Performance Evaluation of 2D vs 4D Surrogate Models of Reflectarray Unit Cells Based on Support Vector Regression

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***Abstract*—Surrogate models of reflectarray unit cells are usually generated employing a number of input variables such as geometrical features of the cell, frequency and angles of incidence. Here we show how surrogate models based on support vector regression can be improved by removing their dependence on the angle of incidence. This is done by grouping the reflectarray elements under a relatively small set of incidence angles. Thus, instead of generating models with the angles of incidence as input variables, models are obtained per angle of incidence pair, reducing dimensionality to improve their performance without significant impact on the reflectarray analysis accuracy.**

***Index Terms*—Machine learning, surrogate model, support vector regression (SVR), angle of incidence, reflectarray antenna**

I. Introduction

A common approach in the literature when generating surrogate models of reflectarray unit cells is to include many input variables such as the geometric features, frequency and angles of incidence [1], [2]. However, as the dimensionality of the model is increased, the number of required samples to maintain a constant sample density increases exponentially. This means that for a higher input space dimensionality, the workload to obtain the surrogate model will be greater.

A strategy to reduce the dimensionality of the surrogate models consists in grouping the reflectarray elements under a reduced set of the angles of incidence (θ, φ). Then, surrogate models of the reflection coefficients are generated per (θ, φ) pair, effectively reducing the dimensionality of the model in two with regard to the aforementioned approach. This approach has already been used [3] for the design and optimization of reflectarray antennas. However, both methodologies have not being compared in terms of accuracy and efficiency. Indeed, although reducing the dimensionality of the surrogate model may translate into a more efficient workload, the discretization of the angles of incidence introduces a distortion in the radiation pattern.

In this work, we compare the two methodologies regarding the use of the angles of incidence in surrogate models using support vector regression (SVR). On the one hand, we generate models of the reflection coefficients considering two geometrical features of a unit cell plus the two angles of incidence, obtaining 4D SVR models. On the other hand, given a set of angles of incidence (θ, φ), we generate models per (θ, φ) pair using as input variables the same two geometrical features as in the previous case, obtaining 2D SVR models. The 4D and 2D SVRs models will be compared in terms of efficiency and accuracy at the model and antenna analysis levels.

II. Problem Statement

A. Introduction

We consider a flat reflectarray comprised of K element, whose matrix of reflection coefficients is:

$$\boldsymbol{R}_k = \begin{pmatrix} \rho_{xx,k} & \rho_{xy,k} \\ \rho_{yx,k} & \rho_{yy,k} \end{pmatrix} \tag{1}$$

with $k = 1, 2, \ldots, K$, where ρ_{xx}, ρ_{yy} are known as direct coefficients and ρ_{xy}, ρ_{yx} are the cross-coefficients. A correct characterization of both copolar and crosspolar far fields requires the full modelling of the four coefficients in (1). The coefficients are obtained with the full-wave analysis method based on local periodicity (FW-LP) described in [4].

These coefficients depend on the geometrical features of the unit cell, substrate characteristics, frequency of operation and angles of incidence. For this work, we will employ a unit cell consisting in two sets of four dipoles shifted half a period from one another. The unit cell, substrate and frequency are detailed in [3]. The length of the dipoles of each set will be proportional, thus obtaining two geometrical features named as T_x and T_y. Grouping these variables with the components of the angle of incidence (θ, φ), we obtain a vector of the input variables to the 4D SVR models, $\vec{x} = (T_x, T_y, \theta, \varphi)$. The input variables for the 2D models are $\vec{x} = (T_x, T_y)$.

B. Support Vector Regression Modelling

The process of surrogate modelling based on SVR is based on a cross-validation procedure using an efficient grid search and it is detailed in [5]. Here we will review the basic concepts.

We consider a set of N inputs $(\vec{x}_i \in \chi \subseteq \mathbb{R}^L, L = 2, 4)$ and outputs $(\rho_i \in \mathbb{R})$, $T = \{\vec{x}_i, \rho_i\}_{i=1,\ldots,N}$, to obtain a function f that estimates the values of ρ for any new input vector $\vec{x} \in \chi$. In this context, ρ_i represents either the real or imaginary parts of any of the reflection coefficients in (1), or the magnitude of the direct coefficients. By following the procedure described in [5], the data set T is divided into three disjoint subsets for training (with $N_r \leq 0.7N$ samples), validation (with $N_v = 0.15N$ samples) and testing (with $N_t = 0.15N$ samples).

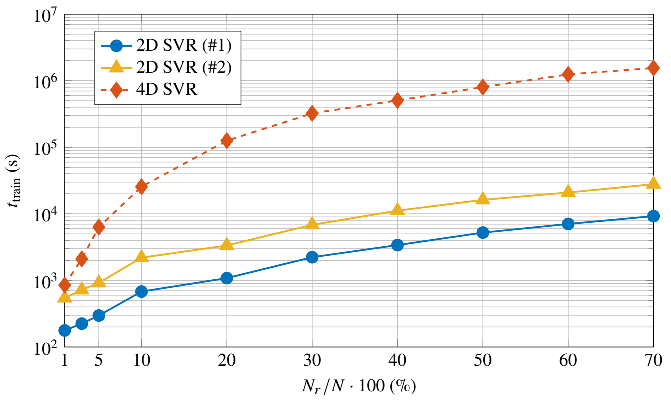

Fig. 1. Total training time, t_{train} (s), vs. the percentage of training patterns with respect to the total number of samples, $N_r/N \cdot 100$ (%), for both the 2D and 4D SVRs. N for 4D SVR and 2D SVR #1 is 65 000, while for 2D SVR #2 is 190 000.

In order to evaluate the accuracy of the surrogate model, the following relative error is employed:

$$\mathrm{RE}_{\mathrm{SVR}} = 20 \log_{10} \left(\frac{\|\vec{e}\|}{\|\vec{\rho}\|} \right) \ (\mathrm{dB}), \tag{2}$$

where $\vec{\rho} = (\rho_1, \rho_2, \ldots, \rho_M)$ is a vector of the actual output of the FW-LP, and $\vec{e} = (e_1, e_2, \ldots, e_M)$ is a vector of the difference between the predicted output of the SVR and the FW-LP output, i.e., $e_i = \rho_i - f(\vec{x}_i)$, $i = 1, 2, \ldots, M$.

To assess the effect of the discretization of (θ, φ) at the antenna level, we define the following relative error:

$$\mathrm{RE}_{\mathrm{FF}} = 100 \cdot \frac{\|G_{\text{FW-LP}} - G_{\mathrm{SVR}}\|}{\|G_{\text{FW-LP}}\|} \ (\%), \tag{3}$$

where G is either the copolar or the crosspolar gain pattern, which was obtained either analysing the antenna with the FW-LP and the real angles of incidence for each element, or by estimating (1) with the SVR and a reduced set of angles.

To characterize the matrix of reflection coefficients in (1) we employ ten models: four for the real part of each coefficient, another four for the imaginary parts, and two more for the magnitude of the direct coefficients. Since the 4D SVR includes the angles of incidence as input variables, we only need 10 surrogate models to characterize $R_k, \forall k = 1, \ldots, K$. On the other hand, for the 2D SVR we need $10M_a$ surrogate models, where M_a is the total number of (θ, φ) pairs ($M_a \leq K$).

III. Cell Modelling Performance

In this section, the cell modelling performance of the 4D and 2D SVRs is assessed by analysing the training performance of each approach and the achieved accuracy in the prediction of the reflection coefficients.

A. Training Performance

The training performance will be measured in terms of time cost and accuracy over the test set with the relative error given by (2). For the 4D SVR we consider a total of $N = 65\,000$ samples, divided into three sets as described in Section II.C.

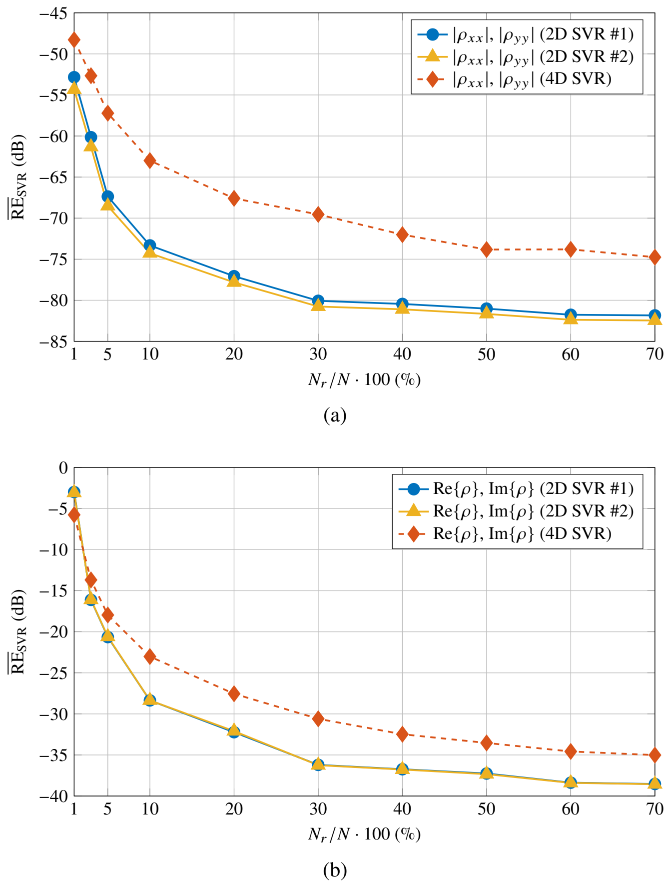

Fig. 2. Average relative test error, $\overline{\mathrm{RE}}_{\mathrm{SVR}}$ (dB), of the reflection coefficients vs. the percentage of training patterns with respect to the total number of samples, $N_r/N \cdot 100$ (%), for both the 2D and 4D SVRs. (a) Magnitude of the direct coefficients, and (b) real and imaginary parts of the direct and cross-coefficients. N for 4D SVR and 2D SVR #1 is 65 000, while for 2D SVR #2 is 190 000.

For the 2D SVR will employ two different discretizations, with $M_{a,\#1} = 26$ and $M_{a,\#2} = 76$, considering in each case 2 500 samples per (θ, φ) pair, giving a total of $N_{\#1} = 65\,000$ and $N_{\#2} = 190\,000$ samples.

Fig. 1 shows the total time training cost of the 4D and 2D SVRs. This training time is the sum of all the training time of the 10 SVR models for the 4D case and of the $10M_a$ models for the 2D SVRs. As it can be seen, the training cost of the 4D SVR is significantly higher than training cost of the 2D SVRs. This is expected if we take into account how the training time depends on the number of training samples N_r. For the usual training strategies in SVR, including the LibSVM employed in this work [6], the time complexity varies between $O\left(N_r^2\right)$ and $O\left(N_r^3\right)$ [7]. Thus, training several 2D SVRs is more computationally efficient than its 4D counterpart.

Fig. 2(a) plots the average value of the relative test error given by (2) of the direct coefficients magnitude, for both 2D and 4D SVRs, versus the percentage of training samples. Similarly, Fig. 2(b) plots the average relative error of the real and imaginary parts of the reflection coefficients. They show that the relative error rapidly decreases with the number of

Table I

AVERAGE RELATIVE ERROR OVER THE TEST SET AND TRAINING TIME OF THE SVR MODEL WITH $N_r = 0.7N$ FOR EACH OUTPUT VARIABLE. N FOR 4D SVR AND 2D SVR #1 IS 65 000, WHILE FOR 2D SVR #2 IS 190 000.

Approach	Variable	$\lvert\rho_{xx}\rvert$	$\lvert\rho_{yy}\rvert$	$\mathrm{Re}\{\rho_{xx}\}$	$\mathrm{Im}\{\rho_{xx}\}$	$\mathrm{Re}\{\rho_{xy}\}$	$\mathrm{Im}\{\rho_{xy}\}$	$\mathrm{Re}\{\rho_{yx}\}$	$\mathrm{Im}\{\rho_{yx}\}$	$\mathrm{Re}\{\rho_{yy}\}$	$\mathrm{Im}\{\rho_{yy}\}$
2D (#1)	$\overline{\mathrm{RE}}_{\mathrm{SVR}}$ (dB)	−81.7	−82.0	−38.1	−38.5	−39.2	−38.6	−38.3	−38.4	−38.6	−38.7
2D (#2)	$\overline{\mathrm{RE}}_{\mathrm{SVR}}$ (dB)	−82.4	−82.5	−38.1	−38.5	−39.2	−38.7	−38.2	−38.3	−38.6	−38.7
4D	$\overline{\mathrm{RE}}_{\mathrm{SVR}}$ (dB)	−74.7	−74.8	−36.5	−35.6	−35.3	−32.7	−34.9	−33.5	−36.7	−35.8
2D (#1)	t_{train} (s)	645	689	1 019	1 027	916	978	990	1 192	838	961
2D (#2)	t_{train} (s)	1 833	2 122	3 067	3 175	2 799	3 044	2 985	3 586	2 457	2 823
4D	t_{train} (s)	133 337	108 833	199 099	212 514	79 121	197 275	125 940	194 132	171 677	139 412

training samples until the sample percentage reaches 30%. This effect is clearer in the 2D cases. An average relative error lower than −30 dB ensures a high degree of accuracy between the SVR-based model and the FW-LP simulations. In light of the above considerations, a good trade-off between training time and relative error may be achieved for $N_r = 0.3N$.

Table I summarizes the results for both training time cost and average relative error over the test set, for all the estimated output variables when using the highest number of training patterns ($N_r = 0.7N$). Results given in this section have been obtained using, in sequential mode, a workstation with 2 Intel Xeon E5-2650v3 CPU at 2.3 GHz and 256 GB of RAM.

B. *Reflection Coefficients*

Fig. 3 shows a comparison between the FW-LP, 2D SVR #1 and 4D SVR of the magnitude and phase for the direct coefficient ρ_{yy} and for the cross-coefficient ρ_{yx} at oblique incidence $(\theta, \varphi) = (36°, 50°)$ and using $N_r = 0.3N$. Only one discretization has been chosen since the error over the test set for both is very similar (see Fig. 2). As it can be seen, there is a high degree of accuracy for both SVRs when compared with FW-LP. In fact, for the curves shown in Fig. 3, the mean absolute deviation (MAD) for the phase of ρ_{yy} is 0.45° and 0.41° for the 2D and 4D SVRs, respectively. Those numbers are 3.81° and 1.93° for the cross-coefficient ρ_{yx}. Regarding the magnitude, the MAD for ρ_{yy} is −77.43 dB and −68.84 dB for the 2D and 4D SVRs, respectively; while for ρ_{yx} they are −56.96 dB and −57.54 dB respectively. Similar results were obtained for the coefficients ρ_{xx} and ρ_{xy} in both magnitude and phase.

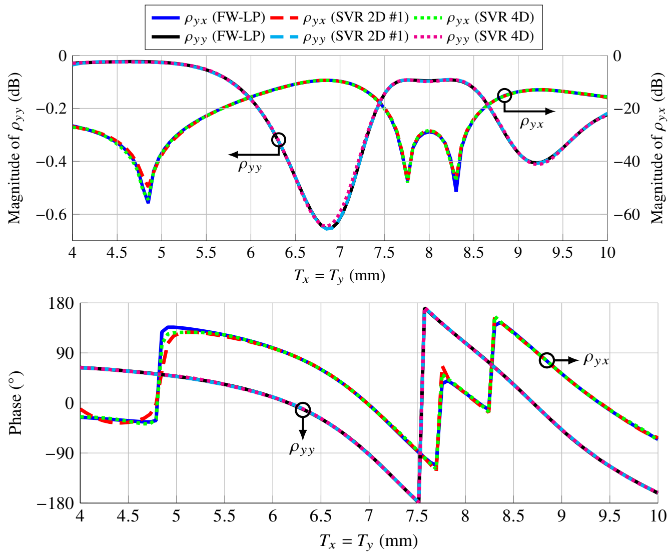

Fig. 3. Comparison between the FW-LP and SVR simulations of the magnitude (top) and phase (bottom) for the direct reflection coefficient ρ_{yy} and the cross-coefficient ρ_{yx} for $(\theta, \varphi) = (36°, 50°)$. The 2D SVR corresponds to discretization #1. SVRs were trained with $N_r = 0.3N$.

IV. RADIATION PATTERNS

Here, the two methodologies will be compared at the antenna level in terms of computational efficiency and accuracy predicting the radiation patterns. To that end, a very large reflectarray for direct-to-home (DTH) applications with European coverage is considered. Details regarding antenna optics and design procedure can be consulted in [3].

A. *Acceleration of Reflectarray Analysis*

Fig. 4 shows the acceleration results for the two 2D SVRs and the 4D SVR for different values of N_r. This acceleration factor is calculated as the ratio between the mean time that takes the FW-LP to analyse a number of unit cells and the mean time that takes the SVR-based model to analyse the same number of unit cells. It can be observed how the SVR becomes slower when N_r increases. Indeed, this is expected since the number of support vectors per coefficient increases to obtain a higher accuracy. Nevertheless, for a similar accuracy as the 2D SVRs, the 4D SVR is much slower (between 10 and 20 times slower for reflectarray analysis), and even the fastest 4D SVR, with $N_r = 0.01N$, is more than two times slower than the slowest 2D SVR, and presenting worse accuracy in the prediction of the reflection coefficients than the 2D SVRs.

In addition, the SVRs were also employed to obtain the layout of a reflectarray antenna following the procedure described in [8]. The results are summarized in Table II. The acceleration achieved in the layout design is smaller than that of a pure analysis since it involves more operations not accelerated by the SVR. Nevertheless, as shown in Table II, the 2D SVRs show superior computational performance than the 4D SVRs.

B. *Accuracy in Radiation Pattern Computation*

Fig. 5 shows the relative error (3) as a function of the number of training samples (N_r) for both the 2D and 4D

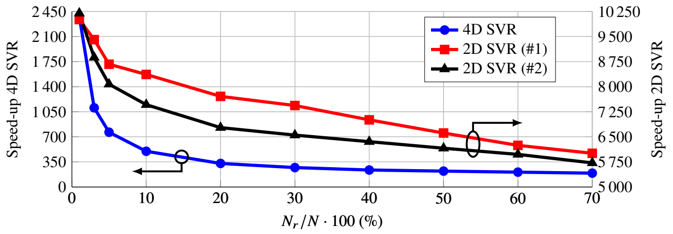

Fig. 4. Reflectarray analysis speed-up of the 4D and 2D SVRs for different values of the number of training samples. N for 4D SVR and 2D SVR #1 is 65 000, while for 2D SVR #2 is 190 000.

Table II
PERFORMANCE OF THE 2D AND 4D SVRS FOR A REFLECTARRAY LAYOUT DESIGN WITH AN INTEL CORE I9-9900 AT 3.10 GHZ. N FOR 4D SVR AND 2D SVR #1 IS 65 000, WHILE FOR 2D SVR #2 IS 190 000.

Tool	**Time (s)**	**Speed-up**
MoM-LP	1574.53	1
2D SVR (#1; $N_r = 0.7N$)	1.37	1149
2D SVR (#2; $N_r = 0.7N$)	1.37	1149
4D SVR ($N_r = 0.1N$)	34.65	45
4D SVR ($N_r = 0.7N$)	91.00	17

SVRs. For all cases, it can be seen that the error rapidly decreases with N_r. For the 4D SVR, the error of the copolar pattern stagnates around 1%, and around 2% for the crosspolar pattern. In contrast, for the 2D SVR the relative error for the copolar pattern is more stable and lower, with a value around 0.3%, but the crosspolar pattern presents a relative error around 11% for discretization #1 and around 4% for discretization #2. It is interesting to note that the error stagnates very quickly when increasing the number of training samples in contrast to the test error shown in Fig. 2. This means that there is a point in which obtaining better accuracy in the prediction of the reflection coefficients does not translate into a lower error in the prediction of the radiation pattern. It is also noteworthy that the test error achieved by the 2D SVR models is lower than the test error for the 4D SVR model, as shown in Table I. This again shows the effect of discretization of the angles of incidence on the prediction of the radiation pattern, that affects more the crosspolar component than the copolar pattern. Since the 4D SVR includes the angles of incidence as input parameters, it does not suffer from this problem. However, the error for the copolar pattern using the 2D SVR is lower than using the 4D SVR.

C. Evaluation of Radiation Patterns

The dual-linear reflectarray with European coverage for DTH applications will be used to graphically compare the two methodologies shown in the present work. Attending to the results shown in Fig. 5, the 4D SVR with $N_r = 0.3N$ and the 2D SVR #2 with $N_r = 0.1N$ will be compared. For this comparison, the total number of samples for both approaches is approximately the same: 19 500 for the 4D SVR and 19 000 for the 2D SVR #2 taking into account all (θ, φ) pairs.

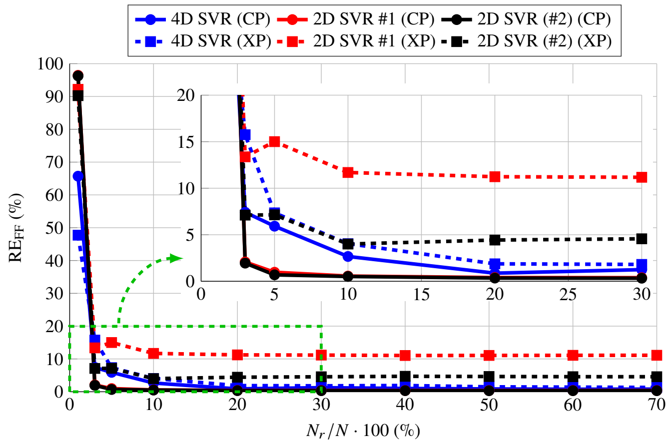

Fig. 5. Relative error in the computation of the far field with the 2D and 4D SVRs for the copolar (CP) and crosspolar (XP) patterns versus the number of the training samples. N for 4D SVR and 2D SVR #1 is 65 000, while for 2D SVR #2 is 190 000.

Fig. 6 shows the comparison with the FW-LP simulation for the copolar and crosspolar patterns. The accuracy in the prediction of the copolar pattern for both SVR is very high, with very slight discrepancies for the 4D SVR for the 10 dBi curves and below. The 4D SVR is only slightly better for the prediction of the crosspolar pattern. The 2D SVR has high accuracy for the high levels of the crosspolar pattern, although presents some minor discrepancies for lower levels.

The comparison has also been carried out with a different radiation pattern: a shaped-beam reflectarray with a sectored beam pattern in azimuth and a squared-cosecant pattern in elevation; and a pencil beam pattern. A summary of these results is shown in Table III. As it can be seen, the 2D SVR models offer lower error in the copolar pattern, although the 4D SVR is slightly better at predicting the crosspolar pattern. However, this higher accuracy is achieved at the expense of greater training time and lower computational efficiency in the analysis of reflectarray antennas. Thus, a compromise might be achieved to greatly accelerate training and analysis time by employing a 2D discretization of the angles of incidence at the expense of slightly decreasing the accuracy of the crosspolar pattern while also maintaining and even increasing the accuracy in the prediction of the copolar pattern.

V. CONCLUSION

In this work, we have carried out a performance evaluation comparison between multidimensional surrogate models based on support vector regression (SVR) for reflectarray design. In particular, two different approaches have been compared: 4D SVR models wherein the angles of incidence and two geometrical variables are input variables, and 2D SVR models using only the two geometrical features as input variables. In the latter case, different angles of incidence are grouped into discrete sets such that 2D SVR models are obtained per discrete set. When both approaches are compared, the 2D SVR is considerably faster than the 4D SVR. In fact, it is

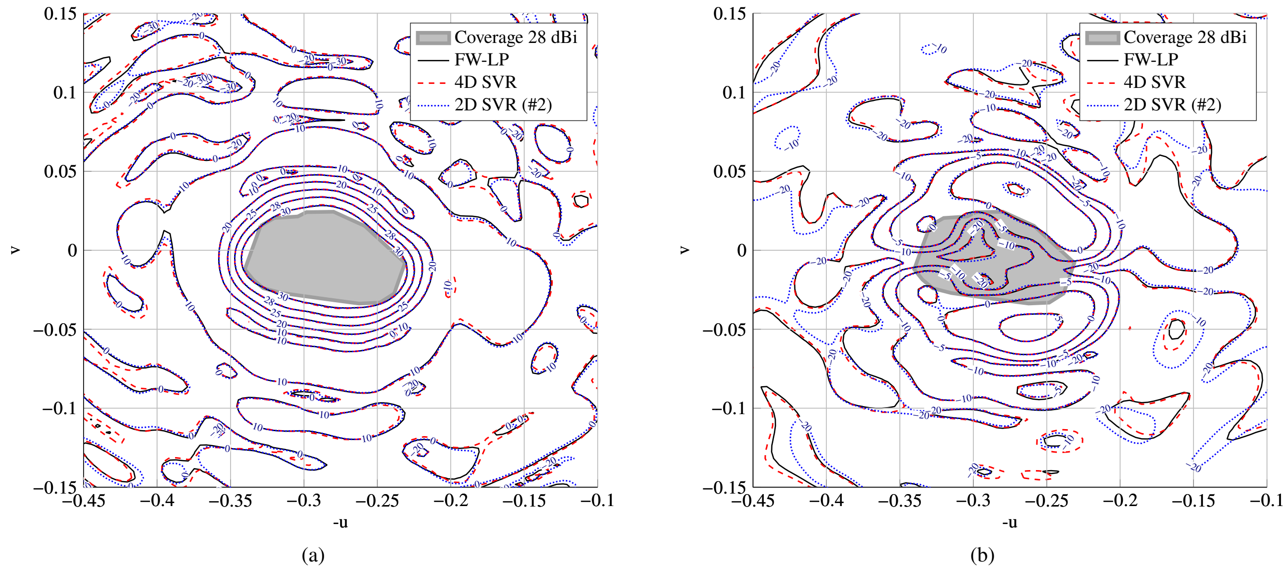

Fig. 6. Comparison of the FW-LP simulation and SVR predictions for the (a) copolar and (b) crosspolar patterns for polarization X of a very large reflectarray with European coverage for DTH application.

Table III
RELATIVE ERRORS (%) IN THE PREDICTION OF THREE RADIATION PATTERNS USING DIFFERENT SVR MODELS FOR THE COPOLAR (CP) AND CROSSPOLAR (XP) PATTERNS.

Tool	**Pencil beam**		**Shaped-beam**		**Contoured-beam**	
	CP	XP	CP	XP	CP	XP
2D SVR (#1)	0.11	7.20	0.91	8.32	0.36	11.54
2D SVR (#2)	0.21	2.14	0.91	3.47	0.51	4.00
4D SVR	0.40	1.01	2.08	2.53	1.25	1.81

between one and two orders of magnitude faster to train and more than one order of magnitude faster to accelerate the reflectarray analysis. Furthermore, a higher degree of accuracy in the prediction of the copolar pattern is achieved by the 2D SVR when compared to FW-LP simulations. Meanwhile, the prediction of the crosspolar pattern is slightly more accurate in the 4D SVR-based simulations, although using the 2D SVR models, errors lower than 4% are achieved. In addition, it has also been shown that there is a point at which obtaining better accuracy in the prediction of the reflection coefficients does not translate into a lower error in the prediction of the radiation pattern.

Finally, even though the two discretizations of the angles of incidence tested in this work show very similar computational performance and accuracy in the prediction of the reflection coefficients, they have a notable effect in the prediction of the crosspolar pattern. Indeed, a poor angle discretization may impact the prediction of the far field and has to be taken into account if 2D SVR surrogate models are to be used for reflectarray design.

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