Enhanced Conjugate-Phase Method for NF-Focusing Using Arrays and RIS

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***Abstract*—A comparison between different Wireless Power Transfer and Maximum SNR methods is achieved showing their equivalences. This study is used to enhance them by using formulation from different scientific areas, hence combining their advantages and reducing their limitations, depending on the final application. Some examples of efficiente Wireless Power Transfer in complex scenarios are presented to illustrate the proposed approach.**

***Index Terms*—Wireless Power Transfer, antenna arrays, Conjugate-Phase, power transfer efficiency.**

I. Introduction

Wireless Power Transfer (WPT) techniques have been proposed to concentrate radiated field over devices to be fed in an efficient manner. In some popular methods, WPT is done using antenna arrays designed to concentrate the power onto a given location in the antenna-radiative near-field (NF) region. It has been found that such concentration of power can be achieved efficiently using the so-called Conjugate-Phase (CP) method [1], an elegant solution based on raw physics and geometry that, however, presents some limitations when dealing with complex scenarios or multiple devices to be fed simultaneously.

On the other hand, different scientific areas have proposed their own design method to optimize the performance of the array. For example, the Power Transfer Efficiency (PTE) maximization method [2], [3] has been proposed under the frame of microwave design, maximizing the ratio between the power received by the device to be fed and the power transmitted by the array antenna, obtaining the optimal set of weights required in a given array. From MIMO communications, the power at the receiver can be maximized when channel state information (CSI) is known by using *Maximal Ratio Combining* (MRC) [4] taking advantage of multi-antenna designs. All these methods are state-of-the-art solutions in their respective areas. In this contribution, we explore their equivalence showing that, under certain circumstances, they can be considered almost (or totally) equivalent, but their formulation is surprisingly different, and may have different advantages or allow their suitability for different kind of problems.

A deeper study of all these formulation schemes allows reformulating the CP to develop an Enhanced Conjugate-Phase method (E-CP) able to consider complex scenarios, such as the active use of a Reconfigurable Intelligent Surface (RIS), so that maximum PTE is achieved over one or more targets, hence overcoming the initial CP limitations. Some illustrative examples show its application to complex scenarios involving the presence of a given RIS, or the actual design of such RIS together with the array. The results will show how the PTE is notably increased, and how a fair distribution of energy between target devices can be achieved.

II. Comparison of methods.

The notation used in this document is the following. The position vector corresponding to any point in the near environment of the antenna is $\vec{r}$, while the position vector corresponding to the n-th element of the array is $\vec{r}'_n$. The electric field radiated by the N elements of the array at any position $\vec{r} = (x, y, z)$ of the antenna-radiative near-field region is given by

$$\vec{E}(\vec{r}) = \sum_{n=1}^{N} \frac{w_n \vec{E}_0(\theta_n, \phi_n) e^{-jk|\vec{r}-\vec{r}'_n|}}{|\vec{r}-\vec{r}'_n|} \tag{1}$$

where $w_n \in \mathbb{C}$ is the weight applied to the n-th element of the array and $k = 2\pi/\lambda$, and $\vec{E}_0(\theta_n, \phi_n)$ is the vector electric field radiated by an array element in the direction defined by (θ_n, ϕ_n) corresponding to the vector $(\vec{r} - \vec{r}'_n)$.

For the sake of simplicity, let us consider the case where the array elements are isotropic, so that $\vec{E}_0(\theta_n, \phi_n) = 1$. The three methods to be evaluated aret he Conjugate-Phase (CP) method [1], MIMO Beamforming for optimal Signal-to-Noise Ratio and the PTE method [2].

A. Conjugate-Phase method

In the CP method, the phase distribution required at the array is directly obtained from raw geometry and physics. The role played by the phase shift applied to each element is to compensate the difference of distance between the element and the focal point, creating a delay so that all the contributions from all the array elements arrive in-phase to the focal point, hence creating a constructive interference.

The phase shift to be applied to the n-th array element is

$$\varphi_n = k|\vec{r} - \vec{r}'_n| \tag{2}$$

The resulting weight is

$$w_n = e^{jk|\vec{r}-\vec{r}'_n|} \tag{3}$$

B. MIMO beamforming

MIMO schemes are usually chosen depending on the availability of CSI in the transmitter. If it is known, a technique usually referred to as *beamforming* is used to maximize the Signal-to-Noise Ratio (SNR) at the receiver. To study the effect of spatial diversity in MIMO, it is usual to study the sub-case of MISO or SIMO systems. A detailed study of the SIMO case allows determining the optimal SNR. Let $\mathbf{h} = [h_1, h_2, ..., h_N]^T$ be the vector of fading-channel coefficients in a SIMO system with N receiving antennas. The received signals, $y_1, y_2, ..., y_N$ may be arranged in a column *received vector* $\mathbf{y} = \mathbf{h}{\cdot}x{+}\mathbf{n}$, where $\mathbf{n}$ is a vector containing N noise samples $n_1, ..., n_N$, with $n_i \sim \mathcal{CN}(0, \sigma_n^2)$. By applying a set of weights in the receiver, $\mathbf{w} = [w_1, ..., w_N]^T$, it can be shown that the resulting SNR is

$$SNR = \frac{|\mathbf{w}^H \mathbf{h}|^2 \cdot P_{tx}}{||\mathbf{w}||^2 \cdot \sigma_n^2} \tag{4}$$

where P_{tx} is the transmitted signal power. It may be maximized by choosing $\mathbf{w} = \frac{\mathbf{h}^*}{||\mathbf{h}||}$ with the n-th weight given by

$$w_n = \frac{h_n^*}{||\mathbf{h}||} \tag{5}$$

The resulting SNR is $SNR = \frac{||\mathbf{h}||^2 \cdot P_{tx}}{\sigma_n^2}$.

This is the expression of the maximum SNR that may be achieved in SIMO or, applying symmetry, MISO systems. The set of weights achieving this maximum SNR is called the *Maximal Ratio Combiner* (MRC) or *optimal beamformer*, and it is used in beamforming in the transmitter of a MISO system when CSI is known. Notice that a NF Focusing problem is actually a MISO system where the only receiver is set at the focal point. Given a focal point, the noise power there is also set, so maximizing the SNR is equivalent to maximizing the signal power in the receiver. The open question here is how to determine the channel coefficients to be considered.

In some sense, if a free-space scenario is considered, the expression for the radiated field distribution in (1) may be considered the factor relating the transmitted and received signals when noise is not considered. In the case of a MISO system, that relation is $y = \mathbf{h} \cdot \mathbf{x} + \mathbf{n}$, where $\mathbf{x} = [x_1, ..., x_N]^T$ is the vector of transmitted signals in each antenna at a given instant. Hence, we may identify that

$$h_n = \frac{e^{-j2\pi|\vec{r} - \vec{r}'_n|/\lambda}}{|\vec{r} - \vec{r}'_n|} \tag{6}$$

and therefore, by applying the optimal beamformer, the weight to be applied to the n-th array element for optimal signal transfer to the focal point is

$$w_n = \frac{h_n^*}{||\mathbf{h}||} = \frac{e^{j2\pi|\vec{r} - \vec{r}'_n|/\lambda}}{|\vec{r} - \vec{r}'_n| \cdot ||\mathbf{h}||} \tag{7}$$

The resulting weight have the same phase than those obtained using the CP method, so it may be assumed that the phase distribution of the weights is, not surprisingly, optimal in the sense of maximum power delivery to the focal point. But there are two differences: a scale factor $||\mathbf{h}||$ that affects to every weight, so it may be disregarded; and a change in the amplitude of each weight according to a factor inversely proportional to the distance between the array element and the focal point.

C. Power Transfer Efficiency method.

In the case of the Power Transfer Efficiency (PTE) method, this efficiency is defined as the ratio between the power delivered to the receiver or focal point, and the transmitted power.

If a receiving array is considered, with M elements located at positions $\vec{r}''_m$, the overall response of the transmitting and receiving arrays is given by:

$$F = \sum_{m=1}^{M} \sum_{n=1}^{N} \omega_n \gamma_m \frac{e^{-jk|\vec{r}''_m - \vec{r}'_n|}}{|\vec{r}''_m - \vec{r}'_n|} \tag{8}$$

where the weights applied in the receiver γ_m have also been included. This expression may be reformulated in matrix-vector form as $F = \mathbf{w}^T \cdot \mathbf{S}^T \cdot \boldsymbol{\gamma}$ with $\mathbf{w} = [\omega_1, \omega_2 ..., \omega_N]^T$, $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, ..., \gamma_M]^T$, and $\mathbf{S}$ is a matrix with elements given by

$$S_{m,n} = \frac{e^{-jk|\vec{r''_m} - \vec{r'_n}|}}{|\vec{r''_m} - \vec{r'_n}|} \tag{9}$$

In [2] it is stated that the maximum PTE is given by the maximum eigenvalue of $\mathbf{S}^H\mathbf{S}$, which can be calculated as

$$T = \frac{|\mathbf{w}^H \mathbf{S}^H|^2}{||\mathbf{w}||^2} = \frac{\mathbf{w}^H \mathbf{S}^H \mathbf{S} \mathbf{w}}{||\mathbf{w}||^2} \tag{10}$$

The corresponding set of weights is the associated eigenvector and, after some manipulation, it can be obtained as

$$w_n = \frac{e^{j2\pi|\vec{r} - \vec{r}'_n|/\lambda}}{|\vec{r} - \vec{r}'_n|} \tag{11}$$

which is the same solution obtained with the MIMO approach but the scale factor.

D. Some conclusions

Although the previous comparison has set up the base for a more complete formulation, the following ideas are specially relevant:

- The three presented methods have been found to be equivalent, at least under certain conditions. Therefore, they are interchangeable, and, for example, the CP method might be formulated as (5) provided that an expression of the channel $\mathbf{h}$ is available. The use of both the amplitude and phase of the weights will be denoted as *Enhanced-CP, ECP*, while the name CP will be reserved for the use of the phase-only method.
- The equivalence found in previous sections indicates that the formulation (1) can be used to represent the channel in free-space conditions.
- The definition of the *Power Transfer Efficiency* depends of the definition of the matrix $\mathbf{S}$, expected to relate received and transmitted power, but its formulation here

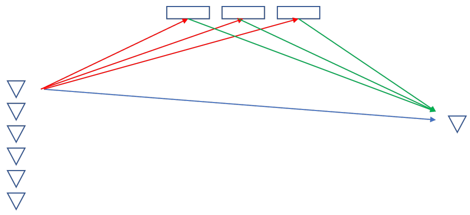

Fig. 1. Scenario with an array, a RIS, and a receiver.

does not provide power at the receiver but field level. Hence, the used PTE is not a power ratio, but it is still valid to evaluate the performance as far as the delivered power is proportional to the radiated field density, i.e., a higher or lower value of this parameter (denoted as Υ in the sequel) still represents a higher or lower PTE as they are proportional, i.e., $\Upsilon \propto$ PTE.

III. ECP for WPT with a given RIS

Figure 1 depicts a scenario where both an N-element array and an M-element RIS contribute to focus the radiated power onto an assigned receiver. According to [5], the channel impulse response can be expressed as:

$$\mathbf{h} = \mathbf{d} + \mathbf{f} \cdot \mathbf{\Theta} \cdot \mathbf{S} \tag{12}$$

where $\mathbf{d}$ is a vector containing the direct channel coefficient $d_n, n = 1...N$ between the N array elements and the target, $\mathbf{f}$ is a vector with the channel coefficientes $f_m, m = 1...M$, between the elements of the RIS and the target (green lines), $\mathbf{S}$ is a matrix whose element $S_{m,n}$ represents the channel between the n-th element of the array and the m-th element of the RIS, and $\mathbf{\Theta}$ is a diagonal matrix whose elements represent the effect of the RIS. Under free-space conditions, these vectors and matrices can be expressed as:

$$d_n = \frac{e^{-j2\pi|\vec{r}-\vec{r}'_n|/\lambda}}{|\vec{r}-\vec{r}'_n|} \tag{13}$$

$$f_m = \frac{e^{-j2\pi|\vec{r}-\vec{r}''_m|/\lambda}}{|\vec{r}-\vec{r}''_m|} \tag{14}$$

$$S_{m,n} = \frac{e^{-j2\pi|\vec{r}''_m-\vec{r}'_n|/\lambda}}{|\vec{r}''_m-\vec{r}'_n|} \tag{15}$$

$$\mathbf{\Theta} = diag\{\theta_1, \theta_2 \dots \theta_M\} \tag{16}$$

where $\vec{r}$ is the position vector corresponding to the point where the field is evaluated, $\vec{r}'_n$ is the position vector for the n-th element of the array, and $\vec{r}''_m$ is the position vector for the m-th element of the RIS. The elements $\theta_m \in \mathbb{C}$ represent the gain (≤ 1) and phase shift of each element of the RIS.

A. Single-target case

Let us consider a case with a pre-defined and known RIS where the array is intended for maximum power transfer on an assigned target. The weights can be obtained using the ECP as (5). In simulation #1 a 32×32-element array with isotropic elements separated 0.7λ and centered in the origin is considered. An 18×18-element RIS is located at $\{20, 0, 40\}\lambda$, also rotated 50º clockwise; the elements are also separated 0.7λ (Fig. 2).

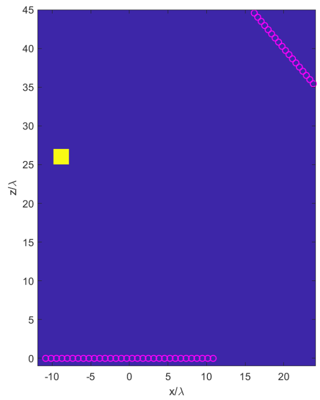

Fig. 2. Array, target, and RIS considered for simulation #1.

The coefficients θ_i have been set to 1 for all $i = 1 \dots M$. A focal point has been assigned at $\vec{r} = \{-8, 0, 26\}\lambda$, also plotted in Fig. 2. By applying the proposed method, the optimum value of Υ is 1.9299, and the corresponding field distribution is represented in Fig. 3.

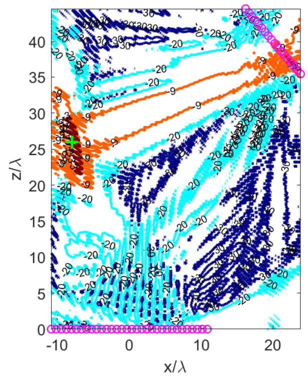

Fig. 3. Field distribution obtained using the proposed method in simulation #1.

B. Multi-target case

In a case where multiple devices have to receive the radiated power, the PTE method has been found to be optimal in the sense of overall PTE. However, it accounts for the sum of all the received power, what may lead to a very dominant transfer of power to only one of the devices, and very small amount of power delivered to the other locations. The ideal result should be delivering an equivalent amount of power to all the devices, what is refered to as *power fairness*.

Let us consider the same simulation carried out in the previous section, but with three devices to be fed, located at $\{-8, 0, 16\}\lambda$, $\{6, 0, 30\}\lambda$, and $\{-3, 0, 34\}\lambda$ (simulation #2). In this case, the dimension of $\mathbf{h}$ is $3 \times N$, so the direct

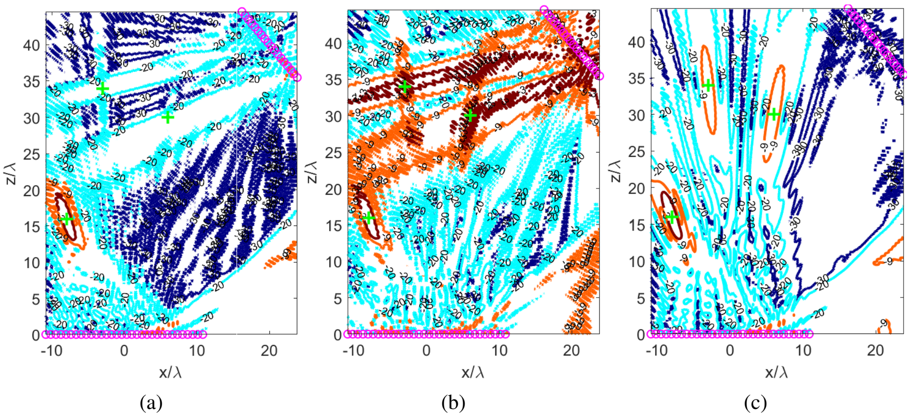

Fig. 4. Field distribution with 3 targets, simulation #2. a) PTE method; b) proposed method: c) not accounting for the RIS.

application of (5) is not feasible, but the PTE method can be applied. The resulting efficiency is $\Upsilon = 2.8613$, but the resulting radiated field distribution (Fig. 4a) shows that one of the targets receives most of the power, while the other two devices barely receive a reasonable power.

The individual PTE for each target can be calculated by considering the channel $\mathbf{h}_i, i = 1, 2, 3$, (with $\mathbf{h} = [\mathbf{h}_1; \mathbf{h}_2; \mathbf{h}_3]$). In this example, the resulting values are 2.7262, 0.0501 and 0.0850 respectively, showing that power fairness is not achieved. It is interesting to notice that the overall PTE can be obtained as the sum of the partial values obtained for each of the D devices to be fed. The PTE is defined as the ratio between the received power and the transmitted power, i.e.:

$$PTE = \frac{P_{rx}}{P_{tx}} = \frac{\sum_{i=1}^{D} P_i}{P_{tx}} = \sum_{i=1}^{D} \frac{P_i}{P_{tx}} = \sum_{i=1}^{D} PTE_i \quad (17)$$

The same applies for $\Upsilon = \sum_{i=1}^{D} \Upsilon_i$. A design method able to achieve power fairness can make use of the superposition principle. Let us consider D designs of radiated field distributions, one for each device. If each radiated field distribution concentrates the field on one of the targets, the sum of the D distributions should present D focal spots. Such sum of distributions can be obtained by considering the sum of the associated synthesized weights, i.e. $\mathbf{w} = \sum_{i=1}^{D} \mathbf{w}_i$. The weights $\mathbf{w}_i$ can be obtained again as $\mathbf{w}_i = \frac{\mathbf{h}_i^*}{||\mathbf{h}_i||}$.

By applying this approach to simulation #2, the overall efficiency is $\Upsilon = 2.4419$, lower than using the PTE method, but the individual efficiencies are 1.1286, 0.6395 and 0.6737, representing a better fairness in the split of power between the three devices, even though the global efficiency is lower. The resulting radiated field distribution is represented in Fig. 4b.

1) Comparison with the method not accounting for the RIS: It is interesting to consder the idea of not accounting for the presence of the RIS, assuming that, maybe, generating a radiated field distribution focused on the three targets would avoid delivering some power to the RIS, hence reducing its effect. Although this idea is reasonable, this approach has been included in the simulation. Fig. 4c shows the resulting field distribution, apparently quite well focused on the targets. However, the quantitative analysis shows that the resulting global efficiency is 1.9842, with individual values 1.6193, 0.2267 and 0.1381, showing that not accounting for the presence of the RIS leads to a lower performance.

IV. Conclusion

An exhaustive comparison between three methods from three different scientific areas to improve the power transmitted to a target has been presented. It allows reformulating the CP method to overcome its more relevant limitations, by simply considering all these method as equivalent. More complicated problems can be addressed, such as considering a complex scenario with a RIS involved, accounting for its effect to optimize the PTE. This approach opens an interesting line to enchace all these three methods, combining them, and creating a complete framework allowing the designed to make use of different formulations and tools depending on the final application or scenario for WPT.

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