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A $\mathcal{N} = 8$ action for multiple M2-branes with an arbitrary number of colors

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Abstract: We obtain a U(M) action for supermembranes with central charges in the Light Cone Gauge (LCG). The theory realizes all of the symmetries and constraints of the supermembrane together with the invariance under a U(M) gauge group with M arbitrary. We do not impose conformal symmetry on the theory since it is not a symmetry of the abelian supermembrane. In distinction, AdS/CFT works with a conformal extension of multiple M2-branes in the low energy approximation. Our theory with the star-product is a full-fledged description of multiple M2-branes minimally immersed. The worldvolume action has (LCG) $\mathcal{N} = 8$ supersymmetry and it corresponds to M parallel supermembranes minimally immersed on the target M_9xT^2 (MIM2). In order to ensure the invariance under the symmetries and to close the corresponding algebra, a star-product determined by the central charge condition is introduced. It is constructed with a nonconstant symplectic two-form where curvature terms are also present. The theory is in the strongly coupled gauge-gravity regime. At low energies, the theory enters in a decoupling limit and it is described by an ordinary $\mathcal{N} = 8$ SYM in the IR phase for any number of M2-branes. We analyze also other limits of the theory.

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1 Introduction

Recently there has been a impressive amount of work trying to extend an effective action for the multiple M2-branes. The original motivation was to prove Maldacena's Conjecture for M-theory [1] according to which M-theory/$AdS_4 \times S^7$ should be dual to a CFT_3 generated by the action of multiple M2-branes in the decoupling limit, that is , for a large number M of M2's. This action was postulated as a low energy description of a SU(M) formulation of the multiple action of multiple M2-branes. The low energy effective action of the multiple M2-branes was expected to correspond to the conformal fixed point of the IR limit of a SYM theory defined in 3D since M-theory is strongly coupled.

In distinction with this line of research, we have a completely different goal. We are going to describe a full-fledged nonabelian extension of the supermembrane action. This action will not be conformal since we are Not interested in characterizing a conformal field theory through AdS_4/CFT_3 correspondance and we will not impose conformality on the theory. In addition we will work in the opposite regime where gauge and gravity sectors are strongly correlated. Due to thes differences our theory should not be expected to be similar to those a la BLG theory or ABJ/M nor in the field content, nor in the symmetries or the gauge interactions. To achieve our aim we will restrict ourselves to a particular sector of the supermembrane.

The supermembrane with a topological restriction associated to an irreducible winding has been shown to have very interesting properties: discreteness of the supersymmetric spectrum [2–4], spontaneous breaking of supersymmetry, stabilization of most of the moduli [5], a spectrum containing dyonic strings plus pure supermembrane excitations [6], formulation on a G2 manifold [7]. This restriction can be seen at algebraic level as a central charge condition on the 11D supersymmetric algebra and geometrically as a condition of being minimally immersed into the target space [8], so from now on, we will denote it as MIM2. The goal of this paper is to consistently extend this action of a single MIM2 to a theory of interacting parallel M2-branes minimally immersed (MIM2's) preserving all of the symmetries of the theory: supersymmetry and invariance under area preserving diffeomorphisms. The theory is not conformal invariant. In the extension, the gauge and gravity sectors are strongly correlated. It corresponds to have a M-theory dual of the Non-Abelian Born-Infeld action describing a bundle of multiple D2-D0 branes,in the same way that a unique supermembrane is the M-theory dual of a D2-brane, so we work in the high energy approximation. When the energy scale is low, the theory decouples and it is effectively described by a $\mathcal{N}=8$ SYM in the IR phase. As the energy scale raises the YM coupling constant becomes weaker and at some point, oscillations modes of the pure supermembrane appear and the theory enters in the strong correlated gauge-gravity sector.

This is completely different to the AdS_4/CFT_3 analysis. In that approach, for the 11D supermembrane it proved to be necessary to introduce non-dynamical gauge fields via a Chern Simons term to avoid breaking the matching of the bosonic and fermionic degrees of freedom. The first attempt was due to Schwarz who developed $N=1$ and $N=2$ cases but could not find a N=8 susy action for supermembranes [9]. The symmetries imposed on the action were the $\mathcal{N}=8$ worldvolume superconformal action and SU(N) symmetry. [10, 11] and independently [12] were the first to obtain a realization of this algebra by imposing fields to be evaluated on a three algebra with positive inner metric, (a particular case of Fillipov algebras).[1] This three-algebra can only be realized in terms of a unique finite dimensional gauge group SO(4) for an inner positive metric, [18] with a twisted Chern-Simons terms, see also [19]. This 3-algebra can also be re-expressed as the tensor product of two $\mathrm{SU}(2)\times\mathrm{SU}(2)$ gauge groups associated each one to a different Chern-Simons term [20]. Consistency checks of BLG in the funnel were indicated in [21] as well as other properties of Lorentzian 3-algebras. To realize this duality in the decoupling limit it is necessary to obtain a large number of supermembranes for an arbitrary number of M2-branes.In order to improve this situation and generalize it for general SU(M) gauge groups, different avenues have been followed: The most sucesfull has been to formulate a Chern-Simons-matter theory $\mathcal{N}=6$ by [22] in which they are able to generalize the theory to an arbitrary SU(N) and recover also BLG theory for the case of $N=2$. The ABJM,

[1] Interesting works on these algebras in relation with the supermembrane theory were obtained long time ago in [13], and it was applied to the M5 in [14]. In those case instead of Fillipov algebras Nambu-Poisson algebras are needed and there are important subtleties concerning its quantization in odd dimensions. See for example [15, 16] and [17] for a cubic matrix description.

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or at least a sector of it, can be also recover from the 3-algebra formulation by relaxing the condition of total antisymmetry of the structure constants [23] In a serie of papers it has also been explored the possibility of obtaining $\mathcal{N} = 8$ models with arbitrary gauge groups by relaxing the positivity condition of the internal metric [24–26]. These models have successfully obtained generic SU(N) gauge groups but to the price of lack of unitarity because of the presence of ghosts. Ghost-free actions have been formulated [27] and has been shown to exactly correspond to a reformulation of a $N = 8$ Super Yang Mills in (2+1)D and not to its IR limit [28]. The relation between multiple M2-branes and D2-branes has been also analyzed in, [29–32]. A non-linear realization of Lorentzian algebras has been proposed in [33]. Massive deformations as for example, [34–36] or [37] for topological twisting have also been considered. These superconformal models can also be obtained by taking the conformal limit of gauge supergravities in 3D [38]. This seems to indicate that the information of all of these theories could be contained on supergravity.[2] This is in some sense surprising for a description that is intended to describe the quantum formulation of multiple M2-branes or even the infrared physics of a Yang-Mills theory. There has been recent advances focused in models with less number of supersymmetries, for example, [39].

We would like again to emphasize that in what follows, we consider, a non abelian extension of the full-fledged theory describing multiple M2-branes in the L.C.G. without imnposing the conformal symmetry not present in the original theory of the M2-brane.

The paper is organized in the following way: In section 2 we make a short summary of the formulation and main properties of the supermembrane minimally immersed. On section 3 we introduce a non abelian extension of the MIM2 that allows to consider different limits, a matrix model regularization with a finite and arbitrary number of colors as well as a condensate of M2's in the large N matricial limit. In order to have a non-abelian formulation in 2+1D respecting all of the symmetries (in particular invariance under diffeomorphisms preserving the area) of the former theory for an arbitrary number of colors a noncommutative star-product has to be included to close the algebra. This is explained in section 5. This noncommutative star product differs from the Seiberg-Witten map since the noncommutative parameter is non constant on the spatial variables. In section 6 we analize its supersymmetry and we show that it has $\mathcal{N} = 8$ supersymmetries. To conclude we finally present our results and main properties, emphasizing its potential phenomenological interest.

2 $D = 11$ supermembrane with central charges on a $M_9 \times T^2$ target manifold

In this section we will make a self-contained summary of the construction of the minimally immersed M2-brane (MIM2). The hamiltonian of the $D = 11$ Supermembrane [40] may be defined in terms of maps X^M, $M = 0, \ldots, 10$, from a base manifold $R \times \Sigma$, where Σ is a Riemann surface of genus g onto a target manifold which we will assume to be 11D Minkowski.

[2] We thank H. Nicolai for comments to this respect.

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The canonical reduced hamiltonian to the light-cone gauge has the expression [41]

$$\mathcal{H} = \int_\Sigma d\sigma^2 \sqrt{W} \left(\frac{1}{2} \left(\frac{P_M}{\sqrt{W}} \right)^2 + \frac{1}{4}\{X^M, X^N\}^2 + \overline{\Psi}\Gamma_- \Gamma_M \{X^M, \Psi\} \right) \tag{2.1}$$

subject to the constraints

$$\phi_1 := d \left(\frac{P_M}{\sqrt{W}} dX^M + \overline{\Psi}\Gamma_- d\Psi \right) = 0 \tag{2.2}$$

and

$$\phi_2 := \oint_{C_s} \left(\frac{P_M}{\sqrt{W}} dX^M + \overline{\Psi}\Gamma_- d\Psi \right) = 0, \tag{2.3}$$

where the range of M is now $M = 1, \dots, 9$ corresponding to the transverse coordinates in the light-cone gauge, C_s, $s = 1, \dots, 2g$ is a basis of 1-dimensional homology on Σ,

$$\{X^M, X^N\} = \frac{\epsilon^{ab}}{\sqrt{W(\sigma)}} \partial_a X^M \partial_b X^N. \tag{2.4}$$

$a, b = 1, 2$ and σ^a are local coordinates over Σ. $W(\sigma)$ is a scalar density introduced in the light-cone gauge fixing procedure. ϕ_1 and ϕ_2 are generators of area preserving diffeomorphisms, see [42]. That is

$$\sigma \to \sigma' \quad \to \quad W'(\sigma) = W(\sigma).$$

When the target manifold is simply connected dX^M are exact one-forms.

The spectral properties of (2.1) were obtained in the context of a SU(N) regularized model [41] and it was shown to have continuous spectrum from $[0, \infty)$.

This property of the theory relies on two basic facts: supersymmetry and the presence of classical singular configurations, string-like spikes, which may appear or disappear without changing the energy of the model but may change the topology of the world-volume. Under compactification of the target manifold generically the same basic properties are also present and consequently the spectrum should be also continuous [43]. In what follows we will impose a topological restriction on the configuration space. It characterizes a $D = 11$ supermembrane with non-trivial central charges generated by the wrapping on the compact sector of the target space [2, 4, 44, 45]. We will consider in this paper the case $g = 1$ Riemann surface as a base manifold Σ on a M_9xT^2 target space. The configuration maps satisfy:

$$\oint_{c_s} dX^r = 2\pi L_s^r R^r \quad r, s = 1, 2. \tag{2.5}$$

$$\oint_{c_s} dX^m = 0 \qquad m = 3, \dots, 9 \tag{2.6}$$

where L_s^r are integers and $R^r, r = 1, 2$ are the radius of T^2. This conditions ensure that we are mapping Σ onto a T^2 sector of the target manifold.

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We now impose the central charge condition

$$I^{rs} \equiv \int_{\Sigma} dX^r \wedge dX^s = (2\pi R_1 R_2) n \epsilon^{rs} \tag{2.7}$$

where ω^{rs} is a symplectic matrix on the T^2 sector of the target and $n = det L^r_i$ represents the irreducible winding.

The topological condition (2.7) does not change the field equations of the hamiltonian (2.1). In fact, any variation of I^{rs} under a change δX^r, single valued over Σ, is identically zero. In addition to the field equations obtained from (2.1), the classical configurations must satisfy the condition (2.7). It is only a topological restriction on the original set of classical solutions of the field equations. In the quantum theory the space of physical configurations is also restricted by the condition (2.7). The geometrical interpretation of this condition has been discussed in previous work [46, 47]. We noticed that (2.7) only restricts the values of L^r_s, which are already integral numbers from (2.5).

We consider now the most general map satisfying condition (2.7). A closed one-forms dX^r may be decomposed into the harmonic plus exact parts:

$$dX^r = M^r_s d\widehat{X}^s + dA^r \tag{2.8}$$

where $d\widehat{X}^s$, $s = 1, 2$ is a basis of harmonic one-forms over Σ and dA^r are exact one-forms. We may normalize it by choosing a canonical basis of homology and imposing

$$\oint_{C_s} d\widehat{X}^r = \delta^r_s. \tag{2.9}$$

We have now considered a Riemann surface with a class of equivalent canonical basis. Condition (2.5) determines

$$M^r_s = 2\pi R^r L^r_s, \tag{2.10}$$

we rewrite $L^r_s = l_r S^r_s$ and $l_1.l_2 = n$. We now impose the condition (2.7) and obtain

$$S^r_t \omega^{tu} S^s_u = \omega^{rs}, \tag{2.11}$$

that is, $S \in \mathrm{Sp}(2, \mathbb{Z})$. This is the most general map satisfying (2.7). See [6] for details, in particular for $n > 1$.

The natural choice for $\sqrt{W(\sigma)}$ in this geometrical setting is to consider it as the density obtained from the pull-back of the Khäler two-form on T^2. We then define

$$\sqrt{W(\sigma)} = \frac{1}{2}\partial_a \widehat{X}^r \partial_b \widehat{X}^s \omega_{rs}. \tag{2.12}$$

$\sqrt{W(\sigma)}$ is then invariant under the change

$$d\widehat{X}^r \to S^r_s d\widehat{X}^s, \quad S \in \mathrm{Sp}(2, \mathbb{Z}) \tag{2.13}$$

But this is just the change on the canonical basis of harmonics one-forms when a biholomorphic map in Σ is performed changing the canonical basis of homology. That is, the biholomorphic (and hence diffeomorphic) map associated to the modular transformation

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on a Teichmüller space. We thus conclude that the theory is invariant not only under the diffeomorphisms generated by ϕ_1 and ϕ_2, homotopic to the identity, but also under the diffeomorphisms, biholomorphic maps, changing the canonical basis of homology by a modular transformation.

Having identified the modular invariance of the theory we may go back to the general expression of dX^r, we may always consider a canonical basis such that

$$dX^r = 2\pi l^r R^r d\widehat{X^r} + dA^r. \tag{2.14}$$

the corresponding degrees of freedom are described exactly by the single-valued fields A^r. After replacing this expression in the hamiltonian (2.1) we obtain,

$$\begin{aligned} H = &\int_\Sigma \sqrt{W} d\sigma^1 \wedge d\sigma^2 \Bigg[\frac{1}{2}\left(\frac{P_m}{\sqrt{W}}\right)^2 + \frac{1}{2}\left(\frac{\Pi^r}{\sqrt{W}}\right)^2 + \frac{1}{4}\{X^m, X^n\}^2 + \frac{1}{2}(\mathcal{D}_r X^m)^2 + \frac{1}{4}(\mathcal{F}_{rs})^2 \\ &+(n^2 \mathrm{Area}^2_{T^2}) + \int_\Sigma \sqrt{W}\Lambda\left(\mathcal{D}_r\left(\frac{\Pi_r}{\sqrt{W}}\right) + \left\{X^m, \frac{P_m}{\sqrt{W}}\right\}\right)\Bigg] \\ &+ \int_\Sigma \sqrt{W}[-\overline{\Psi}\Gamma_-\Gamma_r\mathcal{D}_r\Psi - \overline{\Psi}\Gamma_-\Gamma_m\{X^m, \Psi\} - \Lambda\{\overline{\Psi}\Gamma_-, \Psi\}] \end{aligned} \tag{2.15}$$

where $\mathcal{D}_r X^m = D_r X^m + \{A_r, X^m\}$, $\mathcal{F}_{rs} = D_r A_s - D_s A_r + \{A_r, A_s\}$, $D_r = 2\pi l_r R_r \frac{\epsilon^{ab}}{\sqrt{W}}\partial_a \widehat{X}^r \partial_b$ and P_m and Π_r are the conjugate momenta to X^m and A_r respectively. $\mathcal{D}_r$ and $\mathcal{F}_{rs}$ are the covariant derivative and curvature of a symplectic noncommutative theory [45, 46], constructed from the symplectic structure $\frac{\epsilon^{ab}}{\sqrt{W}}$ introduced by the central charge. The last term represents its supersymmetric extension in terms of Majorana spinors. The physical degrees of the theory are the X^m, A_r, Ψ_α they are single valued fields on Σ.

2.1 Quantum supersymmetric analysis of a single MIM2

We are going to summarize the spectral properties of the above hamiltonian. The bosonic potential of the (2.15) satisfies the following inequality [4] (in a particular gauge condition)

$$\begin{aligned} &\int_\Sigma \sqrt{W} d\sigma^1 \wedge d\sigma^2 \left[\frac{1}{4}\{X^m, X^n\}^2 + \frac{1}{2}(\mathcal{D}_r X^m)^2 + \frac{1}{4}(\mathcal{F}_{rs})^2\right] \\ &\geq \int_\Sigma \sqrt{W} d\sigma^1 \wedge d\sigma^2 \left[\frac{1}{2}(\mathcal{D}_r X^m)^2 + (\mathcal{D}_r A_s)^2\right] \end{aligned}$$

The right hand member under regularization describes a harmonic oscillator potential. In particular, any finite dimensional truncation of the original infinite dimensional theory satisfies the above inequality. We consider regularizations satisfying the above inequality. We denote the regularized hamiltonian of the supermembrane with the topological restriction by H, its bosonic part H_b and its fermionic potential V_f, then

$$H = H_b + V_f. \tag{2.16}$$

We can define rigorously the domain of H_b by means of Friederichs extension techniques. In this domain H_b is self adjoint and it has a complete set of eigenfunctions with eigenvalues accumulating at infinity. The operator multiplication by V_f is relatively bounded with respect to H_b. Consequently using Kato perturbation theory it can be shown that H is self-adjoint if we choose

$$DomH = DomH_b. \tag{2.17}$$

In [2] it was shown that H possesses a complete set of eigenfunctions and its spectrum is discrete, with finite multiplicity and with only an accumulation point at infinity. An independent proof was obtained in [3] using the spectral theorem and theorem 2 of that paper. In section 5 of [3] a rigorous proof of the Feynman formula for the Hamiltonian of the supermembrane was obtained. In distinction, the hamiltonian of the supermembrane, without the topological restriction, although it is positive, its fermionic potential is not bounded from below and it is not a relative perturbation of the bosonic hamiltonian. The use of the Lie product theorem in order to obtain the Feynman path integral is then not justified. It is not known and completely unclear whether a Feynman path integral formula exists for this case. In [4] it was proved that the theory of the supermembrane with central charges, corresponds to a nonperturbative quantization of a symplectic Super Yang-Mills in a confined phase and the theory possesses a mass gap.

In [7] we constructed of the supermembrane with the topological restriction on an orbifold with G_2 structure that can be ultimately deformed to lead to a true G2 manifold. All the discussion of the symmetries on the Hamiltonian was performed directly in the Feynman path integral, at the quantum level, then valid by virtue of our previous proofs.

3 Lessons from naive non-abelian extensions of the MIM2: some interesting limits

In this section we show a first attempt to obtain a non abelian extension of the MIM2-brane. It requires to obtain a U(M) or SU(M) formulation of the MIM2 theory, for an arbitrary number of colors M. We will see that naive extensions are unable to achieve it. Along this section we will characterize the compatibility problem between the non abelian gauge group and the infinite group of diffeomorphisms preserving the area. This will give us a better understanding on how this problem can be overcome, as it is shown in section 5. where a truly non abelian extension can be found. The cases contained in this section correspond to particular limits of the general construction of section 5. Let us introduce first some preliminary definitions that will become of utility along the discussion.

We will denote T_A, $A = (a_1, a_2)$, $a_1, a_2 = -(N-1), \ldots, (N-1)$, $(m, n) \neq (0, 0)$, the generators of the Weyl-Heisenberg group. They satisfy

$$T_A^\dagger = T_{-A} \tag{3.1}$$

$$trT_A = 0 \tag{3.2}$$

$$T_A T_B = N e^{\frac{i\pi(B\wedge A)}{N}} T_{A+B} \tag{3.3}$$

$$T_{(a_1+N, a_2)} = e^{i\pi a_2} T_{(a_1, a_2)} \tag{3.4}$$

$$T_{(a_1, a_2+N)} = e^{i\pi a_1} T_{(a_1, a_2)} \tag{3.5}$$

The algebra $su(N)$ may be realized in terms of T_A. The generators of $su(N)$ may be expressed as $i(T_A + T_A^\dagger), (T_A - T_A^\dagger)$. A real scalar field X with values on $su(N)$ may be expanded as

$$X = X^A T_A = \frac{-i}{2}(X^A + \overline{X}^A) i(T_A + T_A^\dagger) + \frac{1}{2}(X^A - \overline{X}^A)(T_A - T_A^\dagger), \tag{3.6}$$

where $\overline{X}^A$ is the complex conjugate of X^A.

The generators of $su(NM)$ may be realized in terms of $T_A \otimes H_b$, $T_A \otimes \mathbb{I}_M$, $\mathbb{I}_N \otimes H_b$, where T and H are the Weyl-Heisenberg generators associated to $su(N)$ and $su(M)$ respectively. We associate to each member of the above basis the kronecker product of the corresponding matrices. That is, to $H_a \otimes H_b$ the krocnecker product of the matrices T_A and H_b. The bracket of the elements of the basis is the corresponding anticommutator of the matrices. With these definitions $T_A \otimes H_b$, $T_A \otimes \mathbb{I}_M$, $\mathbb{I}_N \otimes H_b$ are the generators of $su(NM)$. $T_A \otimes \mathbb{I}_M$ and $\mathbb{I}_N \otimes H_b$ are the generators of the algebra of the direct product group $\mathrm{SU}(N) \times \mathrm{SU}(M)$, a subalgebra of $su(NM)$.

We have,

$$[T_A, T_B] = f_{AB}^C T_C, \quad f_{AB}^C = 2iN sen\left(\frac{(B \wedge A)\pi}{N}\right) \delta_{A+B}^C$$
$$\{T_A, T_B\} = d_{AB}^C T_C, \quad d_{AB}^C = 2N cos\left(\frac{(B \wedge A)\pi}{N}\right) \delta_{A+B}^C$$

and

$$[T_A \otimes H_a, T_B \otimes H_b] = (f_{AB}^C d_{ab}^c + d_{AB}^C f_{ab}^c) T_C \otimes H_c \tag{3.7}$$

We can extend the range of the index A or a, but not both together, to include $(0,0)$. The corresponding matrix is then defined as usual

$$T_{(0,0)} = N\mathbb{I}_N \quad or \quad H_{(0,0)} = M\mathbb{I}_M. \tag{3.8}$$

The commutation relation (3.7) is then valid for all generators $T_A \otimes H_b$, $T_A \otimes H_{(0,0)}$, $T_{(0,0)} \otimes H_a$.

We will denote

$$\mathbf{F}_{\mathbb{A}\mathbb{B}}^{\mathbb{C}} \equiv f_{AB}^C d_{ab}^c + d_{AB}^C f_{ab}^c \tag{3.9}$$

where $\mathbb{A} = (A,a), \mathbb{B} = (B,b), \mathbb{C} = (C,c)$. $\mathbf{F}_{\mathbb{A}\mathbb{B}}^{\mathbb{C}}$ is totally antisymmetric. It satisfies the Jacobi Identity and has the expression

$$\mathbf{F}_{\mathbb{A}\mathbb{B}}^{\mathbb{C}} = 4iMN sin\left(\frac{(B \wedge A)\pi}{N} + \frac{(b \wedge a)\pi}{M}\right) \delta_{\mathbb{A}\mathbb{B}}^{\mathbb{C}}. \tag{3.10}$$

In the following, we intend to extend the hamiltonian of the supermembrane to include $su(M)$ valued fields, preserving the number of physical degrees of freedom (times the dimension of the internal algebra). The main point is to extend the area preserving constraint

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leaving invariant its first class property. The algebraic structure of the supermembrane hamiltonian is provided by the symplectic bracket,

$$\{X^m, P_m\} = \frac{\epsilon^{ab}}{\sqrt{W}} \partial_a X^m \partial_b P_m. \tag{3.11}$$

We consider an extension of it of the form

$$\{X^{am}, P^b_M\} d^c_{ab} + f^c_{ab} X^{ma} P^b_m \tag{3.12}$$

where f^c_{ab} and d^c_{ab} are respectively the structure constant tensor and the totally symmetric tensor of $su(M)$.

We will perform the analysis on a matrix regularized model. If we now expand the scalars on the base manifold Σ in terms of an orthonormal basis Y_A,

$$X^{ma}(\sigma^1, \sigma^2, \tau) = \sum_{A=-\infty}^{+\infty} X^{maA}(\tau) Y_A(\sigma^1, \sigma^2) \tag{3.13}$$

and define as usual

$$\{Y_A, Y_B\} = g^C_{AB} Y_C \quad Y_A Y_B = \widetilde{d}^C_{AB} Y_C \tag{3.14}$$

where g^C_{AB} is the structure constant of the algebra of APD and $\widetilde{d}_{ABC}$ is the totally symmetric tensor of the APD algebra. We have just re-written the theory in its matrix form, integrating out the spatial dependence captured on the APD structure constants as usual [41], but without regularizing it at this stage. We obtain for (3.12)

$$\widetilde{\mathbf{F}}^{\mathbb{C}}_{\mathbb{AB}} = g^C_{AB} d^c_{ab} + \widetilde{d}^C_{AB} f^c_{ab}. \tag{3.15}$$

The basis Y_A, for a compact torus Σ, may be expressed in terms of the harmonic functions $\widehat{X}^r, r = 1, 2$ of section 2, normalized by $\int_{\mathcal{C}_s} d\widehat{X}^r = 2\pi\delta^r_s$, as

$$Y_{(a_1,a_2)} = e^{i(a_1\widehat{X}^1 + a_2\widehat{X}^2)} \tag{3.16}$$

we then have

$$\frac{1}{Vol(\Sigma)} \int_\Sigma \sqrt{W} Y_{(a_1,a_2)} \overline{Y}_{(b_1,b_2)} = \delta_{(a_1,b_1)(a_2,b_2)}, \tag{3.17}$$

and the APD tensors of the torus are,

$$g^C_{AB} = (B \wedge A) \left(\frac{1}{2}\epsilon_{rs}\epsilon^{ab}\partial_a\widehat{X}^r\partial_b\widehat{X}^s\right) \delta^C_{A+B} = (B \wedge A)\delta^C_{A+B}.$$
$$\widetilde{d}^C_{AB} = \delta^C_{A+B}.$$

We can regularize the model by truncating the infinite expansion and allowing the fields to be valued in the adjoint representation of a SU(N) group, [41]. In the supermembrane theory with a compactified sector of the target space it is not possible to extend directly the regularization to the harmonic sector fields [43]. However in the minimally immersed sector of the compactified supermembrane, a well defined theory by itself, the harmonic sector is completely determined and there exists a consistent regularization of the theory [44]. The harmonic sector is related to a global symmetry $\mathrm{SL}(2,\mathbb{Z})$, realized as a diffeomorphisms not connected to the identity in the infinite dimensional theory and to the center of $\mathrm{SU}(N)$ in the regularized case.

The regularized structure constants for the $\mathrm{SU}(N)$ matrix model are the standard ones [41, 42] in terms of T_A generators with $A = 1,\ldots,N^2-1$. In order to guarantee the appropriate convergence to the original APD structure constants we re-scale d_{ABC} by a factor of $\frac{1}{N}$, and although we will do it, it is not necessary to impose this requirement to the color group since the color index is not the regularized version of a theory in the continuum.

If $B\wedge A$ remains bounded and $N\to\infty$ we have

$$lim_{N\to\infty}\frac{1}{2i}f_{AB}^{C} = g_{AB}^{C}, \quad lim_{N\to\infty}\frac{1}{2N}d_{AB}^{C} = \widetilde{d}_{AB}^{C}. \tag{3.18}$$

This limit was considered in [41–43]. The semiclassical supermembrane subject to an irreducible wrapping was first analyzed in [48]. The large N limit of the spectrum of the regularized $\mathrm{SU}(N)$ model of the semiclassical minimally immersed supermembrane was studied in [4], and it was proven that its eigenvalues $\lambda_N < E$ for a fixed energy E converge to the eigenvalues $\lambda < E$ of the semiclassical supermembrane theory, when $N\to\infty$. The boundness condition of $B\wedge A$ is ensured by the condition that only modes with energy less than E are considered. The large N limit is taken with E fixed. We consider now a regularized model with gauge group $\mathrm{SU}(NM)\supset\mathrm{SU}(N)\times\mathrm{SU}(M)$, with algebraic structure represented as in (3.9), that is

$$[X,P] = X^{\mathbb{A}}P^{\mathbb{B}}\mathbf{F}_{\mathbb{A}\mathbb{B}}^{\mathbb{C}}. \tag{3.19}$$

The first remark is that the first class constraint of the supermembrane theory becomes a first class constraint of the gauge theory. The algebra of the first class constraint is exactly the algebra of the Gauss constraint of a (0+1)Yang-Mills theory with gauge group $\mathrm{SU}(NM)$.

The constraint becomes

$$\phi^{\mathbb{B}} = \lambda_{r\mathbb{A}}^{\mathbb{B}}\Pi^{r\mathbb{A}} + A_r^{\mathbb{A}}\Pi^{r\mathbb{C}}\mathbf{F}_{\mathbb{A}\mathbb{C}}^{\mathbb{B}} + X^{m\mathbb{A}}P_m^{\mathbb{C}}\mathbf{F}_{\mathbb{A}\mathbb{C}}^{\mathbb{B}} + \overline{\Psi}^{\mathbb{A}}\Gamma_-\Psi^{\mathbb{C}}\mathbf{F}_{\mathbb{A}\mathbb{C}}^{\mathbb{B}} = 0 \tag{3.20}$$

$\lambda_{r\mathbb{A}}^{\mathbb{B}}$ is the truncated version of the corresponding APD tensor defined from D_rY_A, since it is a scalar on Σ. It may be decomposed in terms of the basis Y_A,

$$D_rY_A = \lambda_{rA}^{B}Y_B. \tag{3.21}$$

The algebra of the first class constraint is, in terms of parameters ϵ, λ,

$$[<\epsilon,\phi>,<\lambda,\phi>]_{P.B} =<[\epsilon,\lambda]\phi> \tag{3.22}$$

where $<>$ denotes integration on Σ. The constraint contains a linear term on the (0+1)D fields in a similar way as in Yang-Mills theories. This property ensures the elimination from the constraint the gauge degrees of freedom on Π^r and the corresponding one from A_r by

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an admissible gauge fixing condition. We notice that from a supermembrane on a compact base manifold there is no way to fix an angle variable, that is to identify coordinates on the base manifold with coordinates on the target-space since, angle variables are harmonic on the base manifold and there is no gauge freedom on that sector in the supermembrane. In fact, there area preserving constraints generate solely diffeomorphisms homotopic to the identity. In distinction in the minimally immersed M2-brane sector that we are considering there is an additional symmetry which may allow such identification if desired.

The algebra of the first class constraint however does not close for arbitrary values of N and M. It closes for N, M finite or both N, M infinite, and those cases are considered below. The most interesting case corresponding to N infinite (i.e recovering the continuum) with an arbitrary number of colors M does not have a closed algebra in this first construction. The modification needed to hold is done in detail in the next section.

3.1 Some interesting limits

We then have a $\mathrm{SU}(N)\times\mathrm{SU}(M)$ gauge model in $(0+1)$ dimensions, describing the correct number of degrees of freedom. We may now consider different large N,M limits to describe the continuum.

- The first case we consider is when $N = M$, $N \to \infty$, we have

$$lim_{N\to\infty}\frac{1}{4iN}\mathbf{F}^{\mathbb{C}}_{\mathbb{AB}} = \widetilde{\mathbf{F}}^{\mathbb{C}}_{\mathbb{AB}} \tag{3.23}$$

by redefining the interacting terms to eliminate the factor $2iN$ we obtain the algebraic structure of $APD \times APD$. The hamiltonian becomes

$$\begin{aligned} H &\equiv \int d\sigma^2\sqrt{W}\mathcal{H}, \qquad (3.24)\\ \mathcal{H} =& \frac{1}{2}\left(\frac{P^{ma}}{\sqrt{W}}\right)^2 + \frac{1}{2}\left(\frac{\Pi^{ra}}{\sqrt{W}}\right)^2 + \frac{1}{4}(\{X^{mb}, X^{nc}\}\widetilde{d}^{c}_{ab} + X^{mb}X^{nc}g^{c}_{ab})^2 + \frac{1}{2}(\mathcal{D}_r X^{na})^2\\ &+\frac{1}{4}(\mathcal{F}^a_{rs})^2 - \overline{\Psi}\Gamma_-\Gamma_r D_r\Psi - \overline{\Psi}^a\Gamma_-\Gamma_m[\{X^{mb},\Psi^c\}\widetilde{d}^a_{bc} - X^{mb}\Psi^c g^a_{bc}] \end{aligned}$$

and the constraint

$$\mathcal{D}_r\left(\frac{\Pi^{ra}}{\sqrt{W}}\right) + \left\{X^{mb}, \frac{P^c_m}{\sqrt{W}}\right\}\widetilde{d}^a_{bc} + X^{mb}\left(\frac{P^c_m}{\sqrt{W}}\right)g^a_{bc} - \{\overline{\Psi}^b\Gamma_-,\Psi^c\}\widetilde{d}^a_{bc} - \overline{\Psi}^b\Gamma_-\Psi^c g^a_{bc} = 0$$

it is also a first class constraint. In the above expression

$$\mathcal{D}_r\bullet^a = D_r\,\bullet^a + \{A^b_r,\bullet^c\}\widetilde{d}^a_{bc} + A^b_r\,\bullet^c\, g^a_{bc}. \tag{3.25}$$

It is a well-known fact the connection between large $\mathrm{SU}(N)$ groups and the group of diffeomorphims preserving the area [42] so this case emerge naturally when APD is imposed in the action.[3] In order to recover the continuum limit it is implicitly imposed periodicity on the new two arising dimensions, this is always the case when one takes

[3]We thank T. Ortin for suggestive questions about this fact.

the large M limit. The result we obtain can be interpreted as the description of condensate of M2-branes minimally immersed along those compact surface. Since the two APD can be thought as orthogonal directions, one can naturally think that they span a 4D surface related to a M5-brane compactified on a S^1 with some appropriate fluxes and forms that characterize the precise sector that we are considering. The M5-brane hamiltonian on a S^1 expressed in terms Nambu algebras was obtained in [14]. The exhaustive analysis required to determine precisely this statement lies outside the scope of the present paper. In terms of a BLG formulation it was already found a low energy description of a condensate of multiple M2-branes by extending the number of colors to infinite [49]. In their analysis the remaining symmetry is a Diff3-volume diffeomorphisms in distinction with ours that correspond to the product of two APD. Previously, in [18, 19, 50, 51] using the BLG approximation it was also be pointed out its natural connection with M5-brane worldvolume action.

- The SU(2) color case with N infinite is a particular case since $d_{ab}^c = 0$ so we loose the symplectic structure that characterizes the M2-brane. The hamiltonian becomes,

$$\begin{aligned}\mathcal{H}^{\mathrm{SU}(2)} =& \frac{1}{2}\left(\frac{P^{ma}}{\sqrt{W}}\right)^2 + \frac{1}{2}\left(\frac{\Pi^{ra}}{\sqrt{W}}\right)^2 + \frac{1}{4}(X^{mb}X^{nc}\epsilon_{ab}^c) + \frac{1}{2}\left(\mathcal{D}_r X^{na}\right)^2 \\ &+ \frac{1}{4}(\mathcal{F}_{rs}^{\mathrm{SU}(2)a})^2 - \overline{\Psi}\Gamma_-\Gamma_r D_r\Psi - \overline{\Psi}^a\Gamma_-\Gamma_m X^{mb}\Psi^c\epsilon_{bc}^a]\end{aligned}$$

and the constraint

$$\mathcal{D}_r\left(\frac{\Pi^{ra}}{\sqrt{W}}\right) + X^{mb}\left(\frac{P_m^c}{\sqrt{W}}\right)\epsilon_{bc}^a - \overline{\Psi^b}\Gamma_-\Psi^c\epsilon_{bc}^a = 0 \tag{3.26}$$

is the SU(2) first class constraint and

$$\mathcal{D}_r^{\mathrm{SU}(2)}\Pi^{ra} = D_r\Pi^{ra} + A_r^b\Pi^{rc}\epsilon_{bc}^a. \tag{3.27}$$

So the nonabelian product exactly corresponds to a SU(2) SYM without corrections. Note that the covariant derivative D_r in distinction with the ordinary case is not just a partial derivative but inherits the information of the global symplectic bundle through $D_r = 2\partial R_r l_r \epsilon^{ab}\partial_a \widehat{X}_r\partial_b$.

- The third case we consider is when N and M are finite but $N \gg M$ with N large to be a good approximation to the continuum limit although the case $N \to \infty$ does not satisfy the algebra. The structure constant

$$\mathbf{F}_{\mathbb{AB}}^{\mathbb{C}} = NM\sin\left(\tfrac{(b\wedge a)\pi}{M} + \tfrac{(A\wedge B)\pi}{N}\right) \tag{3.28}$$

corresponds roughly speaking to the first two terms contributions of the complete expansion in section 5, however at this regularized level one cannot see the proper decoupling limit, so we leave this analysis to the next section. One can characterize the qualitative properties of the quantum spectrum in this approximation. Since the spectrum of the regularized hamiltonian of a single supermembrane minimally immersed MIM2 is purely discrete at bosonic and supersymmetric level, then it also holds for the $SU(N) \times SU(M)$ case as far as N, M are both finite.

In the next section we would like to go further and analyze rigourously the case for N infinite with an arbitrary number of colors M corresponding strictly to a stack of M parallel MIM2-branes.

4 The multiple MIM2's action with arbitrary number of colors: a generalized star-product

In this section we obtain the $U(M)$ non-abelian extension of the MIM2 for an arbitrary number of colors M. We extend the algebraic symplectic structure of the supermembrane with central charges in the L.C.G in terms of a noncommutative product and a $U(M)$ gauge group. The main point is to show that in such extension the original area preserving constraint preserves the property of being first class.When an abelian gauge group is considered the closure of the area preserving constraints occurs with the complete noncommutative expansion as well as with the first two terms in the product expansion, the exact symplectic structure [52]. In distinction when a $U(M)$ gauge group is considered there is only one possibility, the complete noncommutative expansion. It is not enough to have the symplectic structure tensor $U(M)$ in order to close the algebra of the first class constraint. The complete expansion related to a noncommutative associative product is needed. It is interesting that this argument does not exclude an algebraic extension in terms of a non-associative noncommutative product, which we will discuss elsewhere. The noncommutative product we may introduce is constructed with the symplectic two form already defined on the base manifold Σ:

$$\omega_{ab} = \sqrt{W}\epsilon_{ab}, \tag{4.1}$$

where $\sqrt{W} = \frac{1}{2}Area_{T^2}(\epsilon_{rs}\epsilon^{ab}\partial_a \widehat{X}^r \partial_b \widehat{X}^s)$. In this section, in order to get a better insight on the star product, we use coordinates on the base manifold with length dimension +1 and define the dimensionless $\sqrt{W}$ with the area factor. All results of section 2 are of course valid. The two-form ω define the area element which is preserved by the diffeomorphisms generated by the first class constraint of the supermembrane theory in the Light Cone Gauge, which are homotopic to the identity, and by the $SL(2,\mathbb{Z})$ group of large diffeomorphisms discussed in section 2. The two-form is closed and nondegenerate over Σ. By Darboux theorem one can choose coordinates on an open set $\mathfrak{N}$ in Σ in a way that $\sqrt{W}$ becomes constant on $\mathfrak{N}$. However this property cannot be extended to the whole compact manifold Σ. The noncommutative theory must be globally constructed from a non-constant symplectic ω. The construction of such noncommutative theories, for symplectic manifolds was performed in [53, 54]. The general construction for Poisson manifolds was obtained in [55]. The Fedosov approach was used to construct noncommutative Yang Mills Theories and also noncommutative abelian membrane theories in [52]. A lot of work on noncommutative Yang-Mills theories was developed for constant ω, some of them are [56, 57]. See for example, [58] for an introductory review. We emphasize that our construction is not related

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to a Seiberg-Witten limit of String Theory [59] in which one obtains a noncommutative theory with constant B-field.

The starting point on the Fedosov construction is a symplectic manifold (Σ, ω) were ω is a symplectic two-form, defining a symplectic structure on each tangent space $T_\sigma \Sigma$. The elements of the Weyl algebra are formal series

$$g(\xi, h) = \sum h^k g_{k,\alpha} \xi^\alpha \tag{4.2}$$

where h is a parameter, $\xi \in T_\sigma M$ an α is a multi-index. The associated product is defined by

$$g \circ f = \sum_{k=0}^{\infty} \left(\frac{-ih}{2} \right)^k \frac{1}{k!} \omega^{a_1 b_1} \ldots \omega^{a_k b_k} \frac{\partial^k g}{\partial \xi^{a_1} \ldots \partial \xi^{a_k}} \frac{\partial^k f}{\partial \xi^{b_1} \ldots \partial \xi^{b_k}} \tag{4.3}$$

where the terms are ordered according to the weights $deg(\xi) = 1, deg(h) = 2$. In our construction the h parameter will be identified with the area wrapped by the membrane on the torus. The i factors are exactly the correct ones to reproduce the symplectic bracket with the same coefficients as in the abelian MIM2. The symplectic bracket will appear as the second term in the noncommutative bracket. The Weyl algebra bundle W is the union of the algebras W_σ, $\sigma \in \Sigma$. Its sections are denoted $g = g(\sigma, \xi, h)$ where the coefficients $g_{k,\alpha}(\sigma)$ in the above expansions are now covariant symmetric tensor fields on Σ. Differential q-forms are naturally defined as a section of the bundle $W \otimes \Lambda^q$. They constitute an algebra denoted $C^\infty(W \otimes \Lambda)$. The commutator of two forms $g \in W \otimes \Lambda^{q_1}$ and $f \in W \otimes \Lambda^{q_2}$ is defined as

$$[g, f]_\circ = g \circ f - (-1)^{q_1 q_2} f \circ g. \tag{4.4}$$

On any symplectic manifold Σ there always exist a torsion free connection preserving the tensor ω. The corresponding covariant derivative will be denoted D_a, it satisfies $D_a \omega_{bc} = 0$. Two symplectic connections differ by a completely symmetric tensor. Given a symplectic connection on Σ, a connection on $W \otimes \Lambda$ may be defined as

$$Dg = d\sigma^b \wedge D_b g \tag{4.5}$$

More general connections $\mathcal{D}$ are defined as

$$\mathcal{D}g = Dg + \frac{i}{h}[\mathcal{A}, g]_\circ \tag{4.6}$$

where $\mathcal{A}$ is a section of $W \otimes \Lambda^1$. The next step in the Fedosov construction is to introduce an Abelian connection: $\mathcal{D}$ is Abelian if its curvature is a central form of the algebra, that is

$$[\Omega, a]_\circ = 0 \tag{4.7}$$

for any $a \in C^\infty(W \otimes \Lambda)$, Ω is the curvature of $\mathcal{D}$. There always exist an Abelian connection [54] in the Weyl algebra bundle. The Abelian connection depends explicitly on the Riemann tensor of the symplectic connection. The subalgebra $W_{abelian} \subset C^\infty(W)$ of flat sections, that is the set of $g \in C^\infty(W)$ such that $\mathcal{D}g = 0$, where $\mathcal{D}$ is abelian is called the quantum algebra.

The center Z of W are the elements which do not depend on ξ. For each section $g(\sigma,\xi,h)\in C^\infty(W)$, $\sigma(g)$ denotes the projection onto the center:

$$\sigma(g(\sigma,\xi,h))=g(\sigma,0,h). \tag{4.8}$$

It follows that the map $\sigma: W_{abelian}\to Z$ is bijective. Consequently a star product $*$ on $C^\infty(\Sigma)$ may be defined as

$$\widehat{g}*\widehat{f}=\sigma(\sigma^{-1}(\widehat{g})\circ\sigma^{-1}(\widehat{f})) \tag{4.9}$$

for any $\widehat{g},\widehat{f}\in Z$. The noncommutative star product is as a result of the construction associative. The star product, for the particular case in which ω has constant coefficients and the symplectic connection is trivial, reduces to the Moyal product. In general the star product includes terms depending on the Riemann tensor for the symplectic connection. In particular for the symplectic structure on the base manifold of the supermembrane with central charges, the symplectic connection is necessarily non-trivial. For an explicit construction see [52]. We now extend the above construction and consider the tensor product of the Weyl algebra bundle times the enveloping algebra of $u(M)$. It may be constructed in terms of the Weyl-algebra generators T_A introduced in the previous section, with the inclusion of the identity associated to $A=(0,0)$. This complete set of generators determine an associative algebra under matrix multiplication. The inclusion of the identity allows to realize the generators of the $u(M)$ in terms of T_A matrices, with $A=(a_1,a_2)$ and $a_1,a_2=-(M-1),\ldots,0\ldots M-1$. All the properties of the Fedosov construction remain valid, in particular the associativity of the star product. It is also valid the following Trace property, if $g=g^AT_A$, $f=f^AT_A$, $g^A,f^A\in C^\infty(W)$

$$Tr\int_\Sigma\sqrt{W}\sigma(g\circ f)=\int_\Sigma\sqrt{W}\sigma(g^A\circ f^B)Tr(T_AT_B)=Tr\int_\Sigma\sqrt{W}\sigma(f\circ g)$$
$$Tr\int_\Sigma\sqrt{W}\sigma(g\circ f\circ h)=Tr\int_\Sigma\sqrt{W}\sigma(h\circ g\circ f).$$

We may introduce canonical variables on the Weyl algebra bundle

$$[X^m(\sigma,\xi,h),P_n\sigma',\xi',h)]_{P.B}=\delta^m_n\delta(\sigma'-\sigma)\delta(\xi'-\xi)$$
$$[A_r(\sigma,\xi,h),\Pi^s(\sigma',\xi',h)]_{P.B}=\delta^s_r\delta(\sigma'-\sigma)\delta(\xi'-\xi)$$

It then follows

$$Tr\int_\Sigma\sqrt{W}\sigma(G\circ H\circ[X^n),Tr\int_\Sigma\sigma(P_m]_{P.B}\circ L\circ M)=Tr\int_\Sigma\sqrt{W}\sigma(G\circ H\circ L\circ M)\delta^n_m, \tag{4.10}$$

where we have used $Tr(T_CT_AT_B)Tr(T_DT_ET_F)\eta^{CD}=Tr(T_AT_BT_ET_F)$ and the associativity of the Weyl product. In order to construct the hamiltonian of the theory we consider the following connection on the Weyl bundle [52]

$$\mathcal{D}\diamond=\frac{i}{h}[G_re^r,\diamond]_\circ+\frac{i}{h}[\mathcal{A}_re^r,\diamond]_\circ \tag{4.11}$$

where $G_r, \mathcal{A}_r \in C^\infty(W_{Abelian})$, $\sigma G_r = \delta_{rs} X_h^s$ and $X_h^s = 2\pi R_s l_s \widehat{X}^s$. It corresponds to the harmonic sector of the map to the compact sector of the target space. $\sigma \mathcal{A}_r = A_r$ using the notation of section 2, $e^r = \partial_a \widehat{X}^r d\sigma^a$. Its curvature is given by

$$\Omega = \frac{i}{2h}[G, G]_\circ + \frac{i}{h}[G, \gamma]_\circ + \frac{i}{2h}[\gamma, \gamma]_\circ, \quad \gamma = \mathcal{A}_r^B e^r T_B \tag{4.12}$$

We now consider $(X^m, P_m), (A_r, \Pi^r)$ the canonical conjugate pairs as well as the spinor fields Ψ lifted to the quantum algebra $W_{abelian} \in C^\infty(W)$. In order to simplify the notation we use the same symbols for the lifted quantities. In the presence of a $\circ$ product we refer to the lifted quantities. The constraint is then defined as

$$\begin{aligned}\phi(\sigma, \xi, h) &\equiv \mathcal{D}_r \frac{\Pi^r}{\sqrt{W}} + \frac{i}{h}\left[X^m, \frac{P_m}{\sqrt{W}}\right]_\circ + \frac{i}{h}[\overline{\Psi}\Gamma_-, \Psi]_\circ \\ &= \mathcal{D}_r \frac{\Pi^{rA}}{\sqrt{W}} T_A + \frac{i}{h}\left(X^{mB} \circ \frac{P_m^C}{\sqrt{W}} - \frac{P_m^B}{\sqrt{W}} \circ X_m^C\right) T_B T_C + \frac{i}{h}[\overline{\Psi}\Gamma_-, \Psi]_\circ.\end{aligned}$$

with

$$\begin{aligned}\mathcal{D}_r \frac{\Pi^r}{\sqrt{W}} &= \frac{i}{h}\left[G_r, \frac{\Pi^r}{\sqrt{W}}\right]_\circ + \frac{i}{h}\left[A_r, \frac{\Pi^r}{\sqrt{W}}\right]_\circ \\ &= \frac{i}{h}\left[G_r, \frac{\Pi^{rA}}{\sqrt{W}}\right]_\circ T_A + \frac{i}{h}\left(A_r^B \circ \frac{\Pi^{rC}}{\sqrt{W}} - \frac{\Pi^{rB}}{\sqrt{W}} \circ A_r^C\right) T_B T_C.\end{aligned}$$

We notice that the first two terms of the commutator

$$\left[X^m, \frac{P_m}{\sqrt{W}}\right]_\circ = X^{mB} \frac{P_m^C}{\sqrt{W}} f_{BC}^E T_E + \left(-i\frac{h}{2}\right)\left\{X^{mB}, \frac{P_m^C}{\sqrt{W}}\right\} d_{BC}^E T_E + O((h\omega)^2) \tag{4.13}$$

are the terms which we considered in the previous section as extensions of the algebraic structure of the supermembrane in the Light Cone Gauge. The additional terms arising from the noncommutative product, ensuring an associative product, are relevant in order to close the constraint algebra. In fact using the trace properties discussed above it follows

$$\begin{aligned}&\left[Tr\int \sqrt{W}\sigma(\lambda(\sigma,\xi,h)\circ\phi(\sigma,\xi,h)), Tr\int\sqrt{W}\sigma(\epsilon(\sigma,\xi,h)\circ\phi(\sigma,\xi,h))\right]_{P.B} \\ &\qquad = Tr\int\sqrt{W}\sigma([\lambda,\epsilon]_\circ\circ\phi(\sigma,\xi.h)),\end{aligned} \tag{4.14}$$

$\phi \in W_{abelian}$ is a first class constraint generating a gauge transformation which is a deformation of the original are preserving diffeomorphisms. In particular,

$$\sigma\left[-i\int\sqrt{W}(\lambda\circ\phi), X^m\right]_{P.B} = \frac{i}{h}\sigma[\lambda, X^m]_\circ = \frac{1}{2}\{\lambda, X^m\} + O((h\omega)^2), \tag{4.15}$$

where λ is the infinitesimal parameter of the area preserving diffeomorphisms, valued on the generator $T_{(0,0)}$.

The projection of Ω in (4.12) has the expression [52]

$$\sigma\Omega = -\omega + \mathcal{F} + O(h^2) \tag{4.16}$$

where

$$\mathcal{F} = \frac{1}{2} e^r \wedge e^s \left(D_r A_s - D_s A_r + \frac{i}{h} \{A_r, A_s\}_* \right), \tag{4.17}$$

and $\omega = \frac{1}{2h}\sqrt{w}\epsilon_{ab} d\sigma^a \wedge d\sigma^b$. $O(h^2)$ depend explicitly on the Riemann tensor of the symplectic connection. D_r, D_s are the ones defined in section 2. The hamiltonian of the theory for M multiple parallel M2-branes with U(N) gauge group is then

$$\begin{aligned} Tr\int_\Sigma \mathcal{H} = Tr\int_\Sigma \sqrt{W}\Bigg[& \frac{1}{2}\left(\frac{P^m}{\sqrt{W}}\right)^2 + \frac{1}{2}\left(\frac{\Pi^r}{\sqrt{W}}\right)^2 + \frac{1}{2h^2}(\{X_h^r, X^m\}_* + \{A^r, X^m\}_*)^2 \\ & + \frac{1}{4h^2}\{X^m, X^n\}_*^2 + \frac{1}{2}\Omega_{rs}\Omega^{rs} \\ & - \frac{i}{h}\overline{\Psi}\Gamma_-\Gamma_r(\{X_h^r, \Psi\}_* + \{A^r, \Psi\}_*) - \frac{i}{h}\overline{\Psi}\Gamma_-\Gamma_m\{X^m, \Psi\}_* \Bigg], \end{aligned}$$

where the term $\{X_h^r, X^m\}_* + \{A^r, X^m\}_* = \delta^{rs}\mathcal{D}_s X^m + O(h)$ in the notation of section 2. The hamiltonian is subject to the first class constraint

$$\phi \equiv \left\{X_h^r, \frac{\Pi^r}{\sqrt{W}}\right\}_* + \left\{A_r, \frac{\Pi^r}{\sqrt{W}}\right\}_* + \left\{X^m, \frac{P_m}{\sqrt{W}}\right\}_* - \{\overline{\Psi}\Gamma_-, \Psi\}_* = 0 \tag{4.18}$$

The first terms in the star product expansion are

$$\phi \equiv \mathcal{D}_r \frac{\Pi^r}{\sqrt{W}} + \left\{X^m, \frac{P_m}{\sqrt{W}}\right\} - \{\overline{\Psi}\Gamma_-, \Psi\} + O(h) \tag{4.19}$$

where $\{,\}_*$ has been normalized in a way to be a deformation of $\{,\}$ the symplectic bracket of the supermembrane in the L.C.G. The fields are now $u(M)$ valued. The constraint is a deformation of the $u(M)$ Yang-Mills constraint. The terms $O(h)$ involve the Riemann tensor of the symplectic connection which itself depends on the symplectic two-form introduced by the central charge. The $O(h)$ terms are necessary in order to close the constraint algebra. An explicit expression for the curvature in the abelian case was found in [52],

$$\begin{aligned} \sigma\Omega = & -\omega + \mathcal{F} - \frac{h^2}{96}\left(R_{bcda}\left(D_{\widehat{b}} D_{\widehat{c}} D_{\widehat{d}}\right) A_m - \frac{1}{4} R_{\widehat{b}\widehat{c}\widehat{d}p}\epsilon^{pq} D_q A_m\right)\epsilon^{b\widehat{b}}\epsilon^{c\widehat{c}}\epsilon^{d\widehat{d}} e^a \wedge e^m \\ & - \frac{h^2}{96.8} R_{bcda} R_{\widehat{b}\widehat{c}\widehat{d}m}\epsilon^{b\widehat{b}}\epsilon^{c\widehat{c}}\epsilon^{d\widehat{d}} e^a \wedge e^m + O(h^3)\ldots. \end{aligned}$$

The terms involving the Riemann tensor of the symplectic connection are absent in the Moyal product.

If we make manifest the dimensional dependence of the star-product we can realize that the parameter $[h] = n.Area_{T^2}$, n is the wrapping number. In fact, as it was introduced in the definition of the noncommutative product, h has degree 2 while ξ has degree 1. Since ξ has legth dimension 1 then h must have length dimension 2 in order to have a dimensionless noncommutative product.

The star-product is explicitly given by

$$\begin{aligned}\frac{i}{h}\{f,g\}_*^a &= \frac{i}{h} f^b g^c f_{bc}^a + \{f^b, g^c\} d_{bc}^a + O(h)\\ &= \frac{i}{nArea_{T^2}} f^b g^c f_{bc}^a + \{f^b, g^c\} d_{bc}^a + O(nArea_{T^2})\end{aligned}$$

where $\{f^b, g^c\} = \epsilon^{rs} D_r f^b D_s g^c$. D_r was defined in section 2. The factor $\frac{1}{h}$ ensures that this formalism is a nonabelian extension of the abelian MIM2-brane, since for the abelian case f_{bc}^a vanishes, $d_{bc}^a = 1$, and the algebra closes exactly with the ordinary symplectic bracket corresponding to a single M2 action without further contributions.

The mass square operator may be written as:

$$-\,mass^2 = \int \left(\frac{1}{2} d\widehat{x}^r \wedge d\widehat{x}^s \epsilon_{rs}\right) Tr\left[\frac{1}{2}\left(\frac{P}{\sqrt{W}}\right)^2 + \frac{1}{2}\left(\frac{\Pi}{\sqrt{W}}\right)^2 + (TArea_{T^2}^2)(V_B + V_F))\right] \tag{4.20}$$

where V_B and V_F are the bosonic and fermionic potentials of the Hamiltonian. The scale of the theory is then $T.n.\mathrm{Area}_T^2$. The measure of integration reduces to the dimensionless $\frac{1}{2} d\widehat{x}^r \wedge d\widehat{x}^s \epsilon_{rs}$. The conjugate momenta have mass dimension $+1$, and the corresponding configuration variable mass dimension -1. T has mass dimension $+3$. On the other hand, by considering the contribution to Yang-Mills arising from the first term in the above expansion of the star product and by taking canonical dimensions for the conjugate pairs we get for the coupling constant

$$g_{YM} = \frac{1}{T_{M2}^{1/2}.n.Area_{T^2}}. \tag{4.21}$$

It has dimension of $mass^{1/2}$.

It represents the coupling constant of the first term in the star-product expansion. We assume that the compactification radii is $R_i \gg l_p$ but with the theory still defined at high energies. For a fixed tension and winding number n, the only relevant contribution in the star product at low energies is the U(M) commutator since the natural length is much larger larger that the effective radii $R_{\mathrm{eff}} = n^{1/2}\sqrt{R_1 R_2}$. This is the decoupling limit of the theory since the Yang Mills field strength becomes the coupling constant of the theory and gravitational modes become decoupled. The g_{YM} is very large in this phase and the theory is in the IR phase. It corresponds to have a description of M multiple MIM2-branes as point-like particles, representing M the number of supermembranes. As we raise the energy the g_{YM} coupling constant gets weaker and for energies high enough, comparable with the natural scale of a MIM2-brane with an effective area of $(n.Area_{T^2})$, the oscillation and vibrational modes containing the gauge but also gravity interactions between the supermembranes are no longer negligible so the full star- expansion has to be considered. All terms associated to the supermembrane symplectic structure of the star-bracket contribute while the ordinary SYM contribution vanishes. The point-like particle picture is no longer valid, and it is substituted for that of an extended $(2+1)$D object and the gauge and gravity contributions are strongly coupled. One can define formally and effective physical coupling constant for the ordinary $F_{\mu\nu}$ field strength which it would correspond to $\Lambda = M.g_{YM}$ with M representing the number of supermembranes and then one can try to obtain the 't Hooft coupling expansion in the large M. In this picture however one should take care on the limit. By keeping Λ fixed with M going to infinity, for a fixed tension and a fixed compactification radii, one has to consider the wrapping number n also going to infinity. But $n.Area$ is the order parameter that would also go multiplied by M in the expansion so one enters "faster" in the strong correlated limit where the rest of the terms of the star-product expansion cannot be neglected, moreover, from a physical point of view $n.area_{T^2}$ is related to the size of the MIM2 as an extended object and it cannot be larger than the present energy bounds we have, otherwise it would be in contradiction with our point-particle description at low scales. In order to perform a more accurate analysis one should be working with the nonabelian extension of the MIM2 for 4D noncompact,-it will be considered elsewhere- however we believe that the qualitative arguments presented here should remain valid also in that case.

5 $N = 8$ LCG supersymmetry

In order to analyze the invariance of the MIM2 action under supersymmetry it is convenient to introduce,

$$\begin{aligned}
\mathcal{D}_r \cdot &= [\widetilde{X}_r, \cdot]_* + [A_r, \cdot]_* \\
\mathcal{D}_m \cdot &= [X_m, \cdot]_* \\
\mathcal{D}_0 \cdot &= \partial_\tau \cdot + [A_0, \cdot]_*
\end{aligned}$$

where A_0 is the lagrange multiplier associated to the first class constraint and $\widetilde{X}_r = X_h^s \delta_{rs}$ with $r = 1, 2$. We will denote $A_m = X_m$. We then introduce the index $\mu = 0, r, m$. The $\mathcal{D}_\mu$ satisfy the Leibniz rules

$$\begin{aligned}
\mathcal{D}_\mu F \circ G &= \mathcal{D}_\mu F \circ G + F \circ \mathcal{D}_\mu G \\
\mathcal{D}_\mu [F, G]_\circ &= [\mathcal{D}_\mu F, G]_\circ + [F, \mathcal{D}_\mu G]_\circ
\end{aligned}$$

We also consider the curvatures

$$\begin{aligned}
\Omega_{rs} &= [\widetilde{X}_r, A_s]_* - [\widetilde{X}_s, A_r]_* + [A_r, A_s]_* + [\widetilde{X}_r, \widetilde{X}_s]_* \\
\Omega_{rm} &= \mathcal{D}_r X_m \\
\Omega_{mn} &= [X_m, X_n]_* \\
\Omega_{0r} &= \dot{A}_r - [\widetilde{X}_r, A_0]_* + [A_0, A_r]_* \\
\Omega_{0m} &= \mathcal{D}_0 X_m .
\end{aligned}$$

$\Omega_{\mu\nu}$ satisfy the Bianchi identities

$$\mathcal{D}_\mu \Omega_{\nu\lambda} + \mathcal{D}_\lambda \Omega_{\mu\nu} + \mathcal{D}_\nu \Omega_{\lambda\mu} = 0 \tag{5.1}$$

These relations are valid provided for any associative product. We are considering A_r, X_m valued in the enveloping algebra of $\mathrm{U}(N)$ in terms of the Weyl-Heisenberg generators T_A

of section 3, $\circ$ is the noncommutative associative Weyl product. The tensor product is still an associative product and the property (5.1) is satisfied identically in our construction. The lagrangian of the theory after integration of the momenta P_m, Π^r maybe expressed as

$$\mathcal{L} = -\frac{1}{4}\Omega_{\mu\nu} * \Omega^{\mu\nu} - \overline{\Psi} * \Gamma_- \Gamma^M \mathcal{D}_M \Psi - \overline{\Psi} * \Gamma_- \mathcal{D}_0 \Psi \tag{5.2}$$

where $M = r, m$. The light cone fermionic gauge condition we use is

$$\Gamma_+ \Psi = 0. \tag{5.3}$$

The associated action is invariant under the following supersymmetric transformations with parameter $\epsilon = \Gamma_- \Gamma_+ \epsilon$

$$\begin{aligned}
\delta A_M &= \delta A_M^B T_B = \overline{\epsilon} \Gamma_M \Psi^B T_B \quad M = r, m \\
\delta A_0 &= \delta A_0^B T_B = -\overline{\epsilon} \Psi^B T_B \\
\delta \Psi &= \delta \Psi^B T_B = \frac{1}{4} \Gamma_+ \Omega_{MN}^B \Gamma^{MN} \epsilon T_B + \frac{1}{2} \Gamma_+ \Omega_{0M}^B \Gamma^M \epsilon T_B
\end{aligned}$$

These transformations are a U(N) extension of the SUSY transformations for the supermembrane in the LCG found in [41, 60] and they realize $\mathcal{N} = 8$ supersymmetries on the worldvolume. The invariance of the action arises in a similar way as it does for Super Yang-Mills:

$$\begin{aligned}
Tr\langle \delta \left(-\frac{1}{4} \Omega_\mu * \Omega^{\mu\nu} \right) \rangle &= Tr \langle \delta A_\nu * \mathcal{D}_\mu \Omega^{\mu\nu} \rangle \\
&= Tr \langle 2\overline{\Psi} * \Gamma_- \Gamma^M \mathcal{D}_M \delta\Psi + 2\overline{\Psi} * \Gamma_- \mathcal{D}_0 \delta \Psi \rangle
\end{aligned}$$

and

$$Tr \langle \overline{\Psi} * \Gamma_- \Gamma^M (\delta \mathcal{D}_M) \Psi \rangle = 0, \tag{5.4}$$

where $\langle \cdot \rangle$ denotes integration and Bianchi identities as well as eleven dimensional identities for the Γ matrix have been used.

6 Discussion and conclusions

6.1 Main results

We have obtained a $N = 8$ nonabelian U(M) formulation of the minimally immersed supermembrane for arbitrary number of colors M with all the symmetries of the supermembrane, in the LCG. This corresponds to the M-theory dual of the nonabelian Dirac-Born-Infeld theory, representing a bundle of D2-D0 branes. It is the first time that a nonabelian gauge theory can be directly obtained from a full-fledged sector of M-theory element, so far restricted to String theory: Heterotics and Dp-branes in type II theories. This opens a new interesting window for models in phenomenology. At energies of the order of the compactification scale, the theory has the gauge and gravity sector strongly coupled. It describes all of the oscillations modes of the multiple parallel M2-branes minimally immersed. At low energies the theory enters in a decoupling regime and the physics is then described by a $\mathcal{N} = 8$ SYM theory of point-like particles in the IR phase. We then expect to describe correctly many aspects of phenomenology when realistic gauge groups are considered. From the point of view of the target space the theory has N=1 susy in 9D flat-dimensions. In [5] a N=1 target space, D=4 formulation of a single supermembrane minimally immersed together with a number of interesting phenomenological properties were found. Moreover in [7] a formulation of the supermembrane minimally immersed on a G2 manifold was also obtained. Its quantum supersymmetric spectrum is also purely discrete. The analysis in 4D can be also extended to the nonabelian case following the lines shown in this paper, allowing to obtain models with reduced number of target and worldvolume supersymmetries.

6.2 The case of M2 without central charge condition

We could expect to apply the mechanism we have implemented to the supermembrane without central charges on flat space M_{11}. However this is not possible. In fact the nonabelian formulation of multiple parallel MIM2 has a constraint (6.1) of the Gauss type allowing the elimination of the gauge degrees of freedom a la Yang-Mills. This constraint also generates, when the parameter is valued on the $T_{(0,0)}$ generator, a deformation of an area preserving diffeomorphims. For a single MIM2 the U(1) Gauss constraint and area preserving diffeomorphism constraint are the same,

$$\phi \equiv \left\{\widehat{X}_r, \frac{\Pi^r}{\sqrt{W}}\right\}_* + \left\{A_r, \frac{\Pi^r}{\sqrt{W}}\right\}_* + \left\{X^m, \frac{\Pi^r}{\sqrt{W}}\right\}_* - \{\overline{\Psi}\Gamma_-, \Psi\}_* = 0. \tag{6.1}$$

This does not happen for the 11D supermembrane where the constraint is of the form

$$\{X^m, P_m\}_* + fermions = 0. \tag{6.2}$$

In (6.1), where a linear term is present, it is possible to eliminate exactly one degree of freedom (with internal index) but in (6.2) the gauge degrees of freedom cannot be eliminated correctly, unless an additional assumption is made. It is not possible to impose the static gauge for the supermembrane on compact base manifold [6]. It would imply automatically the vanishing of string-like spikes that are present in the formulation, and the spectrum of the hamiltonian would not be continuous.

6.3 General properties of our construction

- Our action starts from the formulation of the supermembrane with a topological condition and extends it to include nonabelian U(N) interactions while preserving all of the symmetries and constraints of the theory. This means that we work at high energies where gauge fields and supergravity contribution are strongly correlated. The analysis is then valid for any number of M2-branes large or small.

- We are mainly interested in characterizing the M2-branes interactions by themselves. The theory is not scale invariant so there is no a sextic scalar potential in the action, it remains quartic in the fields although now, valued on a U(N) algebra.

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- The single MIM2 action contains a gauge field invariant under symplectomorphisms defined on the (2+1)D worldvolume action of the supermembrane. The first class constraint generating the symplectomorphisms is analogous to the Gauss constraint of SYM theory. It is this property which allows the U(N) formulation without changing the number of degrees of freedom. In the low energy description of the 11D supermembrane the situation is different since it only contains scalar fields in the bosonic sector. For that case, in order to introduce a gauge field without changing the degrees of freedom a Chern-Simons type extension is needed.

- We are describing a supersymmetric theory of the multiple M2-branes (2+1)D (with gauge and gravity sector coupled) strongly coupled immersed in a M_9xT^2 target space with N=1 supersymmetry. Quantum corrections are relevant in this case and its spectral properties are controlled by the bosonic sector. The abelian MIM2 has a well-defined supersymmetric quantum formulation in terms of a Feynman path integral. We have argued that this property remains valid in the nonabelian regularized version with the ordinary product.

- We may perform a matrix regularization of the proposed action of multiple M2-branes minimally immersed in the compactified space. We are able to do it since the central charge condition allow a proper treatment of the harmonic forms present on actions formulated on compactified spaces [44].

- Our approach is not covariant. It is a LCG formulation. However, remarkably, the bosonic sector of the theory corresponds to a non-standard reduction of SYM with the star- product defined in section 5, from 10D to $(2+1)$D. This is so, because the light cone coordinates decouple in our sector of the theory, since we are able to solve completely the constraint for X^- once the gauge fixing condition has been imposed. The global constraint (2.3) guarantee that there is no winding condition on X^-. The LCG supersymmetric algebra has 8 generators, it is related to the one found in [41] for the 11D abelian supermembrane.

6.4 Nonconstant star-product

We would like to emphasize that our approach is not related to a noncommutative Seiberg-Witten (SW) limit of String theory. In order to close the algebra of constrains while keeping the invariance of the action, we introduced a Fedosov star-product. It is not related to the constant B-field of SW. In fact the SW formulation is a local description, by Darboux theorem of the Fedosov star-product. Although our construction is based on a compactification on a 2-torus it does not correspond to the theory on the noncommutative torus done in [61]. For the abelian formulation of the MIM2 we do not need to introduce an star-product since the constraint closes at first order. In the nonabelian case the constraint still closes at first order iff the number of colors go to infinite or iff we restrict our analysis to a regularized case for an arbitrary number of colors. At exact level formulation (not regularized) of the MIM2's with an arbitrary number of colors we need to introduce a Fedosov the star-product to close the algebra. This gives a precise indication on how U(N)

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gauge group is compatible with area preserving diffeomorphisms. The particular case of SU(2) corresponds to the case where the symplectic structure vanishes and it corresponds to a ordinary-type of SYM description. It would be nice to see the connection, if any, between our results and the fact that for low energy descriptions of M2-branes only $N = 8$ models has been only found for $\mathrm{SU}(2) \times \mathrm{SU}(2)$ [11, 20] gauge group or infinite gauge group representing a condensate of M2-branes [49], meanwhile the low energy formulation of a multiple M2 inspired action with an arbitrary number of colors $\mathrm{U}(N) \times \mathrm{U}(N)$ has only been found for less number of supersymmetries $\mathcal{N} = 6$ [22].

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