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# H-FGK formalism for black-hole solutions of $N = 2$, $d = 4$ and $d = 5$ supergravity

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ABSTRACT

We rewrite the Ferrara–Gibbons–Kallosh (FGK) black-hole effective action of $N = 2$, $d = 4, 5$ supergravities coupled to vector multiplets, replacing the metric warp factor and the physical scalars with real variables that transform in the same way as the charges under duality transformations, which simplifies the equations of motion. For a given model, the form of the solution in these variables is the same for all spherically symmetric black holes, regardless of supersymmetry or extremality.



## Introduction

During the past 20 years a huge effort has been made to classify, construct and study black-hole solutions of 4- and 5-dimensional supergravity theories. Most of this work has been devoted to the extremal black-hole solutions and, in particular, to the supersymmetric ones. This was partly due to their very special properties, such as their classical and quantum stability, the attractor mechanism [1], which makes their entropies understandable and computable from a microscopic point of view [2], and partly due to their functional simplicity, which allows for the explicit and systematic construction of all of them (see *e.g.* Ref. [3]).

The territory of non-extremal black holes, which includes as its boundary both supersymmetric and non-supersymmetric extremal solutions, although potentially more interesting, remains largely unexplored.[1] Recently, in Refs. [5,6], we have attempted to make the construction of non-extremal black-hole solutions of $N = 2$, $d = 4, 5$ supergravity coupled to vector supermultiplets as systematic as that of the extremal supersymmetric ones. The proposal is based on a deformation of supersymmetric extremal solutions.

The fields of the supersymmetric black-hole solutions of $N = 2$, $d = 4$ and $d = 5$ supergravity coupled to $n$ vector supermultiplets are given in terms of functions $H^M$, $M = 1, \dots, 2n + 2$ and $H_I$, $I = 1, \dots, n + 1$, respectively, that transform linearly under the generic duality groups $Sp(2n + 2, \mathbb{R})$ and $SO(n + 1)$. The equations of motion are satisfied when these functions are harmonic on the transverse $\mathbb{R}^{d-1}$ space (see Refs. [7,8] and [9]). The functional dependence of the physical fields on the variables $H$ is essentially unique, since the linear action of the duality groups on the $H$'s must produce a fixed non-linear transformation of the scalars and leave the spacetime metric invariant. It is therefore natural to expect that all black-hole solutions of the same model have the same functional form in terms of the variables $H$, although in the non-extremal case or with gaugings these variables will have to satisfy different equations and will no longer be harmonic.[2] A proposal made and checked for several models in Refs. [5,6] suggests that for non-extremal black holes $H$ could be exponential or hyperbolic functions, but it is mandatory to examine both the universality of solutions expressed in $H$ and of the hyperbolic ansatz.

The formalism introduced by Ferrara, Gibbons and Kallosh (FGK) in Ref. [17], and generalized to arbitrary dimensions in Ref. [6], provides us with a convenient setting to investigate these issues. Thanks to a suitable choice of space-time coordinates for spherically symmetric black holes and to the replacement of the vector fields by the charges, the bosonic sector of the supergravity action is reduced to an effective action in ordinary mechanics, whose only remaining dynamical variables are the scalar fields and the metric function $U$; the role of the evolution parameter is played by a radial coordinate. Black holes correspond to stationary points of this effective action.

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[1] For some earlier works on non-extremal black-hole solutions see Refs. [4].

[2] Here we will only deal with the ungauged cases and we will not include hypermultiplets, but experience shows that in the gauged cases [10–14] and in the cases with hyperscalars [15,16] the $H$'s always appear in the same way in the physical fields. In the cases with hyperscalars the $H$-functions are still harmonic, but on spaces more general than $\mathbb{R}^{d-1}$.

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To answer the first question, in this Letter we adapt the FGK formalism by rewriting the FGK effective action in terms of the variables $H$, which will be the new degrees of freedom. The field redefinition is modelled on the functional form of the extremal supersymmetric black-hole solutions, but depends neither on supersymmetry nor extremality. The fact that such a change of variables is possible means that, as functions of the $H$'s, all black-hole solutions of a given model have the same form as the supersymmetric solution. The action, expressed in the new variables, takes a particularly simple $\sigma$-model form with the same Hessian metric[3] occurring both in the kinetic term for the scalars and in the effective potential generated by the gauge fields. For the 5-dimensional case, following a different, more general, formalism developed in Ref. [19], the same results have recently been obtained in Ref. [20]. The analogous results for the 4-dimensional case are presented here for the first time.[4]

Having derived the equations of motion for the variables $H$, we can address the second problem: Are all extremal black-hole solutions given by harmonic functions, also in the non-supersymmetric case? Are the non-extremal black-hole solutions always given by hyperbolic functions? This will be the subject of a forthcoming publication [22].

## 1. H-FGK for $N = 2, d = 5$ supergravity

We find it more natural to present our proposal first in five dimensions. In $N = 2$, $d = 5$ supergravity coupled to $n$ vector multiplets the physical fields defining a black-hole solution with given electric charges $q_I$ $(I = 0, \dots, n)$ are the metric function $U$ and the $n$ real scalars $\phi^x$. Through a field redefinition we will replace them by $n + 1$ variables denoted by $H_I$. We will also define a set of $n + 1$ dual variables $\tilde{H}^I$, which are useful for intermediate steps in our calculation, but can also be used for finding other kinds of solutions [23].

The starting point[5] is the function $\mathcal{V}(h^\cdot)$, homogeneous of third degree in $n + 1$ variables $h^I$,

$$\mathcal{V}\big(h^\cdot\big) \equiv C_{IJK} h^I(\phi) h^J(\phi) h^K(\phi), \tag{1.1}$$

which defines the scalar manifold as the hypersurface $\mathcal{V} = 1$. The dual scalar functions $h_I$ are defined in terms of the $h^I$ by

$$h_I \equiv \frac{1}{3}\frac{\partial \mathcal{V}}{\partial h^I}, \tag{1.2}$$

and are, therefore, homogenous of second degree in the $h^I$.

Clearly, as a function of the $h_I$, the function $\mathcal{V}$ is homogeneous of degree $3/2$; this relation between the homogeneity degree of the function $\mathcal{V}$ when expressed in different variables is a standard implication of a Legendre transform, whence we define a new function $\mathcal{W}(h_\cdot)$ by

$$\mathcal{W}(h_\cdot) \equiv 3 h_I h^I(h_\cdot) - \mathcal{V}\big(h^\cdot\big) = 2\mathcal{V}(h_\cdot), \tag{1.3}$$

which is homogenous of degree $3/2$. The Legendre transform then immediately implies that

$$h^I \equiv \frac{1}{3}\frac{\partial \mathcal{W}}{\partial h_I}. \tag{1.4}$$

Next we introduce two new sets of variables $H_I$ and $\tilde{H}^I$, related to the physical fields $(U, \phi^x)$ by

$$H_I \equiv e^{-U} h_I(\phi), \qquad \tilde{H}^I \equiv e^{-U/2} h^I(\phi), \tag{1.5}$$

and two new functions $\mathsf{V}$ and $\mathsf{W}$, which have the same form in the new variables as $\mathcal{V}$ and $\mathcal{W}$ had in the old, *i.e.*

$$\mathsf{V}(\tilde{H}) \equiv C_{IJK} \tilde{H}^I \tilde{H}^J \tilde{H}^K, \qquad \mathsf{W}(H) \equiv 3\tilde{H}^I H_I - \mathsf{V}(\tilde{H}) = 2\mathsf{V}, \tag{1.6}$$

but which are not constrained. These functions inherit the following properties from $\mathcal{V}$ and $\mathcal{W}$:

$$H_I \equiv \frac{1}{3}\frac{\partial \mathsf{V}}{\partial \tilde{H}^I}, \tag{1.7}$$

$$\tilde{H}^I \equiv \frac{1}{3}\frac{\partial \mathsf{W}}{\partial H_I} \equiv \frac{1}{3}\,\partial^I \mathsf{W}. \tag{1.8}$$

Using the homogeneity properties we find that

$$e^{-\frac{3}{2}U} = \frac{1}{2}\mathsf{W}(H), \tag{1.9}$$

$$h_I = (\mathsf{W}/2)^{-2/3} H_I, \tag{1.10}$$

$$h^I = (\mathsf{W}/2)^{-1/3} \tilde{H}^I. \tag{1.11}$$

We can use these formulae to perform the change of variables in the FGK action for static, spherically symmetric black holes of $N = 2$, $d = 5$ supergravity [6], which in our conventions reads

[3] A metrical manifold is said to be a Hessian manifold if there exists a coordinate system in which the components of the metric can be obtained as the Hessian of some function. Hessian metrics appear quite naturally in supergravity theories [20,21] and also in global supersymmetry [18].

[4] When the present work was being readied for publication, Ref. [21] appeared, with similar results for the 4-dimensional case in a more general framework.

[5] Our conventions are those of Refs. [24,16].

$$\mathcal{I}_{\rm FGK}\big[U,\phi^x\big]=\int d\rho\,\big\{(\dot U)^2+a^{IJ}\dot h_I\dot h_J+e^{2U}a^{IJ}q_Iq_J+\mathcal{B}^2\big\}. \tag{1.12}$$

It can be shown that

$$a^{IJ}=-\frac{2}{3}(\mathsf{W}/2)^{4/3}\partial^I\partial^J\log\mathsf{W}, \tag{1.13}$$

thus the above action, in terms of the $H_I$ variables, takes the form

$$-\frac{3}{2}\mathcal{I}[H]=\int d\rho\left\{\partial^I\partial^J\log\mathsf{W}(\dot H_I\dot H_J+q_Iq_J)-\frac{3}{2}\mathcal{B}^2\right\}. \tag{1.14}$$

The combination $\partial^I\partial^J\log\mathsf{W}$ appearing in the above $\sigma$-model acts as a metric, so we are dealing with a mechanical problem defined on a Hessian manifold. As is well known, the $\rho$-independence of the Lagrangian implies the conservation of the Hamiltonian $\mathcal{H}$. In the FGK formalism, however, not all values of the energy are allowed and there is a restriction called the Hamiltonian constraint. In the new variables this constraint reads

$$\mathcal{H}\equiv\partial^I\partial^J\log\mathsf{W}(\dot H_I\dot H_J-q_Iq_J)+\frac{3}{2}\mathcal{B}^2=0. \tag{1.15}$$

The equations of motion derived from the effective action (1.14) are[6]

$$\partial^K\partial^I\partial^J\log\mathsf{W}(\dot H_I\dot H_J-q_Iq_J)+2\partial^K\partial^I\log\mathsf{W}\ddot H_I=0. \tag{1.16}$$

Multiplying by $H_K$ and using the homogeneity properties of W and the Hamiltonian constraint we get

$$\partial^I\log\mathsf{W}\ddot H_I=\frac{3}{2}\mathcal{B}^2, \tag{1.17}$$

which is equivalent to the equation for the metric factor $U$ that one would obtain from the action Eq. (1.14) expressed in the new variables.

Eqs. (1.16) are all the equations that need to be satisfied, but it can be helpful to first solve the Hamiltonian constraint (1.15) or the equation of motion of $U$, Eq. (1.17).

Observe that in the extremal case $\mathcal{B}=0$, the equations of motion can be always satisfied by harmonic functions $\dot H_I=q_I$.

## 2. H-FGK for $N=2$, $d=4$ supergravity

In the 4-dimensional case we also want to find a convenient change of variables, from those defining a black-hole solution for given electric and magnetic charges $(\mathcal{Q}^M)=(p^\Lambda,q_\Lambda)^{\rm T}$, namely the metric function $U$ and the complex scalars $Z^i$, to the variables $(H^M)=(H^\Lambda,H_\Lambda)^{\rm T}$ that have the same transformation properties as the charges. There is an evident mismatch between these two sets of variables, because $U$ is real. For consistency we will introduce a complex variable $X$ of the form[7]

$$X=\frac{1}{\sqrt{2}}e^{U+i\alpha}, \tag{2.1}$$

although the phase $\alpha$ does not occur in the original FGK formalism. The change of variables will then be well defined, and the absence of $\alpha$ will lead to a constraint on the new set of variables: this constraint is related to the absence of NUT charge, a possibility which in $d=4$ is allowed for by spherical symmetry.

The theory is specified by the prepotential[8] $\mathcal{F}$, a homogeneous function of second degree in the complex coordinates $\mathcal{X}^\Lambda$. Consequently, defining

$$\mathcal{F}_\Lambda\equiv\frac{\partial\mathcal{F}}{\partial\mathcal{X}^\Lambda}\quad\text{and}\quad\mathcal{F}_{\Lambda\Sigma}\equiv\frac{\partial^2\mathcal{F}}{\partial\mathcal{X}^\Lambda\partial\mathcal{X}^\Sigma},\quad\text{we have:}\quad\mathcal{F}_\Lambda=\mathcal{F}_{\Lambda\Sigma}\mathcal{X}^\Sigma. \tag{2.2}$$

Since the matrix $\mathcal{F}_{\Lambda\Sigma}$ is homogenous of degree zero and $X$ has the same Kähler weight as the covariantly holomorphic section

$$\big(\mathcal{V}^M\big)=\begin{pmatrix}\mathcal{L}^\Lambda\\ \mathcal{M}_\Lambda\end{pmatrix}=e^{\mathcal{K}/2}\begin{pmatrix}\mathcal{X}^\Lambda\\ \mathcal{F}_\Lambda\end{pmatrix}, \tag{2.3}$$

where $\mathcal{K}$ is the Kähler potential, we also find

$$\frac{\mathcal{M}_\Lambda}{X}=\mathcal{F}_{\Lambda\Sigma}\frac{\mathcal{L}^\Sigma}{X}. \tag{2.4}$$

Defining the Kähler-neutral, real, symplectic vectors $\mathcal{R}^M$ and $\mathcal{I}^M$ by

$$\mathcal{R}^M=\mathfrak{Re}\,\mathcal{V}^M/X,\qquad\mathcal{I}^M=\mathfrak{Im}\,\mathcal{V}^M/X, \tag{2.5}$$

[6] The equations of motion can be obtained by taking the partial derivative of the Hamiltonian constraint with respect to $H_K$. It goes without saying that having solved the Hamiltonian constraint does not imply having solved the equations of motion.

[7] In this section we will be following the conventions of Ref. [8], where the function $X$ appears as a scalar bilinear built out of the Killing spinors.

[8] We only use the prepotential here to determine quickly the homogeneity properties of the objects we are going to deal with. These properties are, however, valid for any $N=2$ theory in any symplectic frame, whether or not a prepotential exists.

and using the symplectic metric

$$(\Omega_{MN}) \equiv \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix} \tag{2.6}$$

as well as its inverse $\Omega^{MN}$ to lower and raise the symplectic indices according to the convention

$$\mathcal{R}_M = \Omega_{MN}\mathcal{R}^N, \qquad \mathcal{R}^M = \mathcal{R}_N\Omega^{NM}, \tag{2.7}$$

one can rewrite the complex relation (2.4) in the real form

$$\mathcal{R}_M = -\mathcal{M}_{MN}(\mathcal{F})\mathcal{I}^N. \tag{2.8}$$

The symmetric symplectic matrix

$$\mathcal{M}(\mathcal{A}) \equiv \begin{pmatrix} \mathfrak{Im}\,\mathcal{A}_{\Lambda\Sigma} + \mathfrak{Re}\,\mathcal{A}_{\Lambda\Omega}\,\mathfrak{Im}\,\mathcal{A}^{-1|\Omega\Gamma}\,\mathfrak{Re}\,\mathcal{A}_{\Gamma\Sigma} & -\mathfrak{Re}\,\mathcal{A}_{\Lambda\Omega}\,\mathfrak{Im}\,\mathcal{A}^{-1|\Omega\Sigma} \\ -\mathfrak{Im}\,\mathcal{A}^{-1|\Lambda\Omega}\,\mathfrak{Re}\,\mathcal{A}_{\Omega\Sigma} & \mathfrak{Im}\,\mathcal{A}^{-1|\Lambda\Sigma} \end{pmatrix}, \tag{2.9}$$

can be associated with any symmetric complex matrix $\mathcal{A}_{\Lambda\Sigma}$ with a non-degenerate imaginary part (such as $\mathcal{F}_{\Lambda\Sigma}$ and the period matrix $\mathcal{N}_{\Lambda\Sigma}$). The inverse of $\mathcal{M}_{MN}$, denoted by $\mathcal{M}^{MN}$, is the result of raising the indices with the inverse symplectic metric.

It is also immediate to prove the relation

$$d\mathcal{R}_M = -\mathcal{M}_{MN}(\mathcal{F})\,d\mathcal{I}^N. \tag{2.10}$$

From this equality, its inverse and the symmetry properties of $\mathcal{M}_{MN}$ we can derive the following relation between partial derivatives (see *e.g.* [25]):

$$\frac{\partial\mathcal{I}^M}{\partial\mathcal{R}_N} = \frac{\partial\mathcal{I}^N}{\partial\mathcal{R}_M} = -\frac{\partial\mathcal{R}^M}{\partial\mathcal{I}_N} = -\frac{\partial\mathcal{R}^N}{\partial\mathcal{I}_M} = -\mathcal{M}^{MN}(\mathcal{F}). \tag{2.11}$$

Similarly to what we did in five dimensions, we introduce two dual sets of variables $H^M$ and $\tilde{H}_M$ and replace the original $n+1$ fields $X$, $Z^i$ by the $2n+2$ real variables $H^M(\tau)$:

$$\mathcal{I}^M\big(X, Z, X^*, Z^*\big) = H^M. \tag{2.12}$$

The dual variables $\tilde{H}^M$ can be identified with $\mathcal{R}^M$, which we can express as functions of the $H^M$ through Eq. (2.8). This gives $\mathcal{V}^M/X$ as a function of the $H^M$. The physical fields can then be recovered by

$$Z^i = \frac{\mathcal{V}^i/X}{\mathcal{V}^0/X} \quad \text{and} \quad e^{-2U} = \frac{1}{2|X|^2} = \mathcal{R}_M\mathcal{I}^M. \tag{2.13}$$

The phase of $X$, $\alpha$, can be found by solving the differential equation (*cf.* Eqs. (3.8), (3.28) in Ref. [26])

$$\dot{\alpha} = 2|X|^2\dot{H}^M H_M - \mathcal{Q}_\star, \quad \text{where} \quad \mathcal{Q}_\star = \frac{1}{2i}\dot{Z}^i\partial_i\mathcal{K} + \text{c.c.} \tag{2.14}$$

is the pullback of the Kähler connection 1-form

$$\mathcal{Q}_\star = \frac{1}{2i}\dot{Z}^i\partial_i\mathcal{K} + \text{c.c.} \tag{2.15}$$

Having detailed the change of variables, we want to rewrite the FGK action for static, spherically symmetric solutions of $N=2$, $d=4$ supergravity [17], *i.e.*

$$I_{\text{FGK}}\big[U, Z^i\big] = \int d\tau \left\{(\dot{U})^2 + \mathcal{G}_{ij^*}\dot{Z}^i\dot{Z}^{*j^*} - \frac{1}{2}e^{2U}\mathcal{M}_{MN}(\mathcal{N})\mathcal{Q}^M\mathcal{Q}^N + r_0^2\right\}, \tag{2.16}$$

in terms of the variables $H^M$. As in the 5-dimensional case, we start by defining the function $\mathsf{W}(H)$

$$\mathsf{W}(H) \equiv \tilde{H}_M(H)H^M = e^{-2U} = \frac{1}{2|X|^2}, \tag{2.17}$$

which is homogenous of second degree in the $H^M$. Using the properties (2.11) one can show that

$$\partial_M\mathsf{W} \equiv \frac{\partial\mathsf{W}}{\partial H^M} = 2\tilde{H}_M, \tag{2.18}$$

$$\partial^M\mathsf{W} \equiv \frac{\partial\mathsf{W}}{\partial\tilde{H}_M} = 2H^M, \tag{2.19}$$

$$\partial_M\partial_N\mathsf{W} = -2\mathcal{M}_{MN}(\mathcal{F}), \tag{2.20}$$

$$\mathsf{W}\partial_M\partial_N\log\mathsf{W} = 2\mathcal{M}_{MN}(\mathcal{N}) + 4\mathsf{W}^{-1}H_MH_N, \tag{2.21}$$

where the last property is based on the following relation[9]

$$-\mathcal{M}_{MN}(\mathcal{N}) = \mathcal{M}_{MN}(\mathcal{F}) + 4\mathcal{V}_{(M}\mathcal{V}^*_{N)}. \tag{2.22}$$

Using the special geometry identity $\mathcal{G}_{ij^*} = -i\mathcal{D}_i\mathcal{V}_M\mathcal{D}_{j^*}\mathcal{V}^{*M}$, we can rewrite the effective action in the form

$$-I_{\text{eff}}[H] = \int d\tau \left\{\frac{1}{2}\partial_M\partial_N \log \mathsf{W}\left(\dot{H}^M\dot{H}^N + \frac{1}{2}\mathcal{Q}^M\mathcal{Q}^N\right) - \Lambda - r_0^2\right\}, \tag{2.23}$$

where we have defined

$$\Lambda \equiv \left(\frac{\dot{H}^M H_M}{\mathsf{W}}\right)^2 + \left(\frac{\mathcal{Q}^M H_M}{\mathsf{W}}\right)^2. \tag{2.24}$$

The $\tau$-independence of the Lagrangian implies the conservation of the Hamiltonian $\mathcal{H}$

$$\mathcal{H} \equiv -\frac{1}{2}\partial_M\partial_N \log \mathsf{W}\left(\dot{H}^M\dot{H}^N - \frac{1}{2}\mathcal{Q}^M\mathcal{Q}^N\right) + \left(\frac{\dot{H}^M H_M}{\mathsf{W}}\right)^2 - \left(\frac{\mathcal{Q}^M H_M}{\mathsf{W}}\right)^2 - r_0^2 = 0. \tag{2.25}$$

The equations of motion can be written in the form

$$\frac{1}{2}\partial_P\partial_M\partial_N \log \mathsf{W}\left[\dot{H}^M\dot{H}^N - \frac{1}{2}\mathcal{Q}^M\mathcal{Q}^N\right] + \partial_P\partial_M \log \mathsf{W}\ddot{H}^M - \frac{d}{d\tau}\left(\frac{\partial\Lambda}{\partial\dot{H}^P}\right) + \frac{\partial\Lambda}{\partial H^P} = 0. \tag{2.26}$$

Contracting them with $H^P$ and using the homogeneity properties of the different terms as well as the Hamiltonian constraint above, we find the equation (*cf.* Eq. (3.31) of Ref. [26] for the stationary extremal case)

$$\frac{1}{2}\partial_M \log \mathsf{W}\left(\ddot{H}^M - r_0^2 H^M\right) + \left(\frac{\dot{H}^M H_M}{\mathsf{W}}\right)^2 = 0, \tag{2.27}$$

which corresponds to the equation of motion of the variable $U$ in the standard formulation.

Note that in the extremal case ($r_0 = 0$) and in the absence of the NUT charge ($\dot{H}^M H_M = 0$) the equations of motion are solved by harmonic functions $\dot{H}^M = \mathcal{Q}^M$ [25].

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