Influence of initial mathematical competencies on the effectiveness of an intervention

Abstract

Background. Students commonly struggle with mathematics and mathematical problem solving. Therefore, it is necessary to design and implement interventions aimed at improving these essential components of learning. Furthermore, the outcomes of these interventions can vary significantly, and appear to be a function of a student's initial competencies in mathematics.

Aim. The present study attempts to analyze the influence of initial levels of mathematics competency with respect to the benefits of a specific intervention known as the Integrated Dynamic Representation (IDR).

Sample. Participants were 288 students (aged 6-8 years) who were divided according to their levels of mathematics competency (low-medium-high)

Methods. Students were assigned to the two primary intervention groups, Experimental Group (EG; students who received the IDR intervention) and Control Group (CG; students who followed the traditional instructional methods). All participants completed the Test of Early Mathematics Abilities (TEMA-3) both before and after the intervention.

Results and conclusions. Although all the three competency levels of the EG improved, the progression was different for each level. Results showed that students with low competency level improved substantially more than the students with medium and/or high baseline competency level.

Keywords: mathematical competency, intervention, mathematical problem solving, Integrated Dynamic Representation.

Background

Mathematics is a critical component of our core curriculum and is vital to success in today's workplace, and also in everyday living. However, as reflected in international assessment reports, many children experience difficulties in learning basic skills in mathematics (Käser et al., 2013). In this regard, it is crucial to note that approximately 6-14% of school-age children have persistent difficulties with mathematics (Clayton & Gilmore, 2015) and around 20% of the general population have low numeracy skills (Kadosh, Dowker, Heine, Kaufmann, & Kucian, 2013), which highlights the need to increase our understanding of specific strategies that may allow us improve mathematics competencies and, in so doing, also reduce the potential for future difficulties.

Generally, the effectiveness of these interventions is assessed on both a global and a group basis, regardless of the students' previous skills and knowledge. Nevertheless, children start kindergarten with different pre-school levels of mathematics ability (Purpura, Reid, Eiland, & Baroody, 2015), and these differences often predict their later achievement (Bailey, Watts, Littlefield, & Geary, 2014). For example, Navarro et al. (2012) showed that students with a very low score in mathematics competencies at the age of 5 years (third year of kindergarten), also obtained a very low score at the age of 7 years (finishing first grade of Primary Education). This emphasizes the importance of adapting any interventions to an individual student's needs, especially in the face of possible learning difficulties, because these children are more at risk of developing severe mathematics deficits (Powell, Cirino, & Malone, 2017). Given that the acquirement of further knowledge is built upon the acquisition of previous knowledge (and that children present differences at this level) the efficacy of interventions may vary as a function of these initial competencies (Bailey et al., 2014). According to Powell et al. (2017), few if any interventions achieve universal responses, and little is known concerning child characteristics associated with inadequate responses. Thus, the purpose underlying this investigation was to analyze and characterize the improvement profiles of students with initially low, medium and high mathematics competencies after completing a computer-based intervention. We examined the combined results of two previous intervention research samples, one of which included students with mathematics learning difficulties (MLD) and Attention Deficit /Hyperactivity Disorder (ADHD), and the other comprising students without difficulties. Joint analysis of the data from these two samples allowed us to compare the different profiles of students which teachers work with in everyday classes, and also those of students who need specific interventions in order to avoid more severe difficulties in the future (Powell et al., 2017). In this study, the diagnostic profile was deemed to be the covariant variable, especially considering that specific learning difficulties have been labeled differently in the literature (developmental dyscalculia, mathematical difficulties or mathematical learning difficulties, mathematical disabilities, …; Olsson, Östergren, & Träff, 2016). These different terms are used interchangeably, but could potentially describe different children with different performance. Also, following Krawec (2014), low-achieving students and LD students perform similarly (i.e., poorly) on math achievement measures.

Furthermore, the study included a particular focus upon the pre-existing levels of mathematics competencies in subjects with difficulties in order to determine if initial competency status can modulate treatment effect, given that the DSM-5 (Diagnostic and Statistical Manual of Mental Disorders DSM-5; APA, 2013) states that specific learning disorders can vary in severity (mild, moderate and severe) and need different resources and services.

Following Salminen, Koponen, Leskinen, Poikkeus and Aro (2014), an individual's responsiveness to interventions needs to be carefully evaluated. With this in mind, Byrnes and Wasik (2009) carried out a study aimed to determine the factor most strongly associated with mathematics achievement during kindergarten through third grade. Structural equation modeling showed that intrinsic factors (e.g., pre-existing mathematics skills) were the most important determinants of achievement in mathematics. In this respect, although prior research highlights the importance of previous mathematics competency in relation to later achievement, mathematical learning is under-emphasized in the first years of school. For example, kindergarten teachers spend little time on mathematics instruction and cover very basic content, such as counting and shapes (Engel, Claessens, & Finch 2013). However, some basic mathematical skills may be at the basis for later acquisition of more complex ones (Dowker, 2008; Dowker, in press). In this line, although some mathematical competences appear to be easier than others, there does not seem to be a clear hierarchical structure (e.g., children might perform well at supposedly more difficult tasks and worse at supposedly easier tasks) (Dowker, in press). Nonetheless, it must be emphasized that children with less informal mathematics knowledge and competencies are at a clear disadvantage relative to their peers (Jordan, Glutting, & Ramineni, 2010), and show poorer mathematics competency and more difficulties in word-problem solving. The informal mathematics competencies are defined as knowledge that is intuitive or built-up through everyday experiences (e.g., perception of small numbers, quantity perception, enumeration 1 to 5,…) (Libertus, Feigenson, & Halberda, 2013). As students progress through the early elementary-school years, their informal mathematics skills and competencies serve as a platform for the acquisition of formally taught mathematics concepts. The formal competencies include mental calculation, encoding and decoding of the numbers, mechanical operations, etc (Ginsburg & Baroody, 2003). Hence, the importance of teaching and learning arithmetic and mathematics competencies from the very first schooling years.

In this sense, arithmetic is made up of many components, including knowledge of arithmetical facts; ability to carry out arithmetical procedures; understanding and using arithmetical principles such as commutativity and associativity; estimation; and applying arithmetic to the solution of word problems and practical problems (Dowker, 2008).

According to Jitendra, Dupuis and Zaslofsky (2014), the development of general problem-solving skills is facilitated by opportunities for solving word-problems. Word-problems can help students to connect different meanings, interpretations, and relationships concerning mathematical operations. In this sense, schematic representation is an effective strategy which can greatly enhance the processes underlying mathematics word-problem solving (van Garderen, Scheuermann, & Jackson 2012). Although all representational systems (such as mental images, written language, oral language, action movements, symbols) are important for the development of an understanding of mathematical concepts, rigid visual presentations of mathematical equations are commonly used and recommended for mathematics instruction at all grade levels (Pape & Tchoshanov, 2001). Different interventions, with different goals, are used to enhance mathematical skills. Nonetheless, only a few computer-based training programs have been evaluated scientifically (Kadosh et al., 2013). One such example is the computer-based intervention *"Number Race"* (for children with developmental dyscalculia), which enhances the ability to compare numbers and, thereby, strengthens important mental-links between numbers and dimensions (Wilson et al., 2006). *"Rescue Calcularis"* is a set of computer games for primary school children. It aims to improve the construction of numbers representation using the mental lineal-order of numbers (Kucian et al., 2011). *"Elfe and Mathis"* is yet another computer-based training program for elementary school (Lenhard, Lenhard, Schug, & Kowalski, 2011), which has been adapted to the German school curriculum. Alternatively, *"The Integrated Dynamic Representation"* (IDR; González-Castro, Cueli, Cabeza, Álvarez-García, & Rodríguez, 2014) is a computer-based program that is aimed at enhancing not only mathematics competencies, but also mathematically-based word-problem solving abilities. The key to the IDR is that is provides the student with a specific representation structure (a schematic representation) aimed at the word-problem solving, especially in children with learning difficulties since kindergarten (p.e. “I have 2 books and they give me 3 books, how many books do I have now?”). For the training ~~since~~ from kindergarten onwards, the program uses three forms of presentation of the information: 1) only images (iconic presentation); 2) images joined to the words (combined presentation); and 3) only words (symbolic presentation). Also, the IDR is aimed at improving informal and formal mathematics competencies using exercises of word-problem solving (Cueli et al., 2017). The fact that both informal and formal competencies are included allows us to improve the specific skills in which children with or without learning difficulties often have problems, and thereby hopefully prevent future disabilities (Cueli, González-Castro, Rodríguez, Núñez, & González-Pienda, 2018). Cohen-Kadosh, Dowker, Heine, Kaufmann and Kucian (2013) highlight that interventions that focus on the particular components with which an individual child has difficulty are likely to be more effective than those which assume that all children´s arithmetical difficulties are similar.

The strategy IDR has been shown to increase mathematical efficacy in 35 students *without* learning disabilities (González-Castro et al., 2014), and moreover, in 105 students with MLD and ADHD (González-Castro, Cueli, Areces, Rodríguez, & Sideridis, 2016). Taking into account that a child’s development of numerical abilities often occurs at different rates, which can also lead to different mathematical performance profiles being formed (Wilson & Dehaene, 2007), it is necessary to know the benefits of a given program in relation to the underlying competencies of the students enlisted in the program. In this sense, as computer-based trainings can be designed to adapt to children’s cognitive or to performance profiles (Wilson & Dehaene, 2007), the benefits should be analyzed as a function of such profiles or baseline levels.

Therefore, the present study was aimed at determining the influence of initial levels of mathematics competency with respect to the benefits of an IDR intervention in elementary school students (in the first and second grades). The primary research question was: Is the efficacy of the intervention modulated by the competencies levels? The purpose of this study was to determine if initial status influences the treatment effect, with the hypothesis being that students with all three baseline levels of mathematical competency (low, medium and high) will have improved post-intervention. However, given that the IDR is an adaptive intervention, it was also hypothesized that the students with the lowest initial level will have improved the most, and this would be because are they starting from lower scores and thus have more room for improvement in terms of development. Also, it is related with the way in which IDR works, including informal and formal competencies. This hypothesis is supported by previous findings in which students with learning difficulties obtained more benefit from the interventions than students without difficulties (Kucian et al., 2011).

Method

Participants

Participants in this investigation comprised 288 elementary school students (in the 1st and 2nd grades), aged between 6 and 8 years (M = 7.02, SD = 0.68), and were attending 11 schools (22 classrooms). Of these students, 108 were females (37.5%) and 180 were males (62.5%). All subjects included in the sample had been previously analyzed in two earlier published studies. In the first of these, the outcomes of 72 participants were analyzed with the aim to establish the efficacy of the strategy IDR in students without difficulties (González-Castro et al., 2014). In the second study, 216 students participated, and the aim of that study was to specify the benefits of the IDR as a function of the three relevant diagnoses (ADHD; MLD; or ADHD and MLD) in that sample (González-Castro et al., 2016). While these studies showed the effectiveness of the IDR strategy in the improvement of mathematical competence, the outcomes may have varied due to the initial competency, and also given that these learning difficulties can vary in severity (APA, 2013). Hence, taking into account the importance of the initial levels of students in mathematics competency as a predictor of future difficulties, the main goal of the present study was to analyze the combined results of both samples (a total of 288 students), to gauge the effects that initial or baseline levels of mathematics competency may have had on the subsequent efficacy of the IDR intervention.

Students volunteered for the study and presented their parents' informed consent. Initially, as carried out in the above-mentioned earlier studies (González-Castro et al., 2014; González-Castro et al., 2016), a semi-structured interview for parents was applied to rule out other possible learning difficulties or associated disorders, and an intelligence scale for children was used to appraise the possible existence of cognitive deficits or high cognitive capacities. The statistical characteristics of the sample are presented in Table 1.

For the purposes of this study, students were also divided into three levels based on their Math Ability Score (MAS), measured at pre-treatment with the Test of Early Mathematics Abilities (TEMA-3; Ginsburg & Baroody, 2003). The MAS is a standardized score provided by the manual and it is interpreted as medium 100 and standard deviation 15 (Ginsburg & Baroody, 2003). Following the procedure utilized in a recent study by Cueli et al. (2018), three groups were discernable (e.g. low, medium, and high mathematics competency levels) by examination of their 33th and 66th percentiles in the MAS score. The low competency level comprised 103 students with scores below the 33th percentile on the MAS. The medium competency level was made up of 90 students with scores ranging from the 33th to the 66th percentiles on the MAS, and the high competency level included 95 students with scores above the 66th percentile on the MAS. Is necessary to highlight that these three competency-levels specify a student's classification within this particular study-sample, thus a high level is not indicative of a high mathematics proficiency, but rather, a higher level in comparison with the rest of students in this sample.

There were no statistical differences between the groups (EG and CG) in IQ (M = 92.09, SD = 5.973), $F(1, 286) = 0.053$, $p = .819$; or age, $F(1, 286) = 0.050$, $p = .823$. As a function of mathematics competency level, there were also no differences among the three levels (low, medium and high) in terms of IQ, $F(2, 285) = 0.097$, $p = .907$; but there were differences as a function of age, $F(2, 285) = 11.012$, $p < .001$. There were differences in the gender-distribution of males and females in the current sample, $\chi^2(1) = 18.000$, $p < .001$, but not as a function of the diagnosis $\chi^2(3) = 2.778$, $p = .427$.

<Table 1>

Instruments

The *Test of Early Mathematics Abilities TEMA 3* (Ginsburg & Baroody, 2003) is designed to assess children aged 3 years 0 months to 8 years 11 months. The test consists of 72 items designed to assess mathematics competency, and distinguishes between *informal* competencies (41 items) and *formal* competencies (31 items). The *informal* competencies are assessed using four specific subtests: Counting, quantity comparison, informal calculation, and informal concepts.

Similarly, the *formal* competencies are also assessed by means of four specific subtests: Conventionalisms, number facts, formal calculation, and formal concepts. Thus, differentiation among these eight competencies with can allows us to analyze the profiles of students and determine which students have more difficulties, and specifically what those difficulties are. Moreover, the instrument provides a general coefficient, the Mathematical Ability Score (MAS; M = 100, SD = 15). In short, with the TEMA 3, we obtained one general score (MAS), five scores for informal competencies (one general and four specific), and five scores for formal competencies (one general and four specific). According to the examiner's manual, the two-week test–retest reliability of the TEMA-3 is 0.82 and the Cronbach's Alpha for 6, 7 and 8 year-old participants is equal to 0.95 in every case. In the Spanish version, the Cronbach's Alpha for the global sample is 0.92 (0.95 for six years old, 0.94 for seven years old and 0.91 for eight years old students; (Núñez & Lozano, 2010). The total Cronbach´s Alpha for the current sample was 0.91 (0.72 for informal competencies, and 0.92 for formal competencies).

Intervention Program

The Integrated Dynamic Representation (IDR; González-Castro et al., 2014; González-Castro et al., 2016) was the intervention tool implemented. This program has been widely described by Cueli et al. (2017). It consists of four levels of representation, nine levels of working, and three kinds of presentation of the information. This structure allows for working with students with different mathematics competency levels. Also, the way in which it has been developed is designed for students who have had learning difficulties since the first years of school (Cueli et al., 2018).

The administration process was done at four levels of representation: *Representation of Concepts* (selection of the relevant information; Figure 1a, upper panel), *representation of the Links* (situated model; Figure 1a, lower panel), *representation of Questions* (integration of the representations; Figure 1b, upper panel), *reversibility of the Process* (generalization to other contexts; Figure 1b, lower panel).

The program includes 9 main levels in which the activities are sequenced as a function of the degree of difficulty. Every level has 3 secondary sub-levels, making a total of 9 main levels and 27 sub-levels. All of these levels firstly include activities aimed at working on addition-competency, without the need to "carry a number". Then, addition that involves carrying a number, and subtraction *without* carrying a number, are subsequently introduced. Lastly, the competency of combining additions and subtractions is worked on. Importantly, the number-skills are progressively worked on at different intervals (e.g., 1-3, 0-5, 0-9, 0-19, 0-39, etc).

Furthermore, activities are presented following three types of presentation of the information: 1) Iconic presentations (levels 1 to 3: images shown at the first three levels); 2) combined presentation (levels 4 to 6: concepts associated with images/words at the next three levels); and 3) symbolic presentation (levels 7 to 9: statements presented exclusively in linear text at the highest three levels).

The program allows for working with informal and formal competencies to promote the improvement of those aspects in which a student shows more difficulties. The informal competencies are: counting (understanding how the numbers in the program rise and fall as the number of objects increases or decreases); quantity comparison (noting how the numerical data is reflected in the actual number of objects in each of the circles); informal calculation (solving the problem without performing the specific operation, but dragging the objects to the circle providing the final solution); and informal concepts (the child drags the number of objects represented in the numerical data). The formal competencies are: conventionalisms (encoding and decoding of numbers; and how written numbers are symbolic), number facts (mental calculation); formal calculation (performing mechanical operations); and formal concepts (symbolic concept of numbers: how one number can represent the total number of objects). Given that the IDR allows working with those competencies in which students could have more difficulties, is essential to know the efficacy or improvement as a function of the previous level. Also, taking into account that the IDR provides improvement opportunities to students with different mathematics competence level (through the 9 levels and 27 sub-levels included in the IDR).

An example of the intervention steps is shown in Figures 1a and 1b.

Procedure

The sample was randomly selected, using convenience sampling as a function of availability and accessibility, and in cooperation with the Guidance Department of each school (made up of a specialist psychologist with a Master's degree in education). Next, we requested active informed consent from the families. The selected classes were randomly assigned to one of two learning programs: the IDR intervention program or the traditional learning program. The school psychologist (trained in the use of TEMA 3) at each of the 11 schools evaluated the mathematics competencies in both groups. These professionals were supervised by a member of the research team responsible for training in the use of TEMA 3. The intervention program was applied by teachers between the months of January and April (by means of fifty-minute sessions, 4 days per week, with a total of 45 sessions being carried out during the regular mathematics lessons). All the children attended the same number of sessions. These intervention sessions did not replace the regular teaching in mathematics, because IDR is a support or supplement to the regular teaching and cannot replace it. All the mathematics contents officially mandated by the State "classroom program" were worked. The teacher had been previously trained by an expert in the use of the program during two 45-minutes sessions.

On the other hand, the control groups (CG) continued "business as usual" by receiving the traditional curriculum for their grade (involving paper-and-pencil tasks). Traditional instruction involved the teaching of crucial math subject matter (i.e. word problem solving, basic calculations, practice exercises, and reasoning tasks included in the typical reference text-book based on the academic curriculum). Pre- and post-treatment assessments were conducted in the same time frame for both the EG and CG, and all lessons were equivalent in duration for both groups.

Design and data analysis

A quasi-experimental design with a nonequivalent control group was used. A multivariate regression model incorporating the covariates measured at baseline was used to analyze data from factorial design with several dependent random variables. After selecting the most practical model, without ignoring any relationships among the outcome variables, we focused on testing the effects of the fitted model. All the multivariate effects were statistically significant. The next step was to probe the data further to interpret the nature of the specific differences, especially those relating to the interaction effects (group by levels). For that, we concentrated on procedures for locating significant tetrad contrasts or contrast-contrast interactions (both multivariate and univariate) to identify any differences in the functionality of the intervention between the different levels of the program). All analyses presented here assume the treatment effect is constant across different covariates. In addition, partial eta-squared, η_p^2, was used as a measure of effect size because it is the most commonly used parameter in educational research literature (Sun, Pan, & Wang, 2010), although omega-squared is considered a less biased estimator of population variance. Applying Cohen's (1988) classic work to this study's interaction contrasts, a "small" association is defined as η_p^2= .010 (equivalent to Cohen's d = .20), a "medium" association is η_p^2 = .059 (equivalent to Cohen's d = .50), and a "large" association is η_p^2 = .138 (equivalent to Cohen's d = .80). Although Cohen did not explicitly consider multivariate regression models, the same guidelines are also appropriate.

The specific implementation of multivariate regression models was always made fitting unstructured (UN) covariance with parameters obtained by restricted maximum likelihood (REML) estimation as implemented in SAS PROC MIXED (version 9.4, 2014). This model assumes that the outcome measurements follow a multivariate normal distribution and exhibit a common covariance structure. Here we use Mardia's skewness and kurtosis measures, as implemented in SAS Proc Calis, for testing multivariate normality and a likelihood ratio test to assess if the four variances and six covariances are equivalent for the three populations from which the data were sampled. Additionally, to explain the group by levels interaction in a manner consistent with the objectives of the research, we computed the corresponding tetrad contrasts. In order to control the family-wise error (FWE) rate for all possible tetrad contrasts on the two sets of four dependent variables analyzed simultaneously, the Hochberg (1988) step-up Bonferroni inequality was run using the ESTIMATE statement in SAS PROC MIXED and the HOC option in SAS PROC MULTTEST.

Results

The results are presented in two sections: multivariate regression analyses and univariate regression analyses. The means and standard deviations of these variables (based on the direct scores or total of correct responses) are provided in Table 2.

<Table 2>

Multivariate regression analyses

According to the results in Table 3, in the two sets of dependent variables there are significant differences between the EG and CG (when averaging the results of all three levels of competency), in the set of the five dependent variables for informal competencies and formal competencies that were considered simultaneously. Also, we can see that the differences among the conditions of variable levels averaged across the groups in informal competencies, but not in formal competencies by simultaneously considering all dependent variables. Consequently, participants’ performance differs over levels, however, we note that the pattern of change is not deemed the same for the two groups (EG and CG).

<Table 3>

As with many factorial structures, the main task of deduction is to explain the performances of the groups by way of “levels-interactions”, in a manner consistent with the objectives of the research. As can be seen in Table 4, the value of the differences in the average performances of the groups studied (EG and CG) are not the same in all levels of competency (low, medium and high). For this reason, we have used tests of interaction contrasts or tetrad contrasts, which provide with additional information about the relation of the groups with the levels.

<Table 4>

The results reported in Table 4 show that applying Hochberg's sequentially rejective Bonferroni procedure, all tetrad contrasts were statistically significant controlling FWE. One of the most interesting observations of the study was that the differences between the low, medium and high levels were larger in the EG than in the CG (in particular, when the set of informal dependent variables were considered simultaneously). In addition, differences between groups occurred, to a large degree, when considering the contrasts low– high y low–medium of the levels variable. Table 2 shows the proportion of the sample variance accounted for by the six tetrad contrasts.

Univariate regression analyses for each dependent variable

A set of follow-up univariate regression analyses were performed to determine which of the four dependent variables in the two sets (informal and formal) were related to the significant omnibus test of groups by the level-interactions. Table 5 includes the results of the hypotheses tests for each dependent variable. Although we present the results for all fixed effects, we discuss only the interactions. In Table 5 we can see that (with the exception of *'Informal calculation'*) the level of significant differences between the intervention factors was $\geq$ 0.01 for all outcome variables, thus the null hypothesis (no interaction between the intervention factors) was rejected.

<Table 5>

As previously indicated, a useful method of assessing the interaction effect is to perform a series of tetrad contrasts. The present study followed Hochberg's step-up procedure to assess statistical significance, and the tetrad contrasts found to be significant are shown in Table 6. In general, the results provided in this table support those obtained in the multivariate analyses, with the differences between the level also found to be greater in the EG, than in the CG. Table 5 also shows the proportion of population variance accounted for in the series of tetrad contrasts. Notably, the values of η_p^2 for the tetrad contrasts declared statistically significant (controlling for FWE), and ranged from .016 to .113.Although not shown in the table, we should note that the main effect of group accounted for considerably more variance (η_p^2values ranged from .457 to .645 for the set of formal dependent variables, and from .412 to .576 for the set of informal dependent variables) than the group by level interaction.

<Table 6>

Discussion

The chief aim of this study was to determine the influence of initial levels of mathematics competency in relation to performance gains after an intervention with the IDR in elementary school students (first and second grades). More specifically, the fundamental question was: *“Is the efficacy of the intervention modulated by the competencies levels?”*

With this in mind, when taking the initial or baseline levels into account, the results showed that they had a moderate effect on the outcome. Although in the present IDR intervention all the three competency levels of the EG improved, the progression was different for each level. As was hypothesized, students with the lowest level of competency improved more than those with medium and high levels. This reinforces the importance of adapting tasks to the students’ individual levels and needs (Cueli, González-Castro, Krawec, Núñez, & González-Pienda, 2016). It is especially relevant when taking into account that the initial levels of mathematics competency can predict the later achievement levels of students (Purpura, Baroody, & Lonigan, 2013). In any case, one explanation for the improvement in students with low competency levels in the present study could be related with the findings of

Kroesbergen and van Luit (2003), who reported that the most effective techniques in the intervention with students with MLD include direct teaching and strategy-based teaching. These are main features of the IDR, in which the student receives explicit guidance throughout the problem-solving process, and the instructions remain present during each of the steps.

However, it is necessary to highlight that in the CG, students did not improve their informal and formal competencies, especially those students with low initial mathematical competency levels. So, it is imperative that students at risk of MLD (e.g. students who start kindergarten with poor mathematical skills) are given the chance to develop their abilities by means of specific programs aimed at addressing such difficulties (such as the IDR), and thereby also prevent an escalation of these problems in the future.

When considering the between-group differences further, specific variables (e.g. *Counting*, *Quantity comparisons*, and *Informal concepts*) were found to be significantly different between EG and CG, and also between the low, medium and high levels. Moreover, there were significant interactions between these levels. Children usually acquire these skills through spontaneous interaction with their individual environments, which is one of the reasons there are large differences in mathematical abilities by the time children begin school.

Implications for Practice

Bearing this in mind, it is important to highlight the need to implement specific intervention strategies in the daily management of the classroom, as this leads to improving informal and formal mathematical competencies. These strategies must to be adapted to the student's level and the results of the interventions need to be analyzed as a function of it (initial mathematics competency level), given that not all the students have the same needs or academic profiles. Also, taking into account the differentiation stablished in this work (low, medium and high levels) and the improvement of students, is necessary and provides good results, to implement interventions even before of specific diagnosis (as MLD or ADHD), and students with low competencies or lower initial levels will be benefited. The early identification of children at risk of low achievement in mathematics is crucial because it provides the opportunity to mitigate the consequent effects (Navarro et al., 2012) and to analyze the response to the interventions previously to the diagnosis. Implementing interventions at this stage, is significant taking into account that Navarro et al. (2012) in a longitudinal study showed that students with a very low score in mathematics competencies at the age of 5 years, also obtained a very low score at the age of 7 years.

Furthermore, although many new technologies are currently used in schools, in most cases they are simply a digitalization of pre-existing information without any change to traditional teaching methodologies. However, these digitalized programs have the latent potential of being highly effective, as they can easily be adapted to the needs of individual learners by systematically and dynamically providing the scaffolding of key learning processes (Azevedo, Moos, Johnson, & Chauncey, 2010).

Directions for Future Research

In the future it will be interesting to assess the processes that the students carry out, using for example *Think Aloud* protocols in which students describe every thought that they have and task that they do.

With regard to accessing the IDR strategy, it is available to teachers, families and students on a web-site in which also shows the specific instructions necessary to administer this novel intervention tool. Concerning future developments in utilizing this intervention tool, it would be very interesting to keep a record of every “click” of the students, and the period that they required to carry out the exercises. In addition, it would be intriguing to see the potential benefits of promoting this information not only to researchers, but also to teachers, students and families.

Limitations

This study has the following limitations that must be taken into account. One limitation is related to the assessment carried out, in which only the result or outcome of the intervention was considered. In future studies, it would perhaps be appropriate to also assess the processes performed by the students in a step-by-step fashion. The sample selection (as a function of the accessibility) may be a potential limitation of the study, though one might take into account the difficultly of carrying out interventions in schools, given the inevitable disruption to their everyday curricula procedures. Lastly, it is necessary to highlight that the sample included students with specific learning difficulties (MLD and ADHD). Nonetheless, in everyday classes teachers have to work with students with different profiles. Teacher could be a different way to work with IDR strategy, with some of these profiles.

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Table 1. Means and Standard Deviations of age, gender, IQ, and MAS in the three groups

	Total		Mathematics Competency Level								
			Low			Medium			High		
	EG	CG	EG	CG	Total	EG	CG	Total	EG	CG	Total
	n = 140	*n* = 148	*n* = 61	*n* = 42	*n* = 103	*n* = 40	*n* = 50	*n* = 90	*n* = 39	*n* = 56	*n* = 95
Age *M(SD)*	6.83	6.82	6.67	7.00	6.83	7.000	6.50	6.75	6.86	6.83	6.85
	(0.93)	(0.87)	(1.15)	(1.00)	(0.98)	(0.00)	(0.70)	(0.50)	(1.06)	(0.98)	(0.98)
Gender M/F	89 / 51	91 / 57	35 / 26	25 / 17	60 / 43	24 / 16	29 / 21	53 / 37	30 / 9	37 / 19	67 / 28
ADHD	35	37	1	3	4	15	11	26	19	23	42
MLD	40	42	35	22	57	5	14	19	0	6	6
A+M	30	32	21	11	32	9	14	23	0	7	7
No LD	35	37	4	6	10	11	11	22	20	20	40
IQ *M(SD)*	92.007	92.168	91.803	92.375	92.029	92.850	91.220	91.933	91.487	92.875	92.305
	(5.877)	(6.082)	(5.895)	(4.343)	(5.301)	(4.887)	(6.270)	(5.723)	6.774)	(6.959)	(6.882)
MAS *M(SD)*	76.87	81.30	65.67	69.62	67.28	81.10	81.00	81.04	90.05	90.34	90.22
	(11.599)	(9.568)	(6.755)	(6.208)	(3.792)	(2.405)	(2.515)	(2.454)	(3.967)	(4.780)	(4.444)

Note. EG = experimental group; CG = control group; *M* = Mean; *SD* = Standard Deviation; M/F = Male/female; ADHD = Attention Deficit Hyperactivity Disorder; MLD = Mathematics learning difficulties; A+M = Students with ADHD and MLD; No LD = Students without ADHD and MLD.

Table 2. Means and Standard Deviations for direct scores in informal and formal competencies in the EG and CG

	EG						CG						Min/Max (N = 288)	
	Low (*n* = 61)		Medium (*n* = 40)		High (*n* = 39)		Low (*n* = 42)		Medium (*n* = 50)		High (*n* = 56)			
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
	M(SD)	*M(SD)*	*M(SD)*	*M(SD)*	*M(SD)*	*M(SD)*	*M(SD)*	*M(SD)*	*M(SD)*	*M(SD)*	*M(SD)*	*M(SD)*		
Counting	13.96 (3.41)	19.11 (3.24)	15.47 (3.44)	18.90 (2.95)	16.48 (3.53)	18.69 (3.48)	15.59 (2.94)	16.02 (2.91)	15.10 (3.29)	14.90 (3.52)	15.55 (3.67)	15.00 (3.91)	8/22	0/23
Quantity comparison	3.11 (0.91)	4.83 (0.98)	3.80 (1.04)	4.75 (0.80)	4.02 (0.84)	4.92 (1.01)	3.73 (0.66)	3.57 (0.76)	3.52 (0.88)	3.18 (0.96)	3.73 (0.77)	3.73 (0.90)	1/5	1/6
Informal calculation	3.09 (0.78)	4.90 (0.86)	4.02 (0.73)	5.00 (0.71)	4.02 (0.93)	5.00 (1.46)	3.50 (0.59)	3.64 (0.79)	3.82 (0.77)	3.52 (0.90)	4.07 (0.79)	4.00 (0.78)	2/5	2/8
Informal concepts	1.88 (0.63)	3.52 (0.69)	1.97 (0.35)	3.22 (0.61)	2.17 (0.38)	3.28 (0.60)	2.52 (0.74)	2.54 (0.59)	2.40 (0.69)	2.14 (0.70)	2.67 (0.60)	2.53 (0.76)	0/4	1/5
Informal competencies	**22.06 (5.06)**	**32.37 (5.26)**	**25.27 (4.99)**	**31.87 (4.42)**	**26.71 (5.42)**	**31.89 (5.96)**	**25.35 (3.96)**	**25.78 (4.08)**	**24.84 (4.85)**	**23.74 (5.28)**	**26.03 (4.99)**	**25.26 (5.44)**	**11/35**	**9/41**
Conventionalisms	3.83 (1.25)	6.29 (1.46)	4.90 (1.31)	6.12 (1.20)	5.43 (1.50)	5.97 (1.58)	4.40 (0.96)	4.16 (0.93)	4.40 (0.95)	4.20 (1.12)	4.62 (1.05)	4.28 (1.00)	1/8	2/8
Number facts	1.37 (1.39)	3.55 (2.55)	1.30 (1.78)	2.67 (2.09)	2.15 (2.43)	3.25 (3.40)	1.97 (1.88)	1.95 (1.78)	1.74 (1.86)	1.82 (1.87)	1.75 (2.21)	1.51 (1.92)	0/6	0/9
Formal calculation	1.262 (1.13)	3.37 (2.93)	1.00 (0.96)	2.23 (1.77)	1.10 (1.25)	2.64 (2.83)	1.78 (1.60)	1.85 (1.50)	1.26 (1.33)	1.44 (1.43)	1.33 (1.44)	1.39 (1.48)	0/4	0/8
Formal concepts	0.85 (0.67)	2.72 (1.26)	1.10 (0.67)	2.42 (1.03)	1.41 (1.04)	2.69 (1.50)	1.42 (0.70)	1.50 (0.63)	1.38 (0.75)	1.44 (0.90)	1.55 (0.78)	1.58 (0.70)	0/3	0/5
Formal competencies	**7.32 (3.96)**	**15.950 (0.93)**	**8.30 (4.23)**	**13.55 (5.66)**	**10.102 (6.04)**	**14.56 (9.05)**	**9.59 (4.51)**	**9.47 (4.35)**	**8.82 (4.35)**	**8.90 (4.81)**	**9.26 (5.03)**	**8.78 (4.58)**	**1/20**	**2/30**

Note. M = Mean; *SD* = Standard Deviation; EG = experimental group; CG = control group.

Table 3. Results of fitting two multivariate regression model analyses (left panel, first set of dependent variables- Informal competencies; right panel, second set of dependent variables- Formal competencies)

Source	df_N	df_D	F-value	Pr > F	Source	df_D	F-value	Pr > F
Groups	4	270	167.26	< .0001	Groups	271	152.41	< .0001
Levels	8	384	6.30	< .0001	Levels	385	1.49	.1587
Groups × Levels	8	384	4.22	< .0001	Groups × Levels	385	7.37	< .0001
Age	4	270	2.49	.0440	Age	271	3.82	.0048
IQ	4	270	2.65	.0338	IQ			n.s.
Diagnostic	12	468	11.02	< .0001	Diagnostic	470	7.74	< .0001
Counting	4	270	68.36	< .0001	Conventionalisms	271	13.15	< .0001
Quantity comparison	4	270	7.94	< .0001	Number facts	271	31.82	< .0001
Informal calculation	4	270	3.27	.0121	Formal calculation	271	11.82	< .0001
Informal concepts	4	270	13.23	< .0001	Formal concepts	271	26.14	< .0001

Note. The lowest AIC value was selected as the best fit model*;* df_N = numerator degrees of freedom (df); df_D = denominator df.

Table 4. Hochberg's adjusted *p* values for all possible tetrad contrasts by simultaneously considering all dependent variables

Groups	Levels	df_N	df_D	F-value	Pr > F	Adj p	Partial η^2 Sample (Lower−Upper)		
Counting, Quantity comparison, Informal calculation and Informal concepts									
EG *vs* CG	$N_1 - N_3$	4	273	13.82	< .0001	.0001	.168	.086	.237
EG *vs* CG	$N_1 - N_2$	4	273	4.62	.0321	.0366	.063	.011	.114
EG *vs* CG	$N_2 - N_3$	4	273	4.41	.0367	.0367	.061	.009	.111
Conventionalisms, Number facts, Formal calculation and Formal concepts									
EG *vs* GC	$N_1 - N_3$	4	274	31.47	< .0001	.0001	.316	.221	.390
EG *vs* CG	$N_1 - N_2$	4	274	12.82	.0004	.0008	.158	.078	.226
EG *vs* CG	$N_2 - N_3$	4	274	4.12	.0437	.0437	.057	.008	.106

Note. EG = experimental group; CG = control group; N_3 = High level; N_2 = Medium level; N_1= Low level. The lowest AIC value was selected as the best fit model.

Table 5. Results of fitting eight univariate regression model analyses (left panel, first set of dependent variables; right panel, second set of dependent variables)

Source	df_N	df_D	F	Pr > F	Source	df_D	F	Pr > F
Counting					*Conventionalisms*			
Groups	1	277	497.26	< .0001	Groups	277	371.21	< .0001
Levels	2	277	13.29	< .0001	Levels	277	0.90	.4104
Groups × Levels	2	277	9.19	.0001	Groups × Levels	277	17.81	< .0001
Age					Age	277	40.17	< .0001
IQ	1	277	9.07	.0028	IQ			
Diagnostic	3	277	17.26	< .0001	Diagnostic	277	15.51	< .0001
Pre-test	1	277	313.71	< .0001	Pre-test	277	60.42	< .0001
Quantity comparison					*Number facts*			
Groups	1	277	325.04	< .0001		277	211.42	< .0001
Levels	2	277	4.87	.0084		277	0.48	.6201
Groups × Levels	2	277	7.06	.0010		277	10.19	< .0001
Age	1	277	29.09	< .0001		277	38.20	< .0001
Diagnostic	3	277	28.27	< .0001		277	7.43	< .0001
Pre-test	1	277	36.45	< .0001		277	292.61	< .0001
Informal calculation					*Formal calculation*			
Groups	1	280	230.40	< .0001		277	192.97	< .0001
Levels						277	1.20	.3025
Groups × Levels						277	7.69	.0006
Age	1	280	266.78	< .0001		277	41.07	< .0001
IQ	1	280	6.91	.0009				
Diagnostic	3	280	7.34	< .0001		277	7.78	< .0001
Pre-test	1	280	5.16	.0239		277	170.15	< .0001
Informal concepts					*Formal concepts*			
Groups	1	277	296.91	< .0001		277	348.75	< .0001
Levels	2	277	5.38	.0051		277	0.56	.5736
Groups × Levels	2	277	4.87	.0084		277	9.25	.0001

Age	1	277	29.91	< .0001		277	52.74	< .0001
Diagnostic	3	277	8.60	< .0001		277	14.5	< .0001
Pre-test	1	277	65.83	< .0001		277	65.83	< .0001

Note. The lowest AIC value was selected as the best fit model*;* df_N = numerator degrees of freedom (df); df_D = denominator df. Pre-test variables were included like covariates

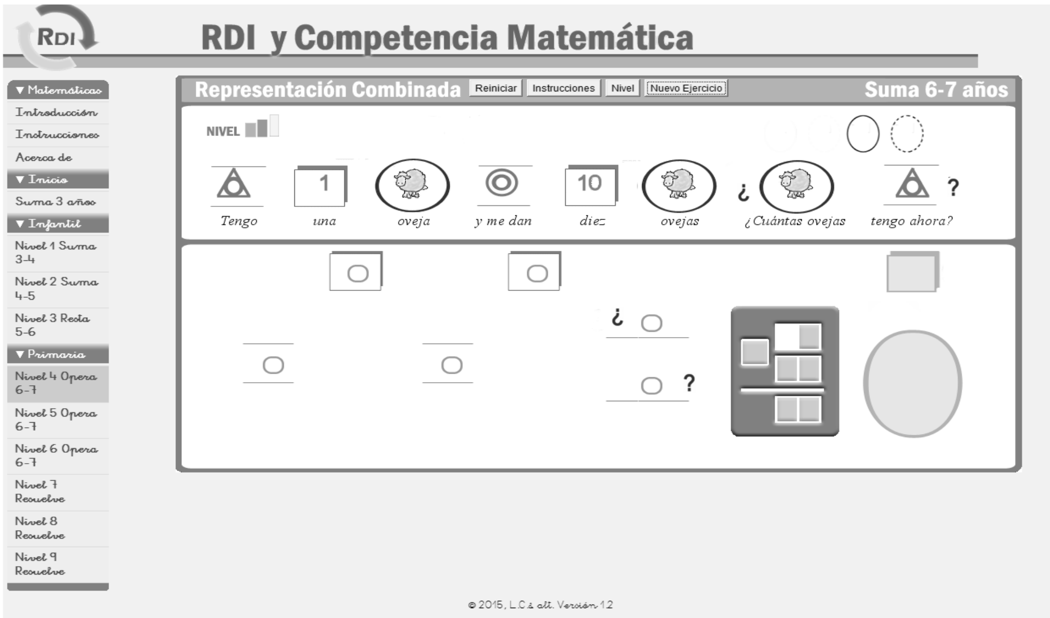
Table 6. Hochberg's adjusted *p* values based on considering all possible tetrad contrasts for each dependent variables

Groups	Levels	df_N	df_D	F-value	Pr > F	Adj p	Partial η^2 Sample (Lower−Upper)		
Counting									
EG *vs* CG	N_1-N_3	1	276	18.40	< .0001	.0001	.063	.013	.125
EG *vs* CG	N_1-N_2	1	276	4.58	.0331	.0355	.017	.000	.058
EG *vs* CG	N_2-N_3	1	276	4.45	.0356	.0355	.016	.000	.057
Quantity comparison									
EG *vs* CG	N_1-N_3	1	276	12.46	.0005	.0015	.044	.008	.101
EG *vs* CG	N_2-N_3	1	276	8.18	.0046	.0093	.029	.003	.078
EG *vs* CG	N_1-N_2	1	276	0.37	.5093	.5093	.000	.000	.000
Informal concepts									
EG *vs* CG	N_1-N_3	1	276	8.52	.0038	.0114	.031	.003	.080
EG *vs* CG	N_2-N_3	1	276	5.86	.0162	.0324	.021	.001	.065
EG *vs* CG	N_1-N_2	1	276	0.20	.6499	.6499	.000	.000	.000
Conventionalisms									
EG *vs* CG	N_1-N_3	1	276	34.46	< .0001	.0001	.113	.051	.185
EG *vs* CG	N_1-N_2	1	276	13.10	.0003	.0006	.046	.010	.102
EG *vs* CG	N_2-N_3	1	276	5.11	.0248	.0248	.018	.000	.061
Number facts									
EG *vs* CG	N_1-N_3	1	276	15.60	.0001	.0003	.054	.014	.114
EG *vs* CG	N_1-N_2	1	276	14.98	.0002	.0004	.052	.013	.111
EG *vs* CG	N_3-N_3	1	276	0.01	.9114	.9114	.000	.000	.000
Formal calculation									
EG *vs* CG	N_1-N_2	1	276	12.75	.0004	.0013	.045	.009	.101
EG *vs* CG	N_1-N_3	1	276	9.49	.0023	.0045	.034	.004	.084
EG *vs* CG	N_2-N_3	1	276	0.27	.6204	.6204	.000	.000	.000
Formal concepts									
EG *vs* CG	N_1-N_3	1	276	16.32	< .0001	.0002	.058	.016	.119
EG *vs* CG	N_1-N_2	1	276	10.76	.0012	.0023	.039	.006	.092
EG *vs* CG	N_3-N_3	1	276	0.25	.4651	.4651	.000	.000	.000

Note. EG = experimental group; CG = control group; N_3 = High level; N_2 = Medium level; N_1= Low level; df_N = numerator degrees of freedom (df); df_D = denominator df.

Upper panel (Step 1)

Representation of concepts (selection of the relevant information) in which the key concepts are presented, associated with drawings, the numerical data that accompany them are framed in squares and the verbs are replaced with pictograms



Lower panel (Step 2)

Representation of the links (iconic-symbolic combination) in which, after the key concepts are identified, they are represented in union-intersection sets, whose number of elements is specified by the numerical data.

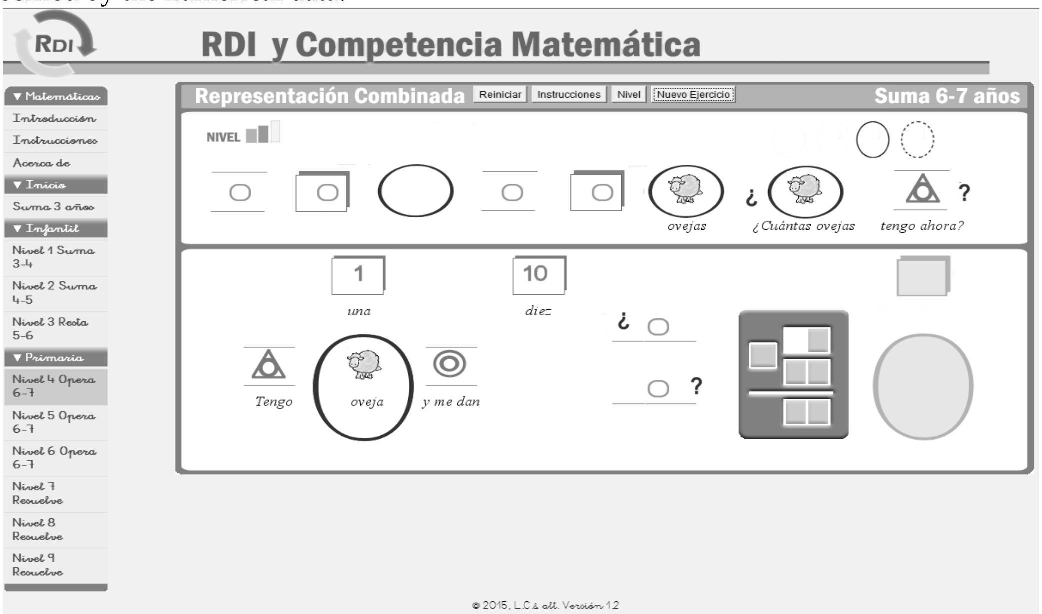
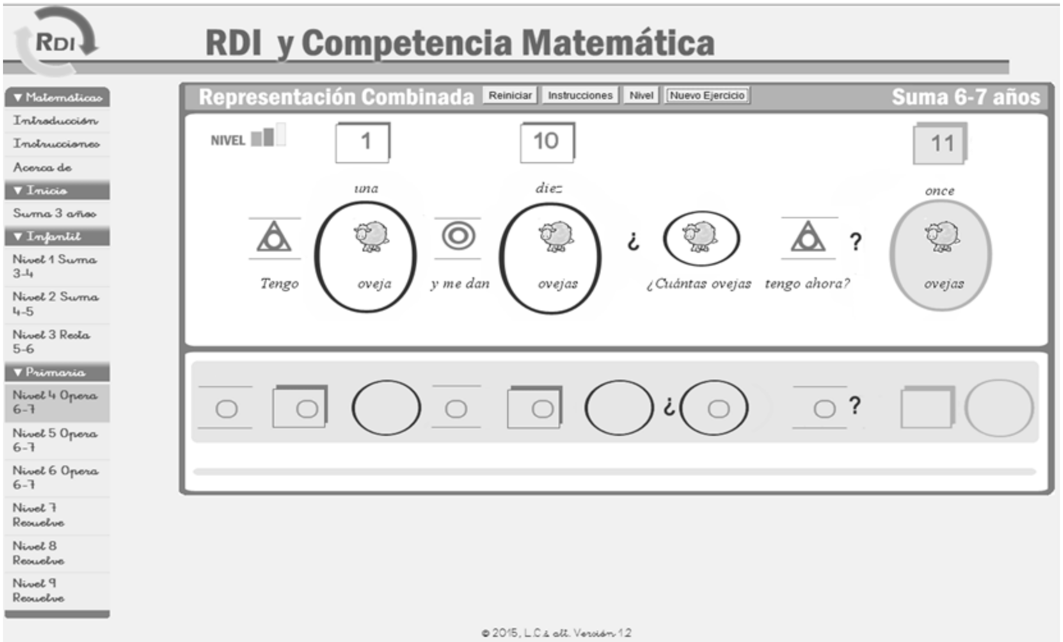


Figure 1a.Description of intervention program for the acquisition of basic math skills; and Steps 1 (upper panel) and 2 (lower panel).

Upper panel (Step 3)

Representation of the questions (integration of the representations). At this level, the representations are connected to each other, depending on the types of relationships of the links to the statement: union (addition) and intersection (subtraction). When the problem is solved the student has a situated model.



Lower panel (Step 4)

Reversibility of the process (generalization to other contexts), where the subject is asked to re-formulate the problem statement, without taking the initial statement into account, on the basis of the integrated representation that leads to the final solution.

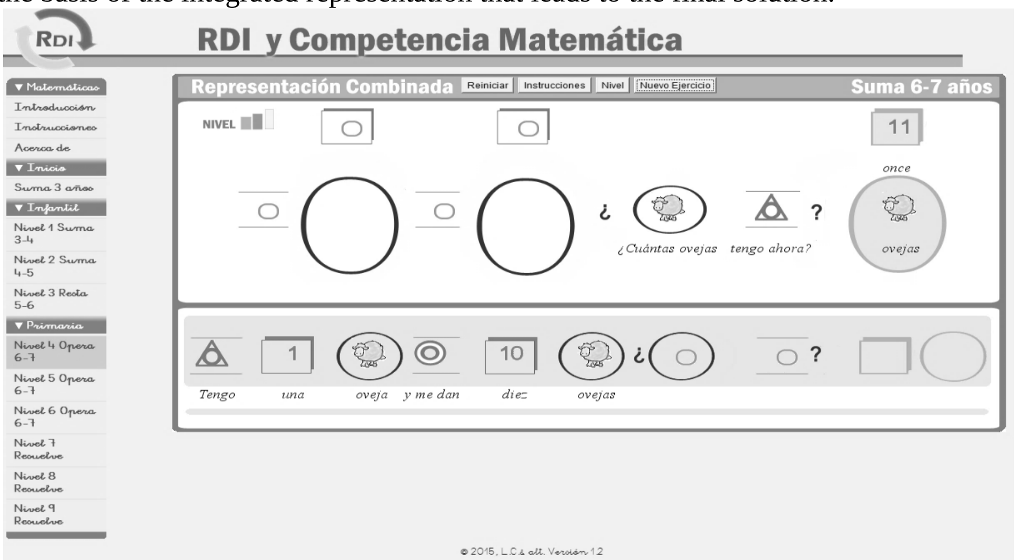


Figure 1b.Description of intervention program for the acquisition of basic math skills; and Steps 3 (upper panel) and 4 (lower panel).