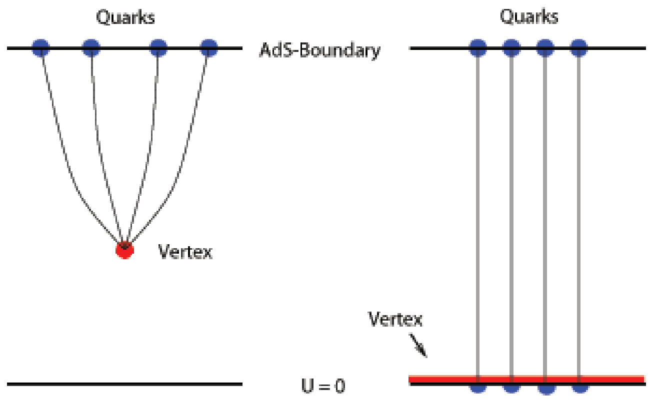


Figure 4.3: When the baryon vertex is located at $u = 0$ and $u' \to \infty$ the strings can move freely at the boundary.

Note that as $g_{xx} = 0$ at the $u = 0$ region (point) any radial string will have the same length and end at the same point, where the D5-brane is (fig. 4.3). These fermionic strings will behave as free quarks, as their position in the AdS boundary will be irrelevant.

Finally, by combining (4.6) and (4.7) equations one can express the total energy of the baryon configuration in terms of hypergeometric functions ([165])[4]

$$E = -\alpha_B N \frac{\sqrt{2g_{YM}^2 N}}{\ell}, \quad \text{where} \quad \alpha_B = ... \simeq 0.007. \tag{4.8}$$

From here one can see that the baryon configuration is stable, as the force $F = dE/d\ell$ is positive. Moreover, the total energy turns out to be proportional to N times that of the quark anti-quark system, as expected from the field theory large N analysis. In terms of the 't Hooft coupling $E \sim -\sqrt{\lambda}/\ell$. Therefore, while the dependence on $1/\ell$ is dictated by conformal invariance, the explicit non-analytical behavior with $\sqrt{\lambda}$ is a non-trivial prediction of string theory for the strong coupling behavior of the gauge theory ([167]). This result is similar to the one found in [161, 162] for the $q\bar{q}$ system.

Baryons with $k < N$ quarks

In [165] a mechanism for modifying the number of quarks was proposed, being possible to construct a more general baryon vertex made of $k < N$ quarks. Although this kind of configurations are not expected in confining theories and excluded in non-supersymmetric

[4]In the same reference, similar expressions were obtained for the $\mathcal{N} = 4$ theory at finite temperature as well as for three and four dimensional non-supersymmetric YM

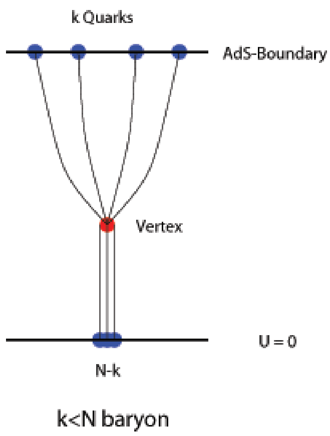


Figure 4.4: A $k < N$ baryon vertex with $N - k$ strings ending at $u = 0$.

ones, such stable k-quarks baryons were surprisingly found in the $\mathcal{N} = 4$ SYM theory.

By letting some of the strings to be stretched from the vertex to $u = 0$ in place of going to the boundary we will be reducing the number of quarks, as shown in fig. 4.4. The calculation of the energy can now be performed in a similar fashion, modifying the boundary equation that now looks

$$\frac{u'}{\sqrt{(u')^2 + u^4/L^4}} = \frac{5N - 4k}{4k}. \tag{4.9}$$

The LHS has to be smaller than 1, from where an upper bound for the number of strings stretching from the vertex to $u = 0$ is obtained

$$\frac{5}{8}N < k \leq N. \tag{4.10}$$

The lower bound corresponds to $u' \to \infty$ and so to radial strings, i.e. free quarks.

The energy of the configuration can be calculated as before, subtracting the energy corresponding to having the D5-brane located at $u = 0$

$$E_k = \frac{u_0}{2\pi}\left[(N-k) + N/4 + k\left(\int_1^\infty dy\left(\frac{y^2}{\sqrt{y^4 - (1 - ((5N-4k)/4k)^2)}} - 1\right) - 1\right)\right]. \tag{4.11}$$

Note that the energy vanishes for the lowest possible value of k, and the location of the D5-branes becomes a moduli of the system. An explanation of what is happening in the non-allowed interval of k was already given in [165]. From the surface relation it can be seen that in that case not all the $N - k$ strings can go radially directed towards $u = 0$. Instead they should come out of the vertex with some finite slope, never reaching $u = 0$ and eventually ending on the boundary. Additionally, in [165] it is also showed that a similar analysis for a confining field theory reveals that these configurations are not possible for such theories, in agreement with the field theory results.

In 2008 Sfetsos and Siampos studied the stability of baryon vertices under fluctuations for a class of curved backgrounds [168]. In this way, in the $AdS_5 \times S^5$ case, they found a more restrictive lower limit for k than the one found by Brandhuber and collaborators [165] and Imamura [166]. Stability restricts the value of k to be larger than a critical value (0.813N, higher than the lower bound 0.625N imposed by the existence of the classical solution). Still, non-singlet configurations are allowed to exist in this background.

Adding magnetic flux

In [169] it was shown how the baryon vertex can be generalized by adding a new quantum number, representing magnetic flux. The key point is to realize that S^5 can be seen as an S^1 bundle over CP^2. The S^1 fiber is a non-trivial $U(1)$ gauge bundle on the CP^2 base, and this allows to switch on a magnetic BI field B on the worldvolume of the D5-brane, proportional to the curvature tensor of the fiber connection. In these S^5 fiber coordinates the $AdS_5 \times S^5$ background (see section 1.5.1) reads (in the following, we will use the convention $\alpha' = 1$)

$$\begin{aligned} ds^2 &= \frac{u^2}{L^2}\eta_{ab}dx^a dx^b \;+\; \frac{L^2}{u^2}du^2 \;+\; L^2\Big((d\chi - B)^2 + ds^2_{CP^2}\Big), \\ C_{abcd} &= L^{-4}u^4\epsilon_{abcd}\,, \qquad\qquad C_{\varphi_2\varphi_3\varphi_4\chi} = \frac{1}{8}L^4\sin^4\varphi_1\sin\varphi_2, \end{aligned} \tag{4.12}$$

where $ds^2_{CP^2}$ stands for the Fubini-Study metric on CP^2 (with coordinates φ_i) and χ is taken along the $U(1)$ fiber [170].

The curvature tensor of the fiber connection introduced as $F = 2n\,dB$ is selfdual and[5]

$$\int_{CP^2} F\wedge F = 8\pi^2 n^2\,. \tag{4.13}$$

It is therefore natural to take the magnetic components living in the CP^2 as proportional F. With this choice for the BI field strength there are no other couplings in the Chern-Simons action besides the one already considered in (4.1). The Born-Infeld action however is given by

$$S_{DBI} \;=\; -T_5\int d^6\xi\;\frac{u}{L}\;\sqrt{\det\Big(g_{\alpha\beta}+F_{\alpha\beta}\Big)} \;=\; -T_5\int d^6\xi\; u\;\sqrt{g_{S^5}}\;\Big(L^4 + 2F_{\alpha\beta}F^{\alpha\beta}\Big) \tag{4.14}$$

[5]One sees that B induces a non-trivial instanton number in CP^2 [170, 171].

Finally, substituting the expression for F in the action and integrating over the S^5 directions the following expression for the energy of the spherical D5-brane is obtained [169]:

$$E_{D5} \;=\; 8\pi^3 T_5\, u\,\Big(n^2+\frac{L^4}{8}\Big). \tag{4.15}$$

Note that this energy consists of two parts: one contribution from the tension of the 5-brane wrapped around the S^5, proportional to L^4, and one from the magnetic flux of the BI vector, proportional to n^2. The magnetic components of F induce a non-zero instanton number n^2 on the worldvolume of the D5-brane, since integrating the Chern-Simons coupling over the CP^2 directions, one obtains

$$S_{CS} \;=\; \frac{1}{2}T_5\int_{\mathbb{R}\times S^5} P[C_2]\wedge F\wedge F \;=\; n^2 T_1\int_{\mathbb{R}\times S^1} P[C_2]\ , \tag{4.16}$$

Even though in $AdS_5\times S^5$ C_2 is zero, this coupling indicates that the magnetic flux is inducing n^2 D-string charge in the configuration. These strings are wound around the fiber direction χ and dissolved in the CP^2. Note that the total energy of the configuration (4.15) is the sum of the energy of the D5 and the D1-branes, which indicates that we are dealing with a threshold BPS bound state.

It is now possible to perform a similar analysis to the one shown in the previous subsection [165], but taking into account the effect of the non-zero magnetic flux on the D5-brane. In this case the equations of motion read

$$\frac{u^4}{\sqrt{(u')^2+\frac{u^4}{L^4}}} \;=\; \text{const}, \qquad \frac{u_0'}{\sqrt{(u_0')^2+\frac{u_0^4}{L^4}}} \;=\; \frac{\pi L^4}{4N}\Big(1+\frac{8n^2}{L^4}\Big), \tag{4.17}$$

for the bulk and the boundary respectively. The equations (4.17) can again be combined into a single one,

$$\frac{u^4}{\sqrt{(u')^2+\frac{u^4}{L^4}}} \;=\; \beta\, u_0^2 L^2, \qquad \text{with} \qquad \beta^2 \;=\; 1-\frac{1}{16}\Big(1+\frac{8\pi n^2}{N}\Big)^2. \tag{4.18}$$

In the absence of magnetic BI flux on the worldvolume, $\beta=\sqrt{15/16}$, as in [165]. However, in general for non-zero n^2, we have to make sure that β is real (as u is real). This implies that $n^2\le 3N/8\pi$, surprisingly finding that there is a bound on the number of D-strings that can be dissolved in the configuration, depending on the number of D3-branes that source the background; in fact, in terms of the gauge theory parameters $n^2\le 3\lambda/32\pi^2 g_s$.

Integrating the equation of motion, the size ℓ and the energy E of the baryon are given by [169]

$$\ell=\frac{L^2}{u_0}\int_1^\infty dy\frac{\beta}{y^2\sqrt{y^4-\beta^2}}, \qquad E=T_1 u_0\Big\{\int_1^\infty dy\Big[\frac{y^2}{\sqrt{y^4-\beta^2}}-1\Big]-1\Big\}, \tag{4.19}$$

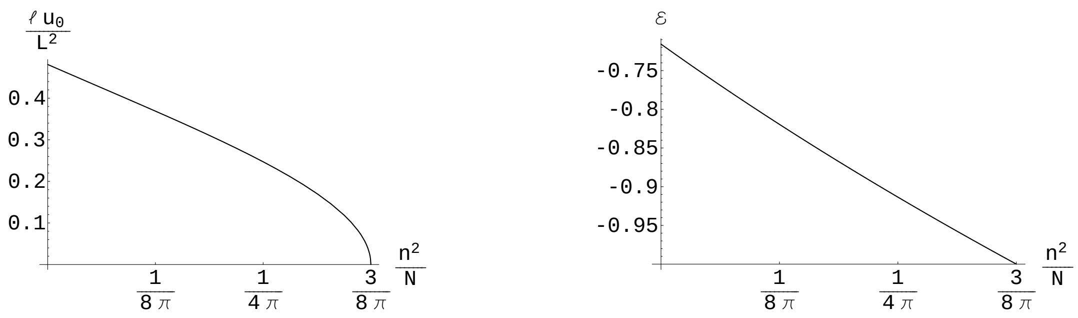


Figure 4.5: Radius ℓ (in units of L^2/u_0) and the energy E (in units of u_0) of the baryon vertex as a function of n^2/N.

with $y = u/u_0$. These integrals can again be solved in terms of hypergeometric functions [165]. In the original fig. 4.5 the radius and the energy of the baryon as a function of n^2/N was plotted [169]. These plots reveal that the size of the baryon vertex goes to zero as we approach the bound on n^2, making it impossible to continue beyond the bound. The expression for the energy has the same form as the expression in [165] and indeed takes the same value for $n = 0$. In particular, the dependence on $\sqrt{g^2 N}$ and on u_0 is unaltered, as expected by conformal invariance. Notice that also here the energy of the configuration is only well defined for n^2 inside the allowed interval.

In [169] it was pointed out that finding a bound on the number of dissolved D1-branes due to the dynamics of the F-strings could probably be related to the stringy exclusion principle of [204]. Their configuration carries a non-zero winding number in the fiber direction of the S^5, which in terms of the dual field theory will manifest itself as a specific charge of the $SU(3)$ R-symmetry group. As these charges are bounded due to conformal invariance, one expects to find a bound on the magnetic flux.

4.1.2 Other particle-like brane configurations

Di-baryons in AdS spaces

In [173] Gubser and Klebanov proposed that, in $AdS_5 \times T^{1,1}$ D3-branes wrapping 3-cycles of the $T^{1,1}$ correspond to baryon-like chiral operators built out of products of N chiral superfields. These baryon-like operators have the form A^N and B^N, being N fully antisymmetrized $SU(N)$ indices. At large N, the dimensions of such operators calculated from the wrapped D3-branes mass, $3N/4$, turned out to be in complete agreement with the dimension of the chiral superfields at the fixed point, which is $3/4$ [173]. After this initial work, Berenstein and Klebanov [174] demonstrated that the previous identification was also true even away from the large N limit, providing a detailed map between a wrapped

D3-brane in AdS backgrounds and a dibaryon operator in the corresponding CFT. In their work, they found a matching between a $U(1)_R$ charge of this wrapped D3-brane with the corresponding R-charge of di-baryons expected from field theory side. It was shown that, in certain *Sasaki-Einstein* geometries, wrapped D3-branes and M2-branes carry an R-charge related to the $U(1)_R$ gauge field emerging from the KK reduction of the full ten or eleven dimensional SUGRA action.

4.2 ABJM theory

As we already mentioned, important progress have taken place in understanding the world volume theory of coincident supermembranes of M-theory in the context of the AdS_4/CFT_3 correspondence. Bagger and Lambert [178] (and independently [179]) constructed a theory conjectured to be related to a specific theory of M2-branes. Based on an algebra with a totally antisymmetric triple product ("3-algebra"), they proposed a field theory model for multiple M2-branes. They then constructed a supersymmetric theory with the required properties. The theory had the 16 supersymmetries and the conformal invariance expected, as well as the $SO(8)$ R-symmetry acting on the eight transverse scalars; however, it only described correctly two M2-brane systems.

Aharony, Bergman, Jafferis and Maldacena (from now on ABJM) constructed and studied a three dimensional superconformal Chern-Simmons-matter theory with $U(N)_k \times U(N)_{-k}$ gauge group[6] and an explicit $\mathcal{N} = 6$ superconformal symmetry [175]. Using brane constructions they argued that the theory described the low energy limit of N M2-branes probing a $\mathbb{C}^4/\mathbb{Z}_k$ singularity. The large N limit of the theory would be then dual to M-theory on an $AdS_4 \times S^7/\mathbb{Z}_k$ background. Taking the 't Hooft limit by holding N/k fixed as $N \to \infty$ it is also possible to arrive to a theory dual to type IIA string theory on $AdS_4 \times \mathbb{P}^3$. Although their construction realized explicitly only six of the eight supersymmetries the theory was conjectured to describe N M2-branes in flat space for $k = 1$. Indeed, in the $N = 2$ case the theory has that missing extra symmetries and reproduced the previous results of [178]. A picture showing schematically all these relations can be shown in fig. 4.6.

[6] The k here is the discrete CS level of the gauge group, which in the ABJM theory is related to the superpotential coupling due to the high degree of SUSY. The requirement that a non-Abelian theory is invariant under large gauge transformations makes this CS level to just take integer values.

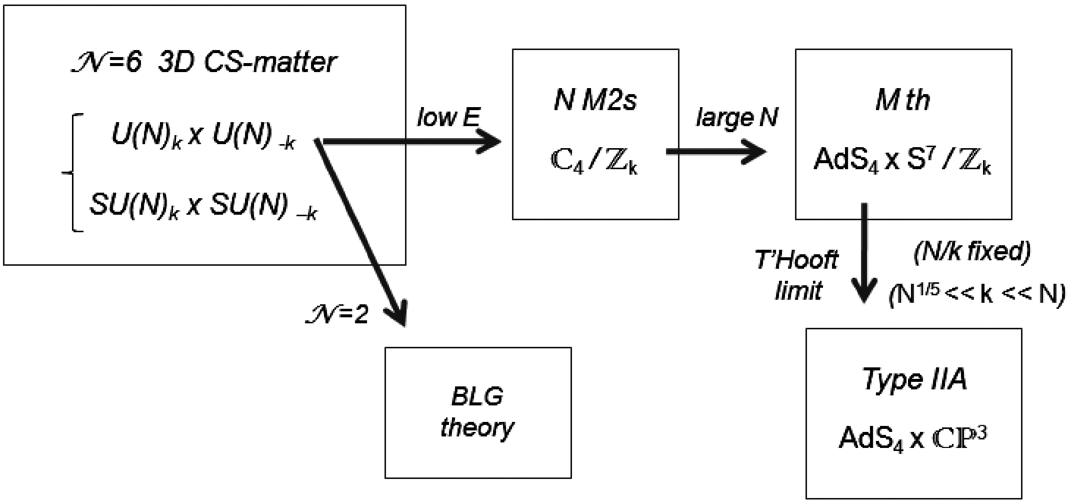


Figure 4.6: ABJM theory is proposed to be equivalent a low energy theory on N M2-branes at a $\mathbb{C}^4/\mathbb{Z}_k$ singularity. For $k >> N$ ABJM is weakly coupled (the superpotential is proportional to the inverse of k) and it has two dual weakly curved gravitational descriptions in M-theory for $k << N^{1/5}$, and in type IIA string theory, for $N^{1/5} << k << N$.

4.2.1 BLG model

Bagger and Lambert [178] and independently, Gustavsson [179], constructed a theory (BLG model) using the 3-algebra previously mentioned. This algebra is defined by means of the following triple product[7]

$$\left[T^a, T^b, T^c\right] = f_d^{abc} T^d \tag{4.20}$$

for a given set of generators T_a and being f_{abcd} a completely antisymmetric tensor. Given this definition, the maximally supersymmetric Chern-Simons lagrangian looks

$$\begin{aligned}
\mathcal{L}_{CS} &= \frac{1}{2}\epsilon^{\mu\nu\lambda}\left(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}\right), \\
\mathcal{L}_{matter} &= -\frac{1}{2}\mathcal{D}^\mu x^{aI}\mathcal{D}_\mu x^I_a + \frac{i}{2}\bar{\psi}^a\Gamma^\mu\mathcal{D}_\mu\psi_a + \frac{i}{4}\bar{\psi}_b\Gamma_{IJ}x^I_c x^J_d\psi_a f^{abcd} \\
&\quad -\frac{1}{12}\,\mathrm{Tr}\left([x^I, x^J, x^K][x^I, x^J, x^K]\right), \qquad I, J = 1, \ldots, 8,
\end{aligned} \tag{4.21}$$

being A^μ_{ab} a gauge field, and ψ_a and $x^I = x^I_a T^a$ matter fields, for gauge indices running from 1 to 4. Indices denoted with capital letters are $SO(8)$ vector indices. For f^{abcd} proportional to the ϵ^{abcd} tensor we obtain an $SO(4)$ gauge symmetry and the theory has manifest

[7]See [180] for a review on M2-branes and AdS/CFT duality.

unitarity and $\mathcal{N} = 8$ SUSY.

This theory has been demonstrated [181, 182] to be equivalent to an $SU(2) \times SU(2)$ CS gauge theory with opposite CS levels.

$$\begin{aligned}
\mathcal{L}_{CS} &= \frac{k}{4\pi}\epsilon^{\mu\nu\lambda}\,\mathrm{Tr}\left(A_\mu\partial_\nu A_\lambda + \frac{2i}{3}A_\mu A_\nu A_\lambda - \hat{A}_\mu\partial_\nu\hat{A}_\lambda - \frac{2i}{3}\hat{A}_\mu\hat{A}_\nu\hat{A}_\lambda\right),\\
\mathcal{L}_{matter} &= -(\mathcal{D}^\mu X^I)^\dagger\mathcal{D}_\mu X^I + i\bar{\Psi}^\dagger\Gamma^\mu\mathcal{D}_\mu\Psi - \frac{4i\pi}{k}\bar{\Psi}^\dagger\Gamma^{IJ}\Big(X^I X^{J\dagger}\Psi + X^J\Psi^\dagger X^I +\\
&\quad +\Psi X^{I\dagger}X^J\Big) - \frac{32\pi^2}{3k^2}\,\mathrm{Tr}\left(X^{[I}X^{\dagger J}X^{K]}X^{\dagger[K}X^J X^{\dagger I]}\right), \qquad (4.22)
\end{aligned}$$

where the covariant derivative is given by $\mathcal{D}_\mu X^I = \partial_\mu X^I + iA_\mu X^I - iX^I\hat{A}_\mu$, and the X^I bifundamental matter fields satisfies $X^I = 1/2(x_4^I\mathbb{I}_{2\times 2} + ix_i^I\sigma^i)$ as well as the reality condition [182]

$$X_a^{\hat{a}} = -\epsilon_{ab}\,X_{\hat{b}}^b\,\epsilon^{\hat{a}\hat{b}}, \qquad \epsilon^{ab} = i\sigma_2^{ab}. \qquad (4.23)$$

A $\mathcal{N} = 2$ formalism can be used by combining the matter fields into bi-fundamental chiral superfields $\mathcal{Z}^A$ [175, 183]. In that case the superpotential can be expressed as

$$W = \frac{\pi}{3k}\epsilon_{ABCD}(\mathcal{Z}^A\mathcal{Z}^{\ddagger B}\mathcal{Z}^C\mathcal{Z}^{\ddagger D}), \qquad (4.24)$$

where

$$\begin{aligned}
Z^{\ddagger A} &= X^{\dagger A} + iX^{\dagger A+4},\\
\bar{Z}_A &= X^A - iX^{A+4}. \qquad (4.25)
\end{aligned}$$

It is possible to check that, although the superpotential (4.24) only has a manifest $U(1)_R \times SU(4)$ invariance, this $\mathcal{N} = 2$ formalism still has the desired $SO(8)_R$ global symmetry [183]. Nevertheless a problem exists when trying to generalize this construction to higher rank gauge groups; the reality condition (4.23) and the "double dagger" operation (4.25) are special to $SU(2) \times SU(2)$.

4.2.2 The ABJM proposal

Aharony, Bergman, Jafferis and Maldacena proposed, in [175], to abandon the manifest global $SU(4)$ invariance by combining the bi-fundamental fields into

$$\begin{aligned}
Z^1 &= X^1 + iX^5, \qquad W^1 = X^{3\dagger} + iX^{7\dagger},\\
Z^2 &= X^2 + iX^6, \qquad W^2 = X^{4\dagger} + iX^{8\dagger}. \qquad (4.26)
\end{aligned}$$

Promoting the fields Z^A and W^A to the chiral superfields $\mathcal{Z}^A$ and $\mathcal{W}^A$, the superpotential can be rewritten as [175, 183]:

$$W = \frac{2\pi}{k}\epsilon_{AC}\epsilon^{BD}\,\mathrm{Tr}\left(\mathcal{Z}^A\mathcal{W}_B\mathcal{Z}^C\mathcal{W}_D\right) \ . \tag{4.27}$$

This superpotential has the same form as that for N D3-branes in the *conifold*[8], and can be easily generalized to higher rank $SU(N)\times SU(N)$ gauge groups. Nevertheless, in the conifold theory the superpotential has a certain $U(1)$ symmetry that becomes global in the IR, meanwhile due to the different dynamics of the case at hand we have to treat this $U(1)$ as a gauge symmetry.

Following this and by using type IIB brane constructions, ABJM proposed $U(N)\times U(N)$ and not $SU(N)\times SU(N)$ as gauge group on N M2-branes [175]. The type IIB brane constructions of ABJM generalized ones of [185, 186] to theories with a $U(N)\times U(N)$ gauge group, CS terms at levels k and $-k$ and matter in the bi-fundamental representation. They showed how these theories flow in the IR to the $\mathcal{N}=6$ superconformal CS introduced before. Finally they lifted the configurations to M-theory by T-duality transformations and relate the low energy of those theories to M2-branes probing a $\mathbb{C}^4/\mathbb{Z}_k$ singularity, supporting the previous argument[9].

We should note that the classical ABJM action possesses a manifest $SU(4)_R\sim SO(6)_R$ symmetry [183], which strongly suggests that, for general N and k, the theory will have at least $\mathcal{N}=6$ SUSY. An explicit demonstration of this $\mathcal{N}=6$ superconformal invariance was presented in [189].

It is remarkable that one can make use of the AdS/CFT duality in order to predict how the correlation functions of protected gauge-invariant operators scale, as their spectrum should be in one-to-one correspondence with the KK harmonics on $S^7/\mathbb{Z}_k$. The result [190] predicts that the number of dof scales as $N^{3/2}$ (not as N^2 as occurs in the D3-brane case) [10]

$$\langle\mathcal{O}_1\mathcal{O}_2\ldots\mathcal{O}_m\rangle \propto (R/l_p)^9 \propto N^{3/2}\ , \tag{4.28}$$

what was later obtained in [205].

4.2.3 The gravitational description

Let us make a lightning review of the $AdS_4\times\mathbb{P}^3$ background while collecting some useful formulae that will be useful in the next chapter. The background created by a stack of

[8] A review on this subject can be found in [184]
[9] Additional geometrical arguments are presented in [187, 188]
[10] For a deeper insight, we recommend looking at the review [191]

N coincident M2-branes is given by (see section 1.5.1). Taking the near horizon limit, the $S^7/\mathbb{Z}_k$ orbifold and reducing to type IIA, the $AdS_4 \times \mathbb{P}^3$ metric reads

$$\begin{aligned} ds^2 &= \frac{4\,\rho^2}{L^2}dx_{1,2}^2 + L^2\frac{d\rho^2}{4\,\rho^2} + L^2 ds_{\mathbb{P}^3}^2 \\ &= L^2\Big(\frac{1}{4}ds_{AdS_4}^2 + ds_{\mathbb{P}^3}^2\Big)\,, \end{aligned} \tag{4.29}$$

with L the radius of curvature in string units,

$$L = \Big(\frac{32\pi^2 N}{k}\Big)^{1/4} \tag{4.30}$$

This is a good description of the gravity dual to the $U(N)_k \times U(N)_{-k}$ CS-matter theory [175] when $N^{1/5} << k << N$.

It is well-known that for $\mathbb{P}^3$ one has $H^q(\mathbb{P}^3) = \mathbb{Z}$ for even q. Indeed, parameterizing the $\mathbb{P}^3$ as (e.g. [206])

$$\begin{aligned} ds_{\mathbb{P}^3}^2 = \quad & d\mu^2 + \sin^2\mu\,\Big[d\alpha^2 + \frac{1}{4}\sin^2\alpha\,\big(\cos^2\alpha\,(d\psi - \cos\theta\,d\phi)^2 + d\theta^2 + \sin^2\theta\,d\phi^2\big) \\ & +\frac{1}{4}\cos^2\mu\,\big(d\chi + \sin^2\alpha\,(d\psi - \cos\theta\,d\phi)\big)^2\Big] \end{aligned} \tag{4.31}$$

where

$$0 \geq \mu,\, \alpha \geq \frac{\pi}{2}\,, \quad 0 \geq \theta \geq \pi\,, \quad 0 \geq \phi \geq 2\pi\,, \quad 0 \geq \psi,\, \chi \geq 4\pi\,, \tag{4.32}$$

there is a $\mathbb{P}^2$ at fixed θ, ϕ, and a $\mathbb{P}^1$ at $\mu = \alpha = \pi/2$ and fixed χ, ψ.

The Kähler form

$$J = \frac{1}{2}d\mathcal{A}\,, \tag{4.33}$$

where $\mathcal{A}$ is the connection in $ds_{S^7}^2 = (d\tau + \mathcal{A})^2 + ds_{\mathbb{P}^3}^2$, which in our coordinates reads:

$$\mathcal{A} = \frac{1}{2}\sin^2\mu\,\Big(d\chi + \sin^2\alpha\,\big(d\psi - \cos\theta\,d\phi\big)\Big)\,, \tag{4.34}$$

satisfies

$$\int_{\mathbb{P}^1} J = \pi\,, \qquad \int_{\mathbb{P}^2} J\wedge J = \pi^2\,, \qquad \int_{\mathbb{P}^3} J\wedge J\wedge J = \pi^3\,. \tag{4.35}$$

Therefore,

$$\frac{1}{6}\,J\wedge J\wedge J = d\mathrm{Vol}(\mathbb{P}^3) \qquad \text{with} \qquad \mathrm{Vol}(\mathbb{P}^3) = \frac{\pi^3}{6}\,. \tag{4.36}$$

The non-vanishing fluxes of this background can then be written as

$$F_2 = \frac{2L}{g_s}J\,, \qquad F_4 = \frac{6}{g_s\,L}\,d\mathrm{Vol}(AdS_4)\,, \qquad F_6 = \frac{6\,L^5}{g_s}d\mathrm{Vol}(\mathbb{P}^3) \tag{4.37}$$

where

$$g_s = \frac{L}{k} \,, \qquad L^4 k = 32\pi^2 N \,. \tag{4.38}$$

The flux integrals read

$$\int_{\mathbb{P}^3} F_6 = 32\,\pi^5\, N \,, \qquad \int_{\mathbb{P}^1} F_2 = 2\pi\, k \,. \tag{4.39}$$

4.3 Particle-like branes in ABJM

In the M-theory description of the ABJM background, $S^7/\mathbb{Z}_k$ has certain non-trivial cycles which branes can wrap, as in the AdS_5 case. More concretely, from the 10-dimensional point of view, in the type IIA description the geometry is $AdS_4 \times \mathbb{P}^3$, so D0, D2, D4, and D6 branes can wrap topologically non-trivial subspaces of the three dimensional complex projective space [175, 192][11]. The D2 and D6 branes develop worldvolume tadpoles as they capture the RR flux of the geometry, meanwhile the D0 and D4 branes do not, so that they should correspond to gauge-invariant operators on the field theory side of the correspondence. A subtle issue affecting these configurations was raised in [193]. Since the D4-branes wrap a non-spin manifold, they carry a half-integer worldvolume magnetic flux due to the Freed-Witten anomaly [194]. On the other hand, matching with the natural interpretation in field theory of such objects as di-baryon-like operators requires to switch a flat half-integer background B-field[12]. More generally, these wrapped branes act as sources to vector fields in AdS_4 arising from the reduction of RR potentials on topologically non-trivial cycles. In turn, vector fields in AdS_4 admit quantization with either of the two possible fall-offs at the boundary [195], which amount to either a dynamical boundary gauge field or to a global current (discussions in this context have appeared recently in [196, 197, 198, 199]). Since a definite quantization must be chosen, it follows that either magnetic or electric sources are forbidden for the corresponding bulk field [195]. This might shed some light on the role of the B-field. Indeed, coming back to the D4-branes, the quantization allowing for the D4-branes to exist should correspond to that where the $U(1)$'s are non-dynamical. Under that assumption, a determinant-like di-baryon dual operator would be gauge invariant by itself and it would have the right dimension to agree with the gravity result. On the other hand, the quantization dual to dynamical $U(1)$'s would forbid the D4-branes, which might suggest that no B-field is needed. However, a

[11]The $\mathbb{P}^3$ space has $H^q(\mathbb{P}^3) = F$ for even $q \leq 6$.

[12]The original argument supporting this B-field in [193] concerns a detailed analysis of the supergravity charges, while the analysis of the D4 worldvolume dynamics arises as a consistency check. For more details we refer to the original paper.

full understanding of this very important point is, at present, still lacking.

Similar comments should hold for the remaining wrapped branes. It has been argued that the D0-brane corresponds to a di-monopole operator in the CFT side. The D6-brane, very much like the baryon vertex in AdS_5, requires N fundamental strings ending on it. Its dual operator should then naturally involve the ϵ tensor of the gauge theory. On the other hand, the D2-brane wrapped on the $\mathbb{P}^1 \subset \mathbb{P}^3$ develops a tadpole that has to be cancelled with k fundamental strings. The dual operator is a monopole 't Hooft operator, realized as a Sym_k product of Wilson lines [175]. As mentioned above, as of today, there is no fully satisfactory understanding of the role of these branes and their dual operators.

Chapter 5

Charged Particle-like Branes in ABJM

In this chapter we are going to present the research work in which we study the effect of adding lower dimensional brane charges to the particle-like brane configurations of $AdS_4 \times \mathbb{P}^3$ described before. These gravitational configurations admit a natural generalization by allowing non-trivial worldvolume gauge fluxes. It is the aim of the work carried out in [23] to generalize the spectroscopy of wrapped branes by adding such non-trivial worldvolume gauge fields. To that matter, we will assume that suitable boundary conditions are chosen in each case such that the discussed branes are possible. These generalized configurations are of potential interest for some AdS/CMT applications (see for instance [200, 201]), for example as candidates for holographic anyons in ABJM, as discussed recently in [202, 203].

Allowing for a non-trivial worldvolume gauge field has the effect of adding lower dimensional brane charges. This modifies how the branes capture the background fluxes in a way that depends on the induced charges, such that, in some cases, additional fundamental strings will be required to cancel the worldvolume tadpoles. From that point of view, the generalized configurations are similar to holographic Wilson loops. We will see that the D2 and D6-branes do not differ much from the zero charge case, although they are stable only if the induced charges lie below some upper bound.

This situation is familiar from the baryon vertex with magnetic flux in $AdS_5 \times S^5$ discussed in chapter 4. In these cases the energy of the bound state increases with the charge that is being induced. However adding charges allows to construct more general baryon vertex configurations. We will see that for the D6-brane the number of quarks that forms the bound state can be increased in this manner. From this point of view adding flux provides an alternative mechanism to that proposed in [165] for modifying the number of quarks. In turn, the D4-brane with flux behaves quite differently from the fluxless case, since it will require fundamental strings ending on it as opposed to the vanishing worldvolume flux case. As we will see, the study of its dynamics reveals that the whole configuration is stable if the magnetic flux lies within a given interval, being maximally stable for an intermediate value, and reducing to free quarks at the boundaries.

5.1 Particle-like branes in $AdS_4 \times \mathbb{P}^3$ with magnetic flux

In this section we generalize the particle-like brane configurations in [175] to include a non-vanishing magnetic flux. We analyze the various brane charges that are dissolved as well as the charges of the different tadpoles induced. Following [193], an important observation is that the dual gravity background might actually involve a non-vanishing but flat B_2 field. It is possible to argue for such a shift by noting that the D4-brane with minimal flux (it will turn out essential for the argument that this minimal flux has to be half-integer due to the Freed-Witten anomaly [194]) should be dual to a di-baryon. In order to review this argument, we will consider first the D4-brane case before turning to the D2 and D6 cases.

5.1.1 The di-baryon

Consider a D4-brane wrapping the $\mathbb{P}^2$ in $\mathbb{P}^3$. This brane lives at fixed θ and ϕ, and since it does not capture any background fluxes it does not require any fundamental strings ending on it.

Since the $D4$-brane wraps a $\mathbb{P}^2$, which is not a spin manifold, it should carry a half-integer worldvolume gauge field flux through the $\mathbb{P}^1 \subset \mathbb{P}^2$, due to the Freed-Witten anomaly [194]. Given that the gauge-invariant quantity in the worldvolume is $\mathcal{F} = B_2 + 2\pi F$, this half-integer worldvolume flux can be cancelled through a shift of B_2. This motivated [193] to include a flat B_2-field in the dual IIA background:

$$B_2 = -2\pi\, J \tag{5.1}$$

which should be considered in addition to the fluxes discussed in the previous section.

We can now consider a more general configuration where we add extra worldvolume flux $F = \mathcal{N}\, J$ on top of the $F_{FW} = J$ required to cancel the Freed-Witten anomaly, such that the total worldvolume flux is $F_T = (\mathcal{N} + 1)\, J$ with even-integer quantization (that is, $\mathcal{N} \in 2\mathbb{Z}$ being $\mathcal{N} = 0$ the minimal case). As noted above, the quantity appearing in the brane worldvolume action is the combination $\mathcal{F} = B_2 + 2\pi\, F_T$ (remember that we are taking $\alpha' = 1$). Putting together the various definitions, we have $\mathcal{F} = 2\pi\, F$, that is, the B_2 shift and the extra half unit of worldvolume flux cancel each other and we can effectively

work as if we had no background B_2-field and $F = \mathcal{N}\, J$.
The DBI action is then given by:

$$\begin{aligned} S_{DBI} &= -\frac{T_4}{g_s}\int d^5\xi\sqrt{-\det(g+2\pi F)} = -\frac{T_4}{g_s}\int d^5\xi\,\sqrt{|g_{tt}|}\sqrt{g_{\mathbb{P}^2}}\left(L^4+2(2\pi)^2F_{\alpha\beta}F^{\alpha\beta}\right) \\ &= -\frac{\pi^2\,T_4}{2\,g_s}\left(L^4+(2\pi\mathcal{N})^2\right)\int dt\,\frac{2\,\rho}{L}\,. \end{aligned} \tag{5.2}$$

Therefore, for non-vanishing magnetic flux the mass of the D4-brane satisfies

$$m_{D4}L = N + k\,\mathcal{N}^2/8 \tag{5.3}$$

From here we can see explicitly that in the minimal flux case, $\mathcal{N}=0$, the background B_2 cancels the half-integer worldvolume flux induced by the Freed-Witten anomaly, such that $m_{D4}\,L = N$; thus naturally admitting an interpretation as a di-baryon.

The D4-brane with magnetic flux captures the F_2 background flux through the coupling

$$\begin{aligned} S_{CS} &= \frac{1}{2}(2\pi)^2\,T_4\int_{\mathbb{R}\times\mathbb{P}^2} P[F_2]\wedge F\wedge A = 2\,(2\pi)^2 T_4\,k\,\mathcal{N}\int_{\mathbb{R}\times\mathbb{P}^2} J\wedge J\wedge A \\ &= k\,\frac{\mathcal{N}}{2}\,T_{F1}\int dtA_t \end{aligned} \tag{5.4}$$

Therefore $k\,\mathcal{N}/2$ fundamental strings are required to end on it in order to cancel the tadpole. Note that, due to the quantization condition for $\mathcal{N}$, this quantity is an integer number. Moreover, the magnetic flux also dissolves D2 charge through the coupling:

$$S_{CS} = 2\pi\,T_4\int_{\mathbb{R}\times\mathbb{P}^2} C_3\wedge F = \frac{\mathcal{N}}{2}\,T_2\int C_3 \tag{5.5}$$

Thus, the number of fundamental strings is k times the number of dissolved D2 branes. In fact, as we will see in the next subsection, a single D2-brane requires k fundamental strings ending on it. Thus, from this perspective, the fundamental strings ending on the D4 are cancelling the tadpole due to the dissolved D2-branes.

We will see in the next section that the D4-brane with the $k\,\mathcal{N}/2$ attached F-strings is stable if the magnetic flux lies in an interval, reducing to $k\,\mathcal{N}/2$ radial fundamental strings stretching from the D4-brane to infinity, i.e. to free quarks, at both ends of the interval.

Given that F is proportional to the Kähler form on the $\mathbb{P}^2$ it satisfies that $\int_{\mathbb{P}^2} F\wedge F = \mathcal{N}^2\pi^2$. Therefore, it also induces D0-brane charge in the configuration, through the coupling:

$$S_{CS} = \frac{1}{2}(2\pi)^2\,T_4\int_{\mathbb{R}\times\mathbb{P}^2} C_1\wedge F\wedge F = \frac{\mathcal{N}^2}{8}\,T_0\int_{\mathbb{R}} C_1\,. \tag{5.6}$$

However, as noted in [193], there are relevant higher curvature corrections [74]

$$\Delta S \sim \int C \wedge e^{\mathcal{F}} \wedge \sqrt{\frac{\hat{\mathcal{A}}(T)}{\hat{\mathcal{A}}(N)}}\,, \tag{5.7}$$

where $\hat{\mathcal{A}}$ is the A-roof genus

$$\hat{\mathcal{A}} = 1 - \frac{\hat{p}_1}{24} + \frac{7\,\hat{p}_1^2 - 4\,\hat{p}_2}{5760} + \cdots \tag{5.8}$$

and the Pontryagin classes are written in terms of the curvature of the corresponding bundle as

$$\hat{p}_1 = -\frac{1}{8\pi^2}\,\mathrm{Tr}\,R^2 \qquad \hat{p}_2 = \frac{1}{256\,\pi^4}\Big((\mathrm{Tr}\,R^2)^2 - 2\,\mathrm{Tr}\,R^4\Big) \tag{5.9}$$

The relevant term in (5.7) is then

$$\Delta S = (2\pi)^4\,T_4 \int C_1 \wedge \frac{1}{48}\,(\hat{p}_1(N) - \hat{p}_1(T)) = -\frac{1}{24}\,T_0 \int C_1 \tag{5.10}$$

Thus, the total D0 charge is

$$\Big(\frac{\mathcal{N}^2}{8} - \frac{1}{24}\Big)\,T_0 \int C_1 \tag{5.11}$$

This equation shows that the D4-brane contains dissolved D0-brane charge even for the minimal flux allowed. Note that the term $k\mathcal{N}^2/8$ in (5.3) can be identified with (L times) the mass of the extra $\mathcal{N}^2/8$ D0-branes dissolved in the worldvolume due to the non-vanishing magnetic flux. Therefore, (5.3) can be interpreted as the energy of a threshold BPS intersection of $\mathcal{N}^2/8$ D0-branes and a D4-brane. We should note however that if we want to study the dynamics of the D4-brane with fundamental strings attached in the probe brane approximation, we need to take the strings distributed uniformly on the D4. Therefore, the Killing spinors preserved by each one of the F1 strings will be different and all supersymmetries will be broken. Nevertheless, since both the wrapped cycle and the worldvolume flux are topologically non-trivial, we expect the system to be at least perturbatively stable.

By making all the F1 strings end in the same point, such that they preserve the same Killing spinor, we expect that a SUSY generalization in terms of a spike can be found. The problem of finding D4-brane spiky solutions in $AdS_4 \times \mathbb{P}^3$ has been addressed recently in [203], although in the ansatz taken there the deformation of the D4-brane due to the electric field is not taken into account. It would be interesting to check if spiky solutions exist for both non-vanishing electric and magnetic fields.

On boundary conditions and dual operators

Given the topology of $\mathbb{P}^3$ it is possible to consider the KK reduction of the 5-form and 7-form respectively on $\mathbb{P}^2$ and $\mathbb{P}^3$ giving rise to vectors in AdS_4. As discussed in [195] and further elaborated in a similar context in [196, 197, 198, 199], the two fall-offs are possible in AdS_4. [1] Choosing one or the other amounts to the dual $U(1)$ symmetry being gauged or not. In turn, from the bulk perspective, this is seen as electric-magnetic duality (the so-called $\mathcal{T}$-operation). It is possible to define a $\mathcal{S}$-operation such that their combined action forms an $SL(2,\mathbb{Z})$ algebra, which then connects *different* boundary CFTs. The action of such algebra is far from being understood. However, one particular implication would be that depending on the boundary conditions that are chosen the allowed sources are either the magnetic or the electric ones. From this point of view, one might argue that the quantization dual to dynamical boundary gauge fields forbids D4, D6 (which are electrically charged under the 5-form and the 7-form respectively), which from the field theory point of view would stand for the non-gauge invariance of the operators $\det A$ and ϵ. On the other hand, the boundary conditions allowing for the D6, D4 would be dual to a certain $SU(N)$ version of the theory, in which the B field would presumably play an important role. Nonetheless, at this point this is no more than a speculation. In particular, the role of the higher curvature couplings, naively coupling the D4 to the 1-form potential (5.11) and thus endowing it with magnetic charge at the same time, remains to be clarified. It should be pointed out that recently a detailed analysis of the field theory has been performed in [207]. Careful analysis of the quantization condition of the $U(1)$ gauge fields suggests that the moduli space of the $U(N) \times U(N)$ gauge theory is a $\mathbb{Z}_k$ cover of the a priori expected $\mathrm{Sym}_N\, \mathbb{C}^4/\mathbb{Z}_k$, thus allowing for determinant-like operators to be gauge-invariant [207] (see also [208]). These operators are naturally dual to the wrapped D4, which suggests that the B field is turned on. It would be very interesting to clarify the role of the B field in this context, and figure out whether a connection to the possibility raised above, namely the subtle role of the quantization of abelian fields in AdS, is possible. Further studies of these issues are well beyond the scope of this study, and are postponed for further work.

In this study we will simply assume that suitable boundary conditions are chosen allowing for the corresponding wrapped objects, and, as we have done for the D4-brane, we will include the effect of the (flat) B-field[2]. The D4-brane with zero flux would be identified with the di-baryon operator $\det A = \epsilon_{i_1 \ldots i_N} \epsilon^{j_1 \cdots j_N} A^{i_1}_{j_1} \ldots A^{i_N}_{j_N}$ in the CFT side, being A one of the bifundamentals in the field theory. It is also natural to ask what could be the dual of the D4-brane with non-minimal flux. Since once the worldvolume flux is

[1] It must be noted that, from an 11d perspective, the $U(1)$ fields discussed here are not related to a topological symmetry as in [198, 199], which makes them more subtle.

[2] The results for a vanishing B-field are simply obtained by tuning the extra worldvolume flux.

turned on extra F1 strings are required, we should expect such dual operator to involve $n_f = \frac{k\mathcal{N}}{2}$ Wilson lines in the fundamental representation of $U(N) \times U(N)$. Indeed, the configuration is reminiscent of the D5 Wilson loop in $AdS_5 \times S^5$ [209], which suggests that these fundamental indices should be antisymmetrized. We will see in the next section that dynamically a bound $n_f^{max} \sim \sqrt{N\,k} \sim \lambda^{-\frac{1}{2}}\,N$, where $\lambda = N/k$ is the 't Hooft coupling, in the number of such fundamental indices appears, which is consistent with the antisymmetrization assumption. It would be interesting to elaborate further on this proposal, and in particular to understand the dependence on the 't Hooft coupling. We postpone such analysis for further work.

5.1.2 The 't Hooft monopole

Let us now consider the D2-brane wrapping the $\mathbb{P}^1$ in $\mathbb{P}^3$, identified in [175] with a ('t Hooft) monopole operator [210, 211, 212].

Since this brane captures the F_2 flux it requires fundamental strings in order to cancel the worldvolume tadpole. Substituting (4.39) in the CS coupling

$$S_{CS} = 2\pi\,T_2 \int_{R\times\mathbb{P}^1} P[F_2] \wedge A = k\,T_{F1} \int dt A_t \tag{5.12}$$

we find that the number of fundamental strings must be $q = k$. Note in particular that this is the anticipated result from the di-baryon case, where the tadpole of a single D2 was expected to be k.

We are now interested in adding worldvolume flux to this configuration. According to the observation in the previous section, there is a background B_2 field given by (5.1) [193]. It is then convenient to split the worldvolume flux as in the previous section $F_T = F + J$, with

$$F = \mathcal{N} J\,. \tag{5.13}$$

We should stress that the D2-brane, wrapping a spin manifold, does not capture the Freed Witten anomaly, and as such, the quantization condition for F_T [213] is

$$\frac{1}{2\pi} \int F_T = \frac{1}{2}\,(\mathcal{N} + 1) \in \mathbb{Z} \tag{5.14}$$

Therefore, the case with minimal magnetic flux corresponds to $\mathcal{N} = -1$.

The D2-brane DBI action then reads

$$S_{DBI} = -\frac{\pi\,T_2}{g_s}\,\sqrt{L^4 + (2\pi\mathcal{N})^2}\,\int dt\,\frac{2\rho}{L}\,. \tag{5.15}$$

Besides, there is D0-brane charge induced in the configuration, since

$$S_{CS} = 2\pi\, T_2 \int_{\mathbb{R}\times\mathbb{P}^1} C_1 \wedge F = \frac{\mathcal{N}}{2}\, T_0 \int_{\mathbb{R}} C_1\,. \tag{5.16}$$

Note that even in the case of minimal magnetic flux, $\mathcal{N} = -1$, there is a non-zero D0-brane charge induced by the shifted B_2.

In this case the charge of the worldvolume tadpole is not modified by the presence of the magnetic flux.

In the next section we will study the dynamics of the configuration formed by the D2-brane plus the k fundamental strings, and show that adding magnetic flux allows to construct more general 't Hooft monopole configurations with charge. This charge will have to lie however below some upper bound for the configuration to be stable in the AdS direction.

In view of (5.16) we see that our system is actually formed by a $D2 - D0$ bound state, which hints to a non-supersymmetric configuration.[3] Thus, one might worry about the stability of the configuration with flux. Nevertheless, since both the cycle wrapped by the brane and the worldvolume gauge field are topologically non-trivial, we expect the configuration to be stable, at least under small perturbations. As discussed in the previous subsection, it is implicit in our probe brane approximation that the strings are uniformly distributed over the D2 worldvolume. Grouping them together in a point would require to consider their backreaction on the D2, which would deform it into a spike, which could in turn be unstable due to the lack of SUSY. Nevertheless, as long as we restrict to the probe approximation, we expect the system to be perturbatively stable.

5.1.3 The baryon vertex

Let us finally consider the D6-brane wrapping the whole $\mathbb{P}^3$. This brane is the analogue of the baryon vertex in $AdS_5 \times S^5$ [163]. In the absence of worldvolume magnetic flux this brane captures the F_6 background flux, and it requires the addition of $q = N$ fundamental strings:

$$S_{CS} = 2\pi\, T_6 \int_{R\times\mathbb{P}^3} P[F_6] \wedge A = N\, T_{F1} \int dt A_t\,. \tag{5.17}$$

Note however that once the shift in (5.1) has been taken into account, the above expression is incomplete, since there are extra contributions to the F1 charge coming from the coupling $\int F_2 \wedge B_2 \wedge B_2$. Nevertheless, once the higher curvature corrections are taken into account,

[3] For this reason conjecturing a dual operator seems much harder.

they cancel out so that the correct expression is actually (5.17). In the case at hand the relevant term in (5.7) is

$$\Delta S = \frac{3}{2}(2\pi)^5\, T_6 \int C_1 \wedge F \wedge \frac{1}{48}\,(\hat{p}_1(N) - \hat{p}_1(T)) \tag{5.18}$$

As shown in [193] this term contributes to the D6-brane action inducing extra F1 charge as

$$\Delta S = -\frac{1}{8}(2\pi)^6\, k\, T_6 \int dt\, A_t\,, \tag{5.19}$$

and this precisely cancels the B_2 contribution to (5.17).

Let us now switch on a gauge flux, $F_T = \mathcal{N} J$. Note that this represents a slight change in the conventions compared to the previous sections, where we split F_T into two pieces one cancelling B. In this case, due to the relevance of the curvature coupling in giving the tadpole of N units in the unfluxed case, it turns out to be more convenient not to do the spliting so that the argument as in [193] goes through. Since $\mathbb{P}^3$ is spin, the appropriate quantization condition is

$$\frac{1}{2\pi}\int F_T = \frac{\mathcal{N}}{2} \in \mathbb{Z} \tag{5.20}$$

The DBI action of the D6-brane becomes:

$$S_{D6} = -\frac{\pi^3\, T_6}{6\, g_s}\left(L^4 + (2\pi\,(\mathcal{N}-1))^2\right)^{3/2} \int dt\, \frac{2\,\rho}{L}\,. \tag{5.21}$$

In this case the magnetic flux modifies the number of fundamental strings that must end on the D6, since it contributes to the worldvolume tadpole through the couplings

$$S_{CS} = \frac{1}{6}(2\pi)^2\, T_6 \int_{\mathbb{R}\times\mathbb{P}^3} P[F_2] \wedge F_T \wedge \Big(2\pi F_T + 3\, P[B_2]\Big) \wedge A = k\, \frac{\mathcal{N}(\mathcal{N}-2)}{8}\, T_{F1} \int dt A_t\,. \tag{5.22}$$

Therefore $q = N + k\,\mathcal{N}(\mathcal{N}-2)/8$. Note that this is always an integer for the quantization condition (5.20). As for the D4-brane, this is the number of fundamental strings required to cancel the tadpole of each of the D2-branes that are dissolved on the D6-brane by the magnetic flux and the B_2 field, through the coupling:

$$S_{CS} = \frac{1}{2}\, T_6 \int_{\mathbb{R}\times\mathbb{P}^3} C_3 \wedge \mathcal{F} \wedge \mathcal{F} \tag{5.23}$$

In this coupling the term proportional to $\int C_3 \wedge B_2 \wedge B_2$ is precisely cancelled with the contribution of the A-roof $\int C_3 \wedge \frac{1}{48}\,(\hat{p}_1(N) - \hat{p}_1(T))$. The other two terms give

$$S_{CS} = \frac{1}{2}(2\pi)^2\, T_6 \int C_3 \wedge F_T \wedge F_T + (2\pi)\, T_6 \int C_3 \wedge F_T \wedge B_2 = \frac{\mathcal{N}(\mathcal{N}-2)}{8}\, T_2 \int C_3 \tag{5.24}$$

Note that the magnetic flux and the B_2 field also induce D0-brane charge in the configuration.

We will study the dynamics of the D6-brane with magnetic flux in the next section. We will see that, similarly to the D2-brane case, adding magnetic flux allows to construct more general baryon vertex configurations in which the charge of the brane can be increased up to some maximum value. In this case, since the number of fundamental strings attached to the D6-brane depends on the magnetic flux, the bound on the magnetic flux imposes as well a bound on the number of F-strings that can end on the brane.

As in the D2 brane case, induced D0 brane charge in a D6 suggests that the system will not be supersymmetric. However, again due to its non-trivial topology, we expect the system to be perturbatively stable.

5.2 Study of the dynamics: Charge bounds

In this section we study the stability in the AdS direction of the brane configurations that we have previously discussed. We follow the calculations in [165] and [161] (see also [169] for similar results for the baryon vertex with magnetic flux in $AdS_5 \times S^5$). We show that the energy of the various configurations is inversely proportional to the distance between the quarks, as predicted by conformal invariance, and that the proportionality constant is negative, so that the configurations are stable against perturbations in ρ. As expected, we find the same non-analytical behavior with the square root of the 't Hooft coupling that was found for the baryon vertex in $AdS_5 \times S^5$ [165] and the $q\bar{q}$ system [161, 162]. This represents a non-trivial prediction of AdS/CFT for the strongly coupled CS-matter theory.

In order to analyze the stability in the ρ-direction we have to consider both the Dp-brane wrapped on the $\mathbb{P}^{p/2}$ and the q fundamental strings stretching between the brane and the boundary of AdS. The action is given by

$$S = S_{Dp} + S_{qF1}\,, \tag{5.25}$$

where S_{Dp} is of the form[4]

$$S_{Dp} = -Q_p \int dt\, \frac{2\rho}{L}\,, \qquad \text{with} \qquad Q_p = \frac{\pi^{p/2}\, T_p}{(\frac{p}{2})!\, g_s}(L^4 + (2\pi\mathcal{N})^2)^{p/4}\,, \tag{5.26}$$

and the action of the strings is given by

$$S_{qF_1} = -q\, T_{F1} \int dtdx \sqrt{\frac{16\rho^4}{L^4} + \rho'^2}\,, \tag{5.27}$$

[4]Note that for the D6-brane $\mathcal{N} \to \mathcal{N} - 1$ in Q_p in order to account for the B_2 field, consistently with the quantization condition (5.20). We will take due care of this shift in section 3.3 below.

where we have parameterized the worldvolume coordinates by (t,x) and the position in AdS by $\rho=\rho(x)$. Following the analysis in [165] the equations of motion come in two sets: the bulk equation of motion for the strings, and the boundary equation of motion (as we are dealing with open strings), which contains as well a term coming from the Dp-brane. One can show easily that these equations of motion are, respectively:

$$\frac{\rho^4}{\sqrt{\frac{16\rho^4}{L^4}+\rho'^2}}=c \tag{5.28}$$

with c some constant, and

$$\frac{\rho_0'}{\sqrt{\frac{16\rho_0^4}{L^4}+\rho_0'^2}}=\frac{2Q_p}{L\,q\,T_{F_1}}\,, \tag{5.29}$$

where ρ_0 is the position of the brane in the holographic direction and $\rho_0'=\rho'(\rho_0)$. As in [165, 169] it is convenient to define

$$\sqrt{1-\beta^2}=\frac{2Q_p}{L\,q\,T_{F_1}}\,, \tag{5.30}$$

where $\beta\in[0,1]$. The two equations of motion can then be combined into just

$$\frac{\rho^4}{\sqrt{\frac{16\rho^4}{L^4}+\rho'^2}}=\frac{1}{4}\,\beta\,\rho_0^2\,L^2\,. \tag{5.31}$$

Integrating the equation of motion we find that the size of the configuration is given by

$$\ell=\frac{L^2}{4\rho_0}\int_1^\infty dz\frac{\beta}{z^2\sqrt{z^4-\beta^2}}\,, \tag{5.32}$$

where $z=\rho/\rho_0$. This expression has the same form as the size of the baryon vertex in $AdS_5\times S^5$ [165][5] and the $q\bar{q}$ system [161, 162], and can also be solved in terms of hypergeometric functions. Note that the dependence on the location of the configuration, ρ_0, and on L^2 is also the same, which is again a prediction of the AdS/CFT correspondence for the strong coupling behavior of the gauge theory.

The total on-shell energy is given by

$$\begin{aligned} E &= E_{Dp}+E_{qF1}=q\,T_{F_1}\,\rho_0\Big(\frac{2Q_p}{L\,q\,T_{F_1}}+\int_1^\infty dz\frac{z^2}{\sqrt{z^4-\beta^2}}\Big) \\ &= qT_{F_1}\rho_0\Big(\sqrt{1-\beta^2}+\int_1^\infty dz\frac{z^2}{\sqrt{z^4-\beta^2}}\Big)\,. \end{aligned} \tag{5.33}$$

[5] And also that of the baryon vertex with magnetic flux constructed in [169].

The binding energy of the configuration can be obtained by subtracting the (divergent) energy of its constituents. When the Dp-brane is located at $\rho_0 = 0$ the strings stretched between 0 and ∞ become radial, and therefore correspond to free quarks. At this location the energy of the Dp vanishes. Therefore, the binding energy is given by:

$$E_{\rm bin} = q\, T_{F_1}\, \rho_0 \Big(\sqrt{1-\beta^2} + \int_1^\infty dz \Big[\frac{z^2}{\sqrt{z^4-\beta^2}} - 1 \Big] - 1 \Big)\,. \tag{5.34}$$

This expression has again the same form than the corresponding expressions in [165, 169, 161, 162].[6]

Notice that for our configurations β is a function of the magnetic flux that is dissolved on the Dp-brane, since from (5.30)

$$\beta = \sqrt{1 - \Big(\frac{2Q_p}{L\, q\, T_{F1}}\Big)^2}\,. \tag{5.35}$$

In particular, in order to find a stable configuration we must have

$$\frac{2Q_p}{L\, q\, T_{F_1}} \le 1\,. \tag{5.36}$$

This imposes a bound on the possible values of the magnetic flux, and therefore on the possible charges that can be dissolved in the Dp-brane. This situation is very similar to the one found in [169] for the baryon vertex in $AdS_5 \times S^5$ with magnetic flux. Moreover, for the di-baryon and baryon vertex configurations, for which the number of fundamental strings required to cancel the tadpole depends on the magnetic flux, there is as well a bound on the number of quarks that can form the bound state.

For the values allowed by (5.36) the binding energy per string is negative and decreases monotonically with β [7]. Therefore, the configuration is stable, becoming less and less stable as β decreases, with the binding energy reaching its maximum value at the bound, $\beta = 0$, where it vanishes. The configuration reduces then to q free radial strings stretching from ρ_0 to ∞, plus a Dp-brane located at ρ_0. Note that this configuration only exists when the magnetic flux is non-vanishing, since only then we can reach $\beta = 0$. When the Dp-brane is charged the configuration corresponding to free quarks is therefore degenerate. It can be realized either as free radial strings stretching from 0 to ∞ plus a charged Dp-brane, with the charge satisfying (5.36), located at $\rho = 0$, or as free radial strings stretching from ρ_0 to ∞ plus a Dp-brane located at ρ_0, with a charge that has to satisfy the equality in (5.36).

[6] In this case we have added the on-shell energy of the Dp-brane.

[7] Its behavior as a function of the magnetic flux depends on the specific function $\beta(\mathcal{N})$ given by (5.35) We will analyze this behavior in the next subsections for the different branes.

In this case the F1's are less energetic due to the fact that they now stretch from ρ_0 to ∞ but this is compensated by the energy of the brane at ρ_0, charged such that $\beta = 0$. Note that the location of the Dp-brane has become a moduli of the system. In both cases since the strings are radial the size of the configuration vanishes.

Note that from (5.34) and (5.32) we have that for all values of the magnetic charge

$$E_{\rm bin} = -f(\beta)\frac{(g_s N)^{2/5}}{\ell} \tag{5.37}$$

with $f(\beta) \geq 0$. Therefore $dE/d\ell \geq 0$ and the configuration is stable. The behavior of $E_{\rm bin}$ as a function of the 't Hooft coupling and the size of the configuration is the same as in $AdS_5 \times S^5$ [161, 162, 165]. As in that case the fact that it goes as $1/\ell$ is dictated by conformal invariance, while the behavior with $\sqrt{\lambda}$ is a non-trivial prediction for the non-perturbative regime of the gauge theory. Note that the same non-analytic behavior with λ is predicted for $\mathcal{N} = 4$ SYM in 3+1 dimensions and for ABJM [214, 215, 216]. In fact, perturbative calculations like those in [160, 217, 218] can explain this behavior when extrapolated to strong coupling, as inferred in [219]. Further, the exact interpolating function between the weak and strong coupling regimes for 1/6 and 1/2 BPS Wilson loops was obtained in [220], using topological strings and the localization techniques applied in [221] to ABJM theories. [8]

We have plotted in fig. 5.1 the behavior of $f(\beta)/qT_{F1}$ as a function of β. We can see that when the number of strings does not depend on β, i.e. for the 't Hooft monopole case, the configuration becomes more stable as β increases. For the di-baryon and baryon vertex configurations the number of strings changes with the magnetic flux, and therefore the stability of the configuration will vary with β in a way which depends on the specific function (5.35). We will analyze this behavior in the next subsections.

We now discuss in some more detail the dynamics of the different configurations discussed in the previous section.

5.2.1 The 't Hooft monopole

In this case

$$Q_2 = \frac{\pi\, T_2}{g_s}\sqrt{L^4 + (2\pi\mathcal{N})^2} \tag{5.38}$$

and $q = k$, so that

$$\beta = \sqrt{1 - \frac{1}{4\pi^2}\Big(1 + \frac{4\pi^2\mathcal{N}^2}{L^4}\Big)}\,. \tag{5.39}$$

[8] See also [222].

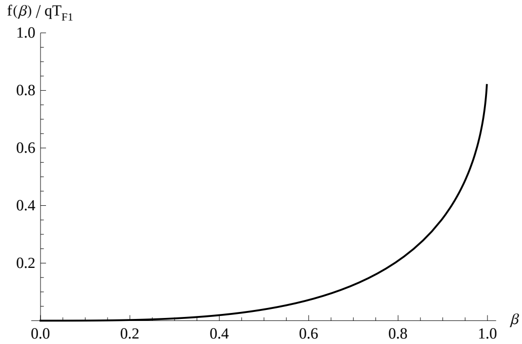


Figure 5.1: Stability of the configuration, for fixed number of strings, as a function of β.

The behavior of the binding energy as a function of $\mathcal{N}$ is shown in fig. 5.2. The minimum binding energy occurs for zero $\mathcal{N}$, for which $\beta = \sqrt{1 - \frac{1}{4\pi^2}}$, and $\beta = 0$ is reached for

$$\frac{\mathcal{N}_{\rm max}}{L^2} = \sqrt{1 - \frac{1}{4\pi^2}}\,, \tag{5.40}$$

for which the monopole is no longer stable and reduces to k radial F1's, stretching from ρ_0 to ∞, plus a spherical D2-brane with D0-charge $\frac{L^2}{2}\sqrt{1 - \frac{1}{4\pi^2}}$, located at ρ_0. As a function of the 't Hooft coupling (5.40) becomes

$$\mathcal{N}_{\rm max} = \sqrt{8\lambda\,(4\pi^2 - 1)} \tag{5.41}$$

which is exactly the same behavior that was encountered in [169] for the maximum value of the magnetic flux that could be dissolved in the baryon vertex in $AdS_5 \times S^5$. We will find this same behavior for the di-baryon and baryon vertex with flux in the next subsections. Although dynamically the origin of the bound is quite clear, pointing at an instability when the magnetic flux makes the energy of the brane too large, its interpretation from the CFT side is not clear to us. We refer to the conclusions for a brief discussion.

5.2.2 Di-baryon

In this case

$$Q_4 = \frac{\pi^2\,T_4}{2g_s}\Big(L^4 + (2\pi\mathcal{N})^2\Big) \tag{5.42}$$

and $q = k\,\mathcal{N}/2$, so that

$$\beta = \sqrt{1 - \frac{L^4}{64\pi^4\mathcal{N}^2}\Big(1 + \frac{4\pi^2\mathcal{N}^2}{L^4}\Big)^2}\,. \tag{5.43}$$

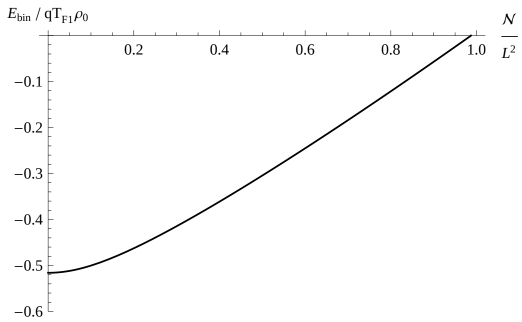


Figure 5.2: Binding energy per string of the 't Hooft monopole (in units of $T_{F1}\,\rho_0$) as a function of $\mathcal{N}/L^2$.

This function has a maximum at $\frac{\mathcal{N}}{L^2}=\frac{1}{2\pi}$, where it reaches $\beta_{max}=\sqrt{1-\frac{1}{4\pi^2}}$. For this value of the magnetic flux the binding energy per string is minimum. Note however that since the number of strings depends also on $\mathcal{N}$ this is not the value for which the configuration is maximally stable (if we define the stability in terms of the function $f(\beta)$ in (5.37)). The allowed values for the magnetic flux are those for which $\beta\in[0,\beta_{max}]$:

$$1-\sqrt{1-\frac{1}{4\pi^2}}\leq\frac{\mathcal{N}}{L^2}\leq1+\sqrt{1-\frac{1}{4\pi^2}}\,. \tag{5.44}$$

At both ends $\frac{\mathcal{N}_\pm}{L^2}=1\pm\sqrt{1-\frac{1}{4\pi^2}}$, $\beta=0$, and the configuration turns into a collection of $q=k\,\mathcal{N}_\pm/2$ free quarks plus a wrapped D4-brane. The behavior of the binding energy per string as a function of $\mathcal{N}/L^2$ is shown in fig. 5.3 (left). Since the total binding energy of the configuration depends on the number of strings, which is a function of the magnetic flux, the behavior of the binding energy is modified as shown in fig. 5.3 (right). The minimum energy occurs now for $\mathcal{N}=1.00\,L^2$. In Figure 4 we have plotted as well $f(\beta)/T_{F1}$ (see (5.37)) as a function of the magnetic flux.

As we have seen, the D4-brane with flux exhibits a very different behavior with the magnetic flux than the D2-brane[9]. The main difference is coming from the fact that now the magnetic flux induces a worldvolume tadpole in the D4-brane that was not present for $\mathcal{N}=0$, and this tadpole has to be cancelled by adding a number of F1's proportional to $\mathcal{N}$. Accordingly, the whole configuration of point-like D4-brane plus fundamental strings only exists for $\mathcal{N}\neq0$, with the allowed interval for the magnetic flux, given by (5.44), implying an allowed interval for the number of fundamental strings ending on the D4-brane:

$$2\pi\,\sqrt{2kN}\Big(1-\sqrt{1-\frac{1}{4\pi^2}}\Big)\leq q\leq2\pi\,\sqrt{2kN}\Big(1+\sqrt{1-\frac{1}{4\pi^2}}\Big)\,. \tag{5.45}$$

[9] And the D6-brane, as we will see next.

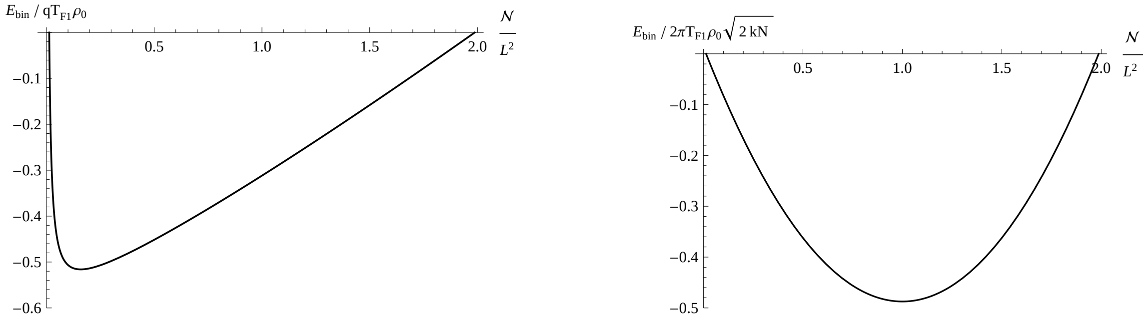


Figure 5.3: Binding energy per string (left) and total binding energy (right) of the di-baryon (in units of $T_{F1}\,\rho_0$ and $2\pi\,T_{F1}\,\rho_0\,\sqrt{2kN}$ respectively) as a function of $\mathcal{N}/L^2$.

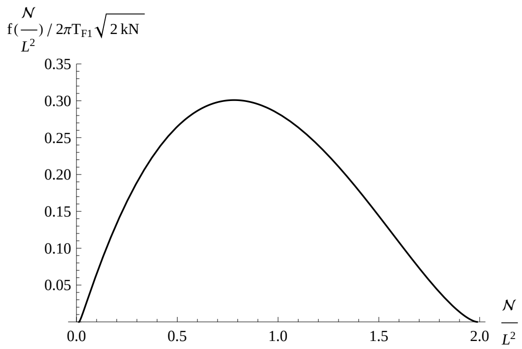


Figure 5.4: $f(\beta)$ for the di-baryon, in units of $2\pi\,T_{F1}\,\sqrt{2kN}$, as a function of the magnetic flux.

At the bounds the strings become radial and the configuration ceases to be stable.

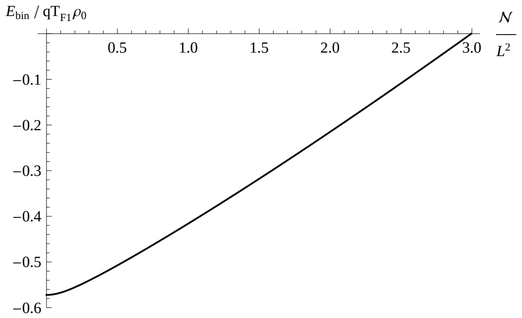
5.2.3 Baryon vertex

In this case

$$Q_6 = \frac{\pi^3\,T_6}{6g_s}\Big(L^4 + (2\pi(\mathcal{N}-1))^2\Big)^{3/2} \tag{5.46}$$

and $q = N + k\,\mathcal{N}(\mathcal{N}-2)/8$, so that

$$\beta = \sqrt{1 - \frac{1}{36\pi^2\Big(1+\frac{4\pi^2\mathcal{N}(\mathcal{N}-2)}{L^4}\Big)^2}\Big(1+\frac{4\pi^2(\mathcal{N}-1)^2}{L^4}\Big)^3}\,. \tag{5.47}$$



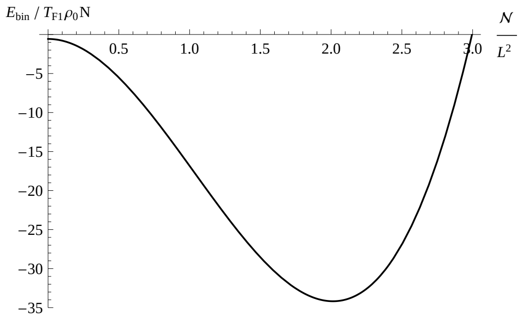


Figure 5.5: Binding energy per string (left) and total binding energy (right) of the baryon vertex (in units of $T_{F1}\,\rho_0$ and $T_{F1}\,\rho_0\,N$ respectively) as a function of $\mathcal{N}/L^2$.

This function decreases monotonically with $\mathcal{N}$, reaching its minimum value $\beta = 0$ when $\frac{\mathcal{N}}{L^2} \sim \frac{\sqrt{36\pi^2-1}}{2\pi}$. Therefore the allowed values of the magnetic flux are

$$\frac{\mathcal{N}}{L^2} \lesssim \frac{\sqrt{36\pi^2-1}}{2\pi} \tag{5.48}$$

We have plotted in fig. 5.5 (left) the binding energy per string as a function of $\mathcal{N}/L^2$. We can see that the qualitative behavior is very similar to the D2-brane case, and also to the charged baryon vertex in $AdS_5 \times S^5$ [169]. In all these examples the binding energy per string increases with the magnetic flux till it becomes zero when the strings are radial and the baryon size vanishes. Note however that in this case the tadpole induced in the worldvolume of the D6-brane depends on the magnetic flux, and therefore the number of quarks that can form the bound state depends on $\mathcal{N}$, as $q = N + k\,\mathcal{N}(\mathcal{N}-2)/8$. This modifies the behavior of the total binding energy as shown in fig. 5.5 (right). Here we can see that the minimum energy configuration occurs for $\mathcal{N}/L^2 \sim 2.01$, and that the configuration loses stability till it reduces to free radial fundamental strings at $\mathcal{N}_{max}/L^2 \sim \frac{\sqrt{36\pi^2-1}}{2\pi}$, for which $q \sim 36\pi^2 N$. The stability of the configuration as a function of the magnetic flux can be seen in fig. 5.6. The analysis in this section shows that the addition of magnetic flux to the D6-brane allows the construction of more general baryon vertex configurations in which the number of quarks can be increased up to $\sim 36\pi^2 N$. A way to construct baryons with $q < N$ number of quarks in $AdS_5 \times S^5$ was considered in [165]. In this background $q = N$ strings are needed in order to cancel the tadpole in the worldvolume of the spherical D5-brane, N being the rank of the gauge group. It is however possible to find more general baryon vertex configurations with $q < N$ quarks if $N - q$ strings stretch between ρ_0 and 0. The study of the dynamics of these configurations reveals that they are stable if the number of quarks satisfies $5N/8 \leq q \leq N$. For the minimum value, $q_{min} = 5N/8$, the strings are radial and the binding energy vanishes, exactly the same behavior that we have found at the limiting values.

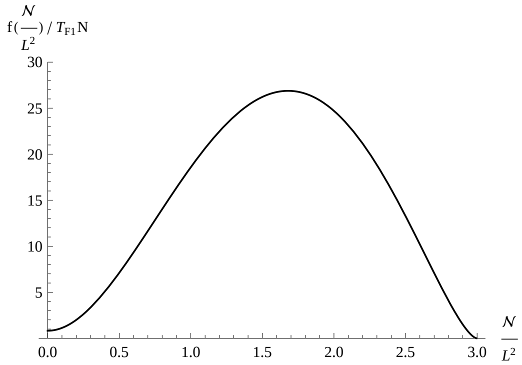


Figure 5.6: $f(\beta)$ for the baryon vertex, in units of $T_{F1}\,N$, as a function of the magnetic flux.

A similar analysis to the one commented in (4.1.1) shows that letting $N-q$ strings stretch between ρ_0 and 0 modifies the boundary equation (5.29) as

$$\frac{\rho_0'}{\sqrt{\frac{16\rho_0^4}{L^4}+\rho_0'^2}}=\frac{N}{6\pi q}\Big(1+\frac{4\pi^2(\mathcal{N}-1)^2}{L^4}\Big)^{3/2}+\frac{1}{q}\Big(N+\frac{k\mathcal{N}(\mathcal{N}-2)}{8}-q\Big)\,, \tag{5.49}$$

from which we can conclude that the number of quarks has to satisfy:

$$\frac{1}{2}(N+\frac{k\mathcal{N}(\mathcal{N}-2)}{8})(1+\sqrt{1-\beta^2})\leq q\leq N+\frac{k\mathcal{N}(\mathcal{N}-2)}{8}\,, \tag{5.50}$$

with β given by (5.47).

Therefore we have found that by combining the addition of magnetic flux and the construction in [165] it is possible to find more general baryon vertex configurations in which the number of quarks differs from N in a way that depends on the magnetic flux dissolved in the D6-brane and the number of strings that end on 0 instead of ∞. Like for all the bounds found in this study, the quarks are free for the minimum and maximum numbers allowed, where the configuration ceases to be stable.

5.3 Adding Romans mass

In this section we briefly discuss how the results of the previous sections for the 't Hooft monopole, di-baryon and baryon vertex configurations are modified by the presence of a non-zero Romans mass F_0.

It was shown in [192] that the CS-matter theory dual to a perturbation of the previous $AdS_4 \times \mathbb{P}^3$ background by a mass term should be a perturbation of ABJM with levels $k_1 + k_2 = F_0$. The simplest way to see this is to realize that a D0-brane in this background develops a tadpole through its CS coupling to the mass [29]:

$$S_{CS} = T_0 \int dt\, F_0\, A_t\,, \tag{5.51}$$

and therefore F_0 fundamental strings should end on it. One can account for these extra indices in the fundamental by modifying the level of one of the gauge groups, such that the di-monopole operator dual to the D0-brane becomes

$$O^{D0} = (\mathrm{Sym_k})_{i_1 \dots i_{k+F_0}} (\overline{\mathrm{Sym}_k})^{j_1 \dots j_k} A^{i_1}_{j_1} \dots A^{i_k}_{j_k}\,. \tag{5.52}$$

It was shown in [192] that indeed ABJM can be deformed in different ways such that the levels do not sum to zero. In all cases the deformed theory flows to a CFT, with different amounts of supersymmetries and global symmetries. The theory that is obtained from ABJM by simply changing the CS levels such that $k_1 + k_2$ is small (in the precise way shown in [192]) breaks all the supersymmetries, but flows to a CFT that respects the SO(6) R-symmetry. This is the theory that can be most simply identified as a deformation of the $\mathcal{N} = 6$ solution by a Romans mass, and the one that we will consider in this section.

The gravity dual of the $\mathcal{N} = 0$ CFT with SO(6) global symmetry discussed in [192] can be constructed as a perturbation of the $\mathcal{N} = 6$ solution, with the usual Fubini-Study metric on $\mathbb{P}^3$, by a small mass $F_0 << k, N$. In that case the F_2 and F_6 fluxes are not modified, and the F_4 flux that has to be introduced along with the mass (see [192]) can be compensated with the coupling of F_2 with a closed B_2 field.This B_2 field will be conveniently absorbed in our definition of F. Note however that it contributes to higher order in the mass to expression (5.53). Therefore we will ignore it in our analysis below. Moreover, as in [192], we will ignore the effect of the Freed-Witten anomaly. The CS coupling to the mass in the D4-brane case, given by equation (5.55) below, suggests that a fractional number of F-strings should be added to the D4-brane in order to cancel the tadpole induced by the mass and the Freed-Witten worldvolume flux. Therefore including this effect requires a more careful study, that we hope to address in a future publication.

In this massive background the D0-brane acquires a tadpole. This is however not the case for the other particle-like branes[10], since the only modification in the action in the massive $AdS_4 \times \mathbb{P}^3$ background is the coupling to the mass [29]

$$S_{CS} = T_p \int F_0 \sum_{r=0} \frac{1}{(r+1)!} (2\pi)^r\, A \wedge F^r \tag{5.53}$$

[10] If we ignore the effect of the Freed-Witten worldvolume flux, as in [192].

in the CS part.

Let us now add a magnetic flux as we did in the previous sections, $F = \mathcal{N} J$. A D2-brane wrapped on S^2 will now develop a tadpole proportional to the mass, given that in the Chern-Simons action:

$$S_{CS} = 2\pi\, T_2 \int F_0\, A \wedge dA = \frac{F_0\, \mathcal{N}}{2} \int dt A_t\,. \tag{5.54}$$

Therefore, for non-zero mass we have to add a number of F1's that is proportional to the product of the mass with the magnetic flux: $q = F_0\, \mathcal{N}/2$.

For a D4 wrapped on the $\mathbb{P}^2$ the relevant coupling is

$$S_{CS} = \frac{1}{3!}(2\pi)^2\, T_4 \int F_0\, A \wedge dA \wedge dA\,, \tag{5.55}$$

therefore for a non-vanishing magnetic flux we need $q = F_0\, \mathcal{N}^2/8$ F1's. Note that this is the number of fundamental strings required to cancel the tadpole of the D0-branes dissolved in the D4-brane in the massive case. Finally, for a D6-brane wrapped on the $\mathbb{P}^3$ the relevant coupling is

$$S_{CS} = \frac{1}{4!}(2\pi)^3\, T_6 \int F_0\, A \wedge dA \wedge dA \wedge dA \tag{5.56}$$

and the number of F1's that must be added for non-zero mass is $q = F_0\, \mathcal{N}^3/48$, which is again F_0 times the number of D0-branes dissolved in the D6-brane.

We have summarized in Table 1 the number of fundamental strings that are required in order to cancel the tadpoles originating from all the background fluxes for each type of wrapped brane.

Dp-brane	Number of F1's
D0	F_0
D2	$k + \frac{F_0 \mathcal{N}}{2}$
D4	$\frac{k\mathcal{N}}{2} + \frac{F_0 \mathcal{N}^2}{8}$
D6	$N + \frac{k\mathcal{N}^2}{8} + \frac{F_0 \mathcal{N}^3}{48}$

Table 5.1: Number of F1's that must end on each Dp-brane in the presence of mass (and magnetic flux).

Note that although $F_0 << k, N$, $\mathcal{N}$ can be sufficiently large so as to make $F_0\mathcal{N} \approx k$. This is certainly the case for the values found in (5.40), (5.44), (5.48). In the next section we study the dynamics of the particle-like brane configurations with these F1's attached.

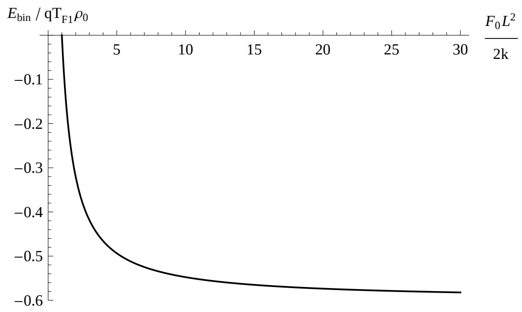


Figure 5.7: Binding energy per string of the di-monopole, in units of $T_{F1}\,\rho_0$, as a function of $\frac{F_0L^2}{2k}$.

5.3.1 Dynamics

The dynamics of the various brane configurations discussed in section 3 is modified in the presence of a non-zero mass due to the fact that the number of F1's attached to the brane depends on the mass as shown in Table 1.

Let us consider first the di-monopole, or D0-brane. Following the analysis in section 3 we have that $Q_0 = T_0/g_s$ and $q = F_0$. Therefore,

$$\beta = \sqrt{1 - \Big(\frac{2k}{F_0L^2}\Big)^2} \tag{5.57}$$

and the bound (5.36) leads to

$$F_0 \geq \frac{2k}{L^2} \tag{5.58}$$

Therefore, the configuration is stable if the mass is sufficiently large. Note that this bound is perfectly compatible, in the regime of validity of the supergravity description, with the fact that $F_0 << k, N$. We have plotted in fig. 5.7 the behavior of the binding energy per string as a function of the mass. Here we see that the configuration is maximally stable when $F_0 \to \infty$, for which $\beta_{\rm max} = 1$, and reduces to F_0 free quarks plus a D0 at the bound, when $\beta = 0$.

The D2-brane with flux turns out to be more stable in the presence of mass. In this case

$$\beta = \sqrt{1 - \frac{1}{4\pi^2(1+\frac{F_0\mathcal{N}}{2k})^2}\Big(1 + \frac{4\pi^2\mathcal{N}^2}{L^4}\Big)} \tag{5.59}$$

This function has a maximum at $\mathcal{N} = \frac{F_0L^4}{8\pi^2k}$. Since the binding energy per string decreases monotonically with β this is the value of the magnetic flux for which the binding energy

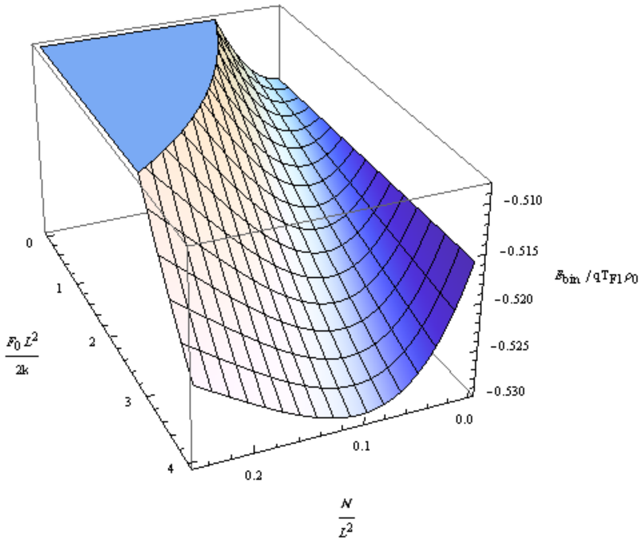


Figure 5.8: Binding energy per string of the 't Hooft monopole, in units of $T_{F1}\,\rho_0$, as a function of the magnetic flux and the mass.

(per string) is minimum.

The values of the magnetic flux for which the configuration can form a bound state depend on the mass. When F_0 satisfies the bound (5.58), required by the stability of the D0-brane, the configuration is stable for all values of the magnetic flux. On the other hand, when $F_0 < \frac{2k}{L^2}$ there is a maximum value for the magnetic flux beyond which the configuration is no longer stable, and reduces to $k+\frac{F_0\mathcal{N}}{2}$ free quarks. As in previous sections this is the value for which $\beta = 0$, which in this case is:

$$\mathcal{N}_{\max} = \frac{kL^2}{\pi(4k^2 - F_0^2 L^4)}\Big(2\pi F_0 L^2 + \sqrt{F_0^2 L^4 + 4k^2(4\pi^2 - 1)}\Big) \tag{5.60}$$

This behavior of the binding energy per string as a function of $\mathcal{N}$ and F_0 can be shown in fig. 5.8.

The D4-brane with flux in the massive background has

$$\beta = \sqrt{1 - \frac{L^4}{4\pi^4(4\mathcal{N} + \frac{F_0}{k}\mathcal{N}^2)^2}\Big(1 + \frac{4\pi^2\mathcal{N}^2}{L^4}\Big)^2} \tag{5.61}$$

This function has a maximum at $\mathcal{N} = \frac{F_0 L^4}{16\pi^2 k}(1+\sqrt{1+\frac{64\pi^2 k^2}{F_0^2 L^4}})$. For this value the configuration is maximally stable. On the other hand $\beta = 0$ is reached when $\mathcal{N} = \frac{L^2}{8\pi^2}$ for $F_0 = \frac{2k}{L^2}$, and $\mathcal{N} = \frac{2kL^2}{2k - F_0 L^2}(1 \pm \sqrt{1 - \frac{2k - F_0 L^2}{8\pi^2 k}})$ for $F_0 \neq \frac{2k}{L^2}$. For these values the configuration is no

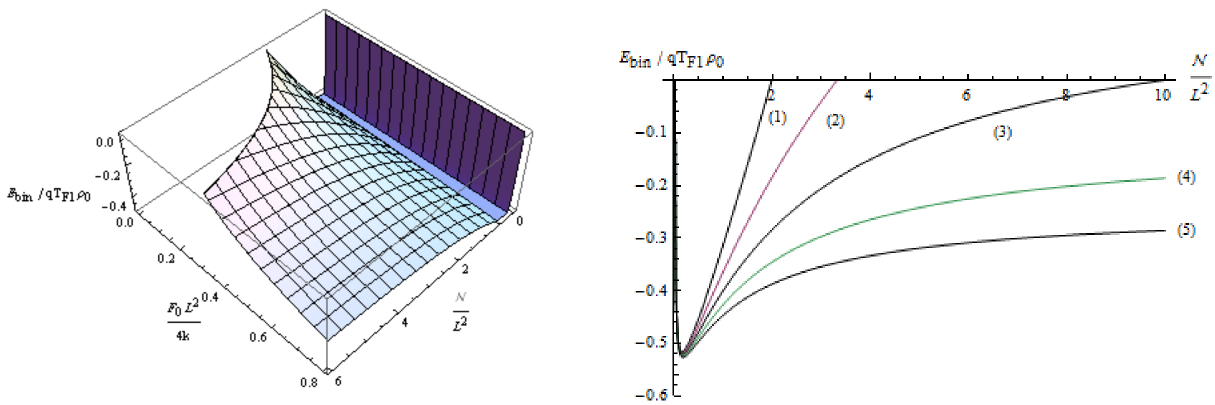


Figure 5.9: Binding energy per string for the di-baryon, in units of $T_{F1}\,\rho_0$, as a function of the magnetic flux and the mass (right: $\frac{F_0 L^2}{4k} = \{0,\ 0.2,\ 0.4,\ 0.6,\ 0.8\}$ for (1)-(5) respectively).

longer stable and reduces to $\frac{k\mathcal{N}}{2} + \frac{F_0\mathcal{N}^2}{8}$ free quarks. In summary the values of the magnetic flux for which the configuration can form a bound state must satisfy:

$$\mathcal{N} \geq \frac{L^2}{8\pi^2} \qquad \text{for} \qquad F_0 = \frac{2k}{L^2}\,, \tag{5.62}$$

$$\mathcal{N} \geq \frac{2kL^2}{F_0 L^2 - 2k}\left(\sqrt{1 + \frac{F_0 L^2 - 2k}{8\pi^2 k}} - 1\right) \qquad \text{for} \qquad F_0 > \frac{2k}{L^2}\,, \tag{5.63}$$

and

$$\frac{2kL^2}{2k - F_0 L^2}\left(1 - \sqrt{1 - \frac{2k - F_0 L^2}{8\pi^2 k}}\right) \leq \mathcal{N} \leq \frac{2kL^2}{2k - F_0 L^2}\left(1 + \sqrt{1 - \frac{2k - F_0 L^2}{8\pi^2 k}}\right) \tag{5.64}$$

for $F_0 < 2k/L^2$. Note that in all cases there is a minimum value required for the magnetic flux, consistently with the fact that also in the massive case a configuration with a D4-brane and fundamental strings attached does not exist for vanishing magnetic flux.

The behavior of the binding energy per string as a function of $\mathcal{N}$ and F_0 is shown in fig. 5.9 (left). Fig. 5.9 (right) exhibits the value of the magnetic flux for which the configuration is maximally stable for different values of the mass.

Finally, the D6-brane with flux has

$$\beta = \sqrt{1 - \frac{1}{36\pi^2\left(1 + \frac{4\pi^2\mathcal{N}^2}{L^4}(1 + \frac{F_0\mathcal{N}}{6k})\right)^2}\left(1 + \frac{4\pi^2\mathcal{N}^2}{L^4}\right)^3} \tag{5.65}$$

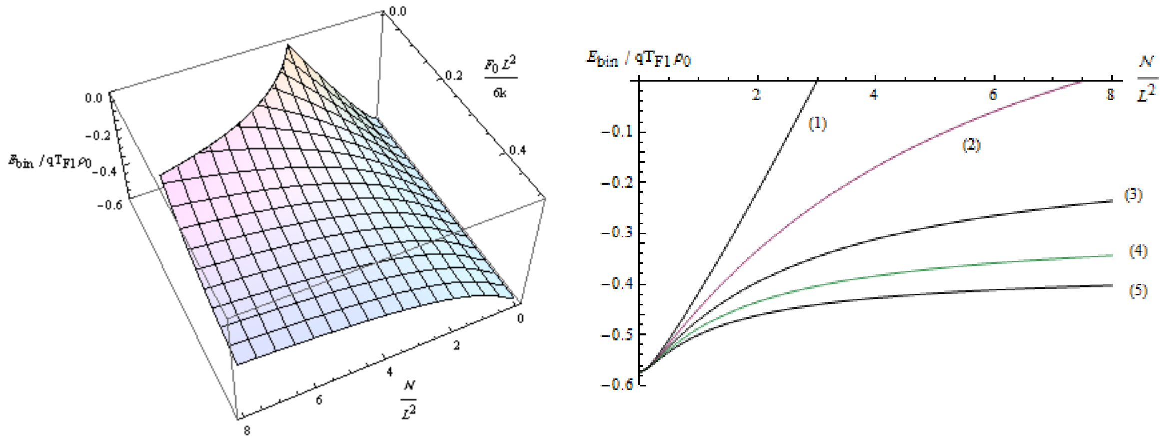


Figure 5.10: Binding energy per string for the baryon vertex, in units of $T_{F1}\,\rho_0$, as a function of the magnetic flux and the mass (right: $\frac{F_0 L^2}{6k} = \{0,\ 0.2,\ 0.4,\ 0.6,\ 0.8\}$ for (1)-(5) respectively).

This function reaches its maximum value when $\mathcal{N} = 0$ for arbitrary mass. On the other hand, $\beta = 0$ is reached for finite $\mathcal{N}$ when $F_0 < \frac{2k}{L^2}$. Beyond this value of $\mathcal{N}$ the configuration is no longer stable and reduces to $N + \frac{k\mathcal{N}^2}{8} + \frac{F_0\mathcal{N}^3}{48}$ free quarks plus a wrapped D6-brane.

The behavior of the binding energy per string as a function of $\mathcal{N}$ and F_0 is shown in fig. 5.10 (left). Fig. 5.10 (right) exhibits more clearly the behavior of the binding energy with the magnetic flux for various values of F_0.

5.4 Conclusions

In this study we have analyzed various configurations of particle-like branes in ABJM, focusing on the study of their dynamics. This study has revealed that new and more general monopole, di-baryon and baryon vertex configurations can be constructed if the particle-like branes carry lower dimensional brane charges.

We have seen that a new di-baryon configuration with external quarks can be constructed out of the D4-brane wrapped on the $\mathbb{P}^2 \subset \mathbb{P}^3$. In the presence of a non-trivial magnetic flux $F = \mathcal{N} J$, with J the Kähler form of the $\mathbb{P}^3$, this brane develops a tadpole that has to be cancelled with $k\mathcal{N}/2$ fundamental strings. The study of the dynamics of the D4-brane plus the $k\mathcal{N}/2$ F-strings reveals that the configuration is stable for $1 - \sqrt{1 - \frac{1}{4\pi^2}} \leq \frac{\mathcal{N}}{L^2} \leq 1 + \sqrt{1 - \frac{1}{4\pi^2}}$. Dynamically, the upper bound arises because if the energy of the D4-brane with flux is too large the F-strings cannot form a bound state with it. For this value the strings become radial, and the configuration reduces to free $k\mathcal{N}_{\rm max}/2$ quarks plus the charged D4-brane. We have found as well a minimum value for the magnetic flux, that has to do with the fact that if the magnetic flux is too small the number of F-strings ending on the D4-brane is not enough to form a bound state. The existence of this lower bound was expected in this case given that the whole configuration of D4-brane with fundamental strings attached can only exist in the presence of flux. When this value is reached the configuration reduces to free $k\mathcal{N}_{\rm min}/2$ quarks plus a D4-brane. It is perhaps significant that the value of the magnetic flux for which the configuration is maximally stable is that for which the (off-shell) energy of the $\mathcal{N}^2/8$ D0-branes dissolved in the D4-brane equals the (off-shell) energy of the D4-brane. This seems to point at some kind of degeneracy for the ground state. It would be interesting to find an explanation for this phenomenon.

The D2 and D6-brane (monopole and baryon vertex) configurations exist already for vanishing magnetic flux. Consistently, no minimum value is found in the study of their dynamics. In these cases the effect of the magnetic flux is to allow the construction of more general monopole and baryon vertex configurations. The simplest case is the D2-brane, for which the charge of the tadpole is not modified by the magnetic flux and the number of attached F-strings is still k. We have seen that the configuration formed by the bound state D2-D0 plus the k F-strings is stable for $\mathcal{N}/L^2 \leq \sqrt{1-\frac{1}{4\pi^2}}$, reducing to k free quarks plus a D2-brane with $\frac{L^2}{2}\sqrt{1-\frac{1}{4\pi^2}}$ D0-brane charge when the upper bound is reached. The D6-brane in turn is the analogue of the baryon vertex in $AdS_5\times S^5$ [163]. The generalization of the later to include a non-vanishing magnetic flux was studied in [169]. In that reference it was found that the magnetic flux had to be bounded from above in order to find a stable configuration, like for the D2 and D6 branes considered in this study. For the D6-brane the number of F-strings depends as well on the magnetic flux, but this fact does not modify substantially its dynamics.

As we have mentioned, all the configurations that we have considered reduce to free quarks when the magnetic flux reaches the highest possible value (also the lowest for the D4-brane). For this value the brane can be located at an arbitrary position in AdS, with the free radial strings stretching from there to ∞. This is different from the free quark configuration of [165], where the D5-brane is located at $\rho_0 = 0$, where it has zero-energy, and the radial strings stretch from 0 to ∞. For the maximum (and minimum, if applicable) value of the magnetic flux the D-brane is located at an arbitrary ρ_0, where it has some energy which is compensated by the lower energy of the strings stretching between ρ_0 and ∞. In the presence of magnetic flux the location of the Dp-brane has therefore become a moduli of the system.

We have already stressed that in the probe brane approximation used in this study all supersymmetries are broken. However, in analogy with the baryon vertex construction in $AdS_5 \times S^5$ we expect that, at least when the charged branes are supersymmetric, some supersymmetries will be preserved when the strings join the brane at the same point. In this case we would have to consider the full DBI problem and look for spiky solutions [223, 224]. The description of the baryonic brane in $AdS_5 \times S^5$ in terms of a single D5-brane developing a spike was done in [225]. This configuration is BPS, and this is reflected in the fact that its binding energy is zero. An attempt to find similar spiky solutions in $AdS_4 \times \mathbb{P}^3$ has been made recently in [203], with rather negative results even for the D6-brane with zero magnetic flux, which should be analogous to [225]. We hope that some spiky configurations can still be found in this background by relaxing some of the ansatze taken in [203]. We will report on these issues in a future publication.

It is significant that for all the configurations that we have discussed the binding energy is non-analytic in the 't Hooft coupling, with this non-analyticity being of the precise form of a square-root branch cut, like in $AdS_5 \times S^5$. This hints at some kind of universal behavior based on the conformal symmetry of the gauge theory.

An important question that remains open is what are the field theory realizations of the Dp-branes with charge that we have considered. Since we do not expect that the D2 and D6 brane configurations are supersymmetric it is hard to have an intuition about the interpretation of the new charges in the field theory side. It is interesting to note that the number of extra fundamental strings required to cancel the worldvolume tadpole is that required to cancel the tadpole on the dissolved D2 branes. This might suggest that the dual operators are doped versions of the original ones with an operator representing the D2 branes. It is hard to be more precise, in particular due to the expected lack of SUSY. However, for the D4-brane with D0-charge one can expect that a supersymmetric spiky solution exists, in which case it makes sense to try to interpret the bounds on the magnetic flux in the gauge theory dual. In field theory language the bound (5.36) would read:

$$N + \frac{\mathcal{N}^2}{8}\, k \leq 2\pi\, n_f \sqrt{2\lambda}\,, \tag{5.66}$$

where n_f is the number of external quarks, which is a function of the magnetic flux: $n_f = k\mathcal{N}/2$, and λ is the 't Hooft coupling. Therefore, at strong 't Hooft coupling we expect a bound on the baryon charge of (generalized) di-baryon configurations with n_f external quarks. This should be related in some way to the stringy exclusion principle of [204], although we have not been able to find a direct interpretation. Note that for all branes the bound on the magnetic flux exhibits the same non-analytic behavior with λ as the binding energy, which seems to indicate that the bounds should have its origin in the conformal symmetry of the gauge theory.

Finally, the role of the B field, and its potential relation to the Abelian part of the gauge symmetry, remains to be understood. We postpone these investigations for further work.

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