



Universidad de Oviedo
Facultad de Formación del Profesorado y Educación

**Indagación sobre la probabilidad en el aula
de primaria: experimentos con
probabilidad intuitiva y subjetiva**

**Inquiry about probability in the primary classroom:
experiments with intuitive and subjective probability**

TRABAJO FIN DE GRADO

GRADO EN MAESTRO EN EDUCACIÓN PRIMARIA BILINGÜE

Lorena Rodríguez Suárez
Tutor: Luis J. Rodríguez Muñiz
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1. Introducción

Este trabajo fin de grado se centra en el papel de la probabilidad en el ámbito educativo. El ámbito de la probabilidad es un contenido relativamente nuevo en el currículo de Educación Primaria. No es hasta 2006, con la Ley Orgánica de Educación (LOE), cuando aparecen conocimientos relacionados con la probabilidad. Actualmente, en la Ley Orgánica para la Mejora de la Calidad Educativa (LOMCE) en la asignatura de Matemáticas, el bloque 5 está dedicado a la estadística y a la probabilidad.

El trabajo se encuentra dividido en dos partes: el marco teórico y una propuesta de intervención educativa. Primero se realiza en español una breve justificación del trabajo, así como los objetivos generales y específicos y la metodología que se plantearon para llevar a cabo este trabajo. No obstante, estos apartados se volverán a presentar en inglés más adelante.

Respecto al marco teórico se presentan dos campos de estudios. Por un lado, los distintos significados de la probabilidad que se pueden encontrar en el currículo de Educación Primaria: significado clásico, intuitivo, frecuencial y subjetiva. También se menciona el significado axiomático de la probabilidad, aunque este no se imparte hasta Educación secundaria. Por otro lado, se explican los diferentes tipos de lenguaje probabilístico que puede usar el alumnado para expresar la probabilidad de un suceso o los grados de creencia de que algo suceda.

En la segunda parte se presenta la intervención educativa que consiste en tres actividades llevadas a cabo en dos clases de 5º de primaria. En esta sección se presentan las actividades diseñadas teniendo en cuenta su organización, los materiales necesarios, las bases teóricas en las que se fundamentan y las fases en las que se dividen. A continuación, se presentan objetivamente los resultados tanto orales como escritos de la intervención. Para la recogida de datos escritos se usarán tablas y se presentarán aquellas respuestas que tengan un mayor interés. Una vez presentadas las respuestas se llevará a cabo su análisis basándose en los significados de la probabilidad y el lenguaje probabilístico. Por último, se extraerá una conclusión de la intervención educativa.

2. Justificación

Alsina y Vásquez (2016) destacan la necesidad de enseñar probabilidad para que el alumnado adquiera las herramientas para analizar críticamente aquellas situaciones de incertidumbre. Un ejemplo claro relacionado con la actual situación sanitaria derivada de la COVID-19 lo muestran Alsina *et al.* (2020) donde “para informar sobre el número de contagios, diversos países han contabilizado exclusivamente los casos testados con PCR (*Polymerase Chain Reaction*) u otras pruebas diagnósticas, sin considerar los casos asintomáticos o con sintomatología leve que no han necesitado hospitalización” (pág. 100). Garantizar una formación que permita al alumnado analizar críticamente datos en este tipo de situaciones es esencial. No obstante, los contenidos relacionados tanto con la probabilidad como con la estadística suelen ser los que se enseñan menos. Esto se debe a la falta de formación del profesorado en este ámbito lo que deriva en inseguridades a la hora de enseñarlos (Alsina, 2016).

En este contexto debemos preguntarnos si el alumnado logra alcanzar la base necesaria para poder comprender la probabilidad. Además de si son conscientes de la existencia de la probabilidad subjetiva, la cual se encuentra presente en nuestro día a día. Esto implica que todo el mundo, aunque carezca de una formación explícita sobre probabilidad, es capaz de establecer un grado de ocurrencia o de creencia de que un suceso tenga lugar, pudiendo usar tanto un lenguaje matemático como verbal.

3. Objetivos

Objetivo general

- Llevar a cabo una secuencia de actividades para observar el nivel de comprensión que tiene el alumnado de 5º de Educación Primaria sobre la probabilidad.

Objetivos específicos

- Alentar al alumnado a razonar sobre la probabilidad.
- Presentar la probabilidad subjetiva a los estudiantes.
- Relacionar la probabilidad con situaciones de la vida cotidiana, como, por ejemplo, con los deportes.

4. Metodología

A continuación, se va a exponer la metodología que se siguió para la elaboración de este trabajo, así como la población con la que se ha trabajado y otros aspectos relacionados.

4.1 POBLACIÓN

La población se ha dividido en dos grupos y ha tenido que llevarse a cabo en dos sesiones diferentes. La razón principal fue la imposibilidad de unir ambas clases en una misma aula debido al COVID-19. En ambas sesiones el alumnado procedía de un colegio público de Oviedo. El primer grupo A está formado por dieciocho estudiantes, mientras que el segundo, B, es más pequeño con trece. Son de 5º curso, lo que significa que tienen once o incluso doce años. Esto supone que el alumnado pertenezca a la etapa de operaciones concretas en la teoría de Piaget del desarrollo cognitivo que comprende edades entre siete y once años. Según Fischbein (1975), "entre los mecanismos adaptativos del niño debe incluirse la capacidad de estimar intuitivamente las proporciones y hacer predicciones" (p. 18). Los estudiantes de entre nueve y diez años pueden resolver problemas que requieren la comparación entre probabilidades de un mismo suceso en dos experimentos diferentes siempre y cuando los casos favorables y desfavorables sean los mismos en ambos experimentos (Cañizares, 1997). Aunque no se ha descubierto el momento exacto en que se adquieren intuiciones probabilísticas, en esta etapa, los niños comienzan a entender lo que es la aleatoriedad. Sin embargo, todavía creen que pueden controlarlo. Además, durante esta etapa, el alumnado aumenta las estrategias utilizadas para comparar probabilidades considerando no solo casos favorables, sino también casos desfavorables (Alsina *et al.*, 2021).

En cuanto al lenguaje probabilístico, Fischbein y Gazit (1984) han llegado a la conclusión de que los estudiantes de entre 9 y 14 años tienen más dificultades con el

término "seguro" que con "probable". La razón es porque se relacionan "seguro" con un resultado único, mientras que "probablemente" está relacionado con más resultados. En resumen, el alumnado relaciona el concepto de "seguro" con la singularidad. Además, como se basan en sus experiencias subjetivas, relacionan el término "poco probable" con "imposible" y luego, "imposible" con "incertidumbre".

4.2 INSTRUMENTO

La información se recopila con dos fuentes principales. Por un lado, tenemos las hojas escritas donde se presentan las preguntas y los enunciados (Anexo). El alumnado ha contestado en dichas hojas. Este tipo de documentación sería una fuente primaria. Por otro lado, una fuente personal en la que se escribieron notas durante la sesión. Estas notas están relacionadas con el debate oral que ha tenido lugar antes y después de las preguntas. En este caso sería una fuente secundaria, ya que no ha sido registrada por los propios autores sino escrita por una tercera persona. Como los estudiantes son españoles los enunciados están escritos en español, así como sus respuestas. Aunque el alumnado está aprendiendo inglés en la escuela, en estas ocasiones es mejor que usen su lengua materna para que les sea más fácil expresar sus pensamientos e ideas. Por lo tanto, durante el trabajo estos dos aspectos serán traducidos al inglés.

Hay que tener en cuenta que una fuente escrita no refleja las interacciones sociales entre los estudiantes. Solo representa el producto final sin mostrar el proceso que se ha llevado a cabo para alcanzarlo. Por ello, en esta investigación es esencial también recopilar información sobre la parte oral del experimento. Una recopilación de la fuente oral permite al investigador observar los cambios que se producen durante un período de tiempo concreto (López-Noguero, 2002).

4.3 METODOLOGÍA DE ANÁLISIS

La metodología que seguiremos para analizar los resultados se llama análisis cualitativo de contenido. Sin embargo, también se conoce como paradigma cualitativo, naturalista, fenomenología o interpretativo. Aunque cada metodología sigue diferentes enfoques, todos ellos comparten los mismos principios (Bisquerra, 1996). Según Berelson (1952), el análisis del contenido es una técnica que debe expresar el contenido de una manera objetiva, cuantitativa y sistemática.

Algunas características de este paradigma según López-Noguero (2002) son las siguientes:

- Los datos son recogidos por el investigador. Esto podría derivar en un análisis subjetivo. Por ello, es fundamental que el investigador adquiera una disciplina personal que requiere ser autoconsciente y una reflexión continua.
- Este paradigma se usa para producir teorías e hipótesis. De ningún modo se utiliza para verificar hipótesis.
- Se caracteriza por ser un análisis discursivo: "el diseño de la investigación es emergente: se va elaborando a medida que avanza la investigación" (López-Noguero, 2002, p.169).

5. Introduction

This end-of-degree work focuses on the role of probability in the educational field. The scope of probability is relatively new content on the Primary Education curriculum. It was not until 2006, with the Organic Law on Education (LOE), that knowledge related with probability appeared. Currently in the Organic Law for the Improvement of Educational Quality (LOMCE) in the subject of Mathematics, the block 5 is dedicated to statistics and probability.

The work is divided into two parts: the theoretical framework and a proposal for educational intervention. Nevertheless, first there will be a brief justification about the work, as well as the general and specific goals that were set to carry out this work.

Two fields of study are presented regarding the theoretical framework. On the one hand, the different meanings of probability that can be found in the Primary Education curriculum: classical meaning, intuitive meaning, frequential meaning and subjective meaning. The axiomatic meaning of probability is also mentioned even though it is not taught until High School. On the other hand, the different types of probabilistic language that students can use to express the probability of an event or the degrees of belief that something will happen.

The second part presents the educational intervention consisting of three activities carried out in two classes of 5th grade. This section presents the activities designed considering the organization, the necessary materials, the theoretical bases and the phases in which they are divided. Then, the oral and written results of the intervention are presented objectively for its following analysis based on the meanings of probability and probabilistic language. Tables will be used for the collection of written data and answers that are of greatest interest will also be shown. Finally, it will be a conclusion about the educational intervention.

6. Justification

Alsina and Vasquez (2016) emphasize the need to teach probability so that students acquire the necessary tools to critically analyze those situations of uncertainty. A clear example related to the current health situation arising from COVID-19 is shown by Alsina *et al.* (2020) where "to report on the number of infections, several countries have accounted exclusively for cases tested with PCR (*Polymerase Chain Reaction*) or other diagnostic tests, without considering asymptomatic or mild symptomatology cases that have not needed hospitalization"(p. 100). Therefore, training that allows students to critically analyze data in such situations is essential. However, content related to both probability and statistics is often the least taught. This is due to the lack of teacher training in this area which leads to insecurities when teaching them (Alsina, 2016).

In this context, we should wonder if students get to achieve the base needed to understand probability. As well as if they are aware of the existence of subjective probability, which is present in our daily life. This implies that everyone, even if they lack explicit probability training, can establish a degree of occurrence or belief that an event takes places, being able to use both mathematical and verbal language.

7. Goals

General goal:

- To carry out a sequence of activities to observe the level of understanding students from 5th grade of Primary Education have about probability.

Specific goals:

- To encourage the students to reason about probability.
- To make students deal with subjective probability.
- To relate probability with real-life situations such as sports.

8. Theoretical framework

Chance is present in our daily life at some degree. Batanero and Godino (2002) distinguish four fields where random events take place: biology, the environment, social and political areas. Within biology inheritability is the clearest example. Many biological factors such as weight, sex or hair color cannot be predicted beforehand. Another popular example nowadays is the probability of infection during the COVID-19 pandemic or the success and possible secondary effects of a vaccine. Regarding the environment, it is always changing which is a perfect context for random events to appear. It is the case of meteorology. Despite the technological progresses, it is hard to predict with perfect accuracy the weather in a specific place and time. Moreover, humans live in society creating a wide variety of uncertain situations. For instance, we cannot predict the number of family members, the delay of a plane or the results of the lottery. Lastly, in politics the decisions depend on uncertain events that need to be researched about. This is the reason why it is needed to carry out census, and interviews to obtain specific data. As we have observed probability appears in many situations of our daily life.

8.1 MEANINGS OF PROBABILITY

There are different meanings of probability (classical, frequentist, subjective, *etc.*) according to the given interpretation of probability. Taking as reference some ideas by Batanero and Díaz (2007), we recapitulate below those different meanings of probability:

- **Intuitive meaning:** to develop intuitive ideas, it is not needed a specific education in probability. Even children can express their degree of belief in the occurrence of a specific event by using oral language. Fischbein (1975) distinguish two types of intuitions. The first one is called anticipatory referring to a global vision developed from a problem. The other one is called affirmatory which are explanations of the interpretations. Within the anticipatory intuition there are primary which are acquired outside school without the need for an explicit instruction; and secondaries which are acquired thanks to education. Therefore, the social contexts of the students have a great importance within intuitive meaning. A student, whose social context has many random experiences, would be more likely they would develop primary notions of intuition (Cañizares, 1997). Fischbein (1975) also says that “the germ of intuitive reasoning about probability lies in natural “experiments” with stochastic results, which involve predictions and random draws or other equivalent actions.” (p.17)

- **Classical meaning:** probability is related with games of chances being probability the expectations to win. The first person to propose a formal definition of probability is De Moivre (1718) in “*The Doctrine of Chances*”. However, it was redefined by Laplace by only considering those events in which the cases have the same probability to happen. Laplace defined probability as “a fraction whose numerator is the number of favorable cases and whose denominator is the number of all cases possible” (Laplace, 1985/1814, 28). Therefore, the classical meaning of probability given by Laplace cannot be applied when the cases are infinite or when cases are not equiprobable (Batanero *et al.*, 2015).
- **Frequentist or experimental meaning:** probability is assigned from the use of relative frequencies which are obtained after a high number of repetitions of a specific experiment. Jacques Bernoulli was the first to give proof of the *Law of Large Numbers* demonstrating this meaning of probability. Von Mises (1952/1928) defined probability as the hypothetical number towards which the relative frequency tends after a high number of repetitions. Therefore, the higher the sample, the higher the reliability as there is more variability in smaller samples than in bigger ones (Batanero & Godino, 2002). Nevertheless, many events are almost impossible to repeat with the same circumstances in such as high number of times (Batanero & Díaz, 2007).
- **Subjective meaning:** as its own name reflects, this approach does not give an objective view of probability, but a subjective one. De Finetti (1974) defined probability as the personal belief of the happening of an event. Subjective probability depends on the previous knowledge and experiences of a person. Therefore, randomness can differ from one person to another and cannot be expressed quantitatively with mathematical expressions. From a didactic point of view, this approach develops the intuitive idea of learning by doing (Batanero *et al.*, 2015; Blanco-Fernández *et al.*, 2016; Kazak & Leavy, 2018)
- **Axiomatic meaning:** due to its high level of abstraction, this meaning of probability is common in high school education, therefore, it will be not considered in this work.

8.2 PROBABILISTIC LANGUAGE

The previous meanings of probability must be expressed by using a specific type of language. Each meaning of probability has a concrete way to communicate randomness. Gómez *et al.* (2013) have distinguished five main groups for the acquisition of probabilistic language which are the following:

- **Verbal expressions:** are those terms or expressions used to talk about mathematics. According to Shuard and Rothery (1984) we can distinguish between three categories of verbal expressions. The first category includes words used only in the mathematical fields such as “estimate”, “favorable cases” or “event”. The second category comprehends those words whose meaning change depending on if they are in a mathematical context or in a daily life situation. For instance, the term “impossible” in mathematics is used to indicate an event whose probability is zero whereas in quotidian language “impossible” can refer to an

event close to zero. Lastly, those terms whose meanings are the same in both contexts like “randomness” or “likely”. By considering the analysis done by Gómez *et al.* (2013), we can determine that in textbooks there are more expressions related with quotidian language. Those expressions specific of probability and randomness do not appear until third and fourth grades.

- **Numerical language:** entail the quantitative possibility of an event to happen. The numerical data can be whole numbers (since first cycle), fractions (since second cycle) or decimal numbers (since third cycle). This type of language is heavily related with classical meaning mainly due to the Laplace’s formula which requires the use of numerical data. Nevertheless, it is also found in subjective probability. Expressing a probability just by using verbal language cannot be too precise. Therefore, to assign probability to events in subjective probability they can give a numerical value between zero and one (Batanero & Godino, 2002). Being zero an event that is never going to happen and one to an event that is always going to happen in a specific experiment. We can also combine verbal with numerical language. As it has been confirmed by a study done by Witteman and Renooij (2002), students find less difficulties when the data is presented in a verbal-numerical probability scale.
- **Symbolic language:** comprehends the use of symbols or signs to express mathematical concepts or properties. Those symbols are called “significances”. The meaning of the symbols has been accepted and introduced by mathematics as an institution (Camos & Rodríguez, 2015). Most of the symbols used in Primary are in common with other areas of mathematics. It is the case of the symbol from the subtraction (-) or the equality sign. The symbolic language specific of probability is not introduced until higher grades of Primary (Contreras *et al.*, 2013). Nevertheless, due to their complexity the symbols used are just a few.
- **Graphic language:** refers to the variety of graphical representations used to summarize, organize, and communicate the information obtained (Alsina *et al.*, 2020). Graphic language is related with the frequential meaning of probability. Depending on the type of variable (qualitative or quantitative), the graphic used can change. For instance, a pie chart is used to represent absolute frequencies of a qualitative variable while a histogram represents probabilities from a quantitative continuous variable (Contreras *et al.*, 2013).
- **Tabular language:** is the use of tables to represent data. As graphic language, tabular language is mostly used in frequential meaning due to its need to organize the recollected data. Tables are applied to the representation of relative frequencies and their estimation of probability (Vásquez & Alsina, 2017).

9. Methodology

Below, we will present the methodology that was followed for the elaboration of this work as well as the population involved and other related aspects.

9.1 POPULATION

The population has been divided in two groups and had to be carried out in two different sessions. The main reason behind this choice was the impossibility to join both classes due to the COVID-19. In both sessions the students were from a public school in Oviedo. The first group A consists of eighteen students while the second one, B, is smaller with thirteen. They are from 5th grade which means they are eleven or even twelve years old. This means students belong to the stage of concrete operations in Piaget theory of cognitive development which comprehends ages between seven and eleven years old. According to Fischbein (1975), “among the child’s adaptive mechanisms should be included the ability to intuitively estimate proportions and make predictions” (p. 18). Students between nine and ten years old can solve problems that require the comparison between probabilities of a same event in two different experiments when the favorable and unfavorable cases are the same in both experiments (Cañizares, 1997). Although it has not been discovered the exact moment when probabilistic intuitions are acquired, in this stage, children begin to understand what randomness is. Nevertheless, they still believe they can control it. Moreover, during this age, students increase the strategies used to compare probabilities by considering not just favorable cases but also unfavorable cases as well (Alsina *et al.*, 2021).

Regarding probabilistic language, Fischbein and Gazit (1984) have analysed that student between 9 and 14 years old have more difficulties with the term “sure” than with “likely”. The reason is because they relate “sure” with a unique result while “likely” is related with more results. To sum up, students relate de concept of “sure” with uniqueness. Furthermore, as they are bases on their subjective experiences, they link unlikely with impossible and then, impossible with uncertainty.

9.2 INSTRUMENT

The information is gathered with two main sources. On the one hand, we have the answers written by the students in the sheets where the questions and wordings are presented. This type of documentation would be a primary source. On the other hand, a personal source in which I had written notes during the session. Those notes are related with the speaking debate that has taken place before and after the questions. In this case it would be a secondary source as it has not been recorded but written by a third person. As the students are Spanish the wordings are written in Spanish and also their answers. Although the students are learning English in the school, in these occasions is better that they use their mother tongue, so it is easier for them to express their thoughts and ideas. Therefore, during the work those two aspects will be translated to English.

A written source does not reflect the social interactions between the students. It only represents the final product without showing the process to reach it. Because of this, in this research is essential to also gather information about the oral part of the experiment. An oral source allows the researcher to observe the changes that take place during a concrete period of time (López-Noguero, 2002).

9.3 ANALYSIS METHODOLOGY

The methodology we will follow to analyze the results is qualitative analysis of content. Nevertheless, is also known as qualitative paradigm, ethnomethodology, naturalist, phenomenology or interpretative. Although each methodology follows different approaches, all of them share the same principles (Bisquerra, 1996). According to Berelson (1952), analysis of content is a technique that should express the content in an objective, quantitative and systematic way.

Some characteristics of this paradigm according to López-Noguero (2002) are the following:

- The data is sorted by the researcher. As this could lead to a subjective analysis, the researcher must keep a disciplined subjective (López-Noguero, 2002). This requires to be self-conscious and a continuous reflection.
- This paradigm is used to produce theories and hypothesis. Under no circumstances it is used to prove a given hypothesis.
- It is characterized by a recursive analysis which means the research is held to rules that are recurrent. The research is designed while the research is progressing (López-Noguero, 2002).

10. Description and sequence of the activities

In order to achieve the objectives established before, I have designed three activities following a logical question order to make students reason about different situations related with probability, following approaches based on game contexts and contextualized situations (Muñiz-Rodríguez *et al.*, 2014, 2021). Students must answer some questions which will allow us to perceive their level of understanding of probability. Furthermore, after each question, students must debate the answer so they can share their point of view and discuss their reasoning with the rest of their classmates.

10.1 SWEET BOX

The first activity is called the ‘sweet box’ and lasts for around twenty minutes. In the next sections it will be exposed how it is organized, the materials needed, the theoretical basis and the different phases to put the task in practice.

10.1.1 Organization

The activity is developed individually due to the actual COVID-19 situation. Students are organized in an individual way to reduce the interaction between each other and reduce the probability of infection. Each student will be given a sheet with several questions that will arise during the session. After each one writes down their answers, there will be a debate in which students will have to listen to their partners’ opinions and defend their owns. During the debate is crucial that students respect each other speaking time and listen carefully.

10.1.2 Materials

It is needed a box where there will be inside: two red sweets, one yellow sweet, five green sweets and three orange sweets (Figure 1). All sweets must be from the same brand to guarantee they are all the same size. In this way all cases are equally likely. If one kind of sweets were bigger, the problem could not be resolved following the classical approach of probability. The other resources needed are the sheet with the questions, a computer, and a projector to project the explanation.



Figure 1: image of the sweet box used in activity 1.

10.1.3 Theoretical basis

This first activity is a game mainly based in Laplace's rule of probability which belongs to the classical meaning of probability. Laplace's formula can only be used when the cases are equally likely which means there is the same probability to take one sweet or another. Therefore, by using Laplace's formula the probability could be calculated mathematically if needed. This methodology is the most used in school when students need to solve problems in the subject of mathematics (Alsina *et al.*, 2020). However, in this specific task there are also some questions related with subjective and frequentist probability. Subjective probability is present because students can justify their answers according to their likes or beliefs. Although the students are not aware, many of them will follow a frequentist meaning of probability during these questions. This is because they might change their answers due to the previous results from the specific event (grabbing a sweet).

The language used in this activity can vary depending on the previous knowledge of the students about probability. Due to the design of the wording, the types of language that are more likely to happen are the verbal language and numerical language. Within verbal language all three categories distinguished by Shuard and Rothery (1984) might appear. During the first phase there is no numerical data which means it is impossible for students to use numerical language. Nevertheless, in the following questions, numerical data will appear gradually.

10.1.4 Stages of the activity

First, it will be explained which flavors of sweet are in the box and the total number of sweets in the box. The information they will not have yet is the number of sweets of each flavor. As questions go by, students will be given more information. The main goal of having different sections with questions is to observe if the students' answers change as new information is given; or, on the contrary, if they stay with the same reasoning despite of the debate and the given data.

Before the extraction, students must answer the following questions in the sheet:

- Which flavor sweet will come out? Why?
- Which flavor sweet will not come out? Why?
- Which flavor sweet will you like to come out? Do you think it will come out or not? Why?

These questions have the aim to know the reason behind the prediction of the students about which flavor sweet will come out and which not. Moreover, we can analyze, if they consider impossible to guess properly the answer without farther information.

After the first extraction, the sweet will be put into the box again to maintain the original number of sweets in the box. Now that they have a lived experience of the experiment, students will be giving more data. In this case they will know the total number of green sweets, which is the flavor with a greater number of sweets. The principal reason behind why this flavor has the highest number of sweet is because most of the students might dislike this flavor. Therefore, if they are guided by their own likes, they will not choose this flavor despite having a high number of sweets. After the extraction and having been given the data, students will answer again the following questions:

- Which flavor sweet will come out? Why?
- Which flavor sweet will not come out? Why?
- Have you changed your answer regarding the previous section? Why?

The main objective is to perceive whether the students have changed their answers due to the lived experience or the quantitative information received.

Then, a second extraction will take place and as in the previous one, the sweet will be put into the box again. Students will be given the number of yellow sweets, one; and orange sweets, three. Although the number of red sweets it is not given explicitly, students can calculate with an addition and a subtraction. In this way they will know the number of all flavors. Now with all the quantitative data students must answer again the following questions:

- Which flavor sweet will come out? Why?
- Which flavor sweet will not come out? Why?
- Have you changed your answer regarding the previous section? Why?

As the flavor with more sweets is the green one, those who believe green is the one which will come out, means they have based their answers on mathematical reasons as it

is the flavor with a higher probability. On the contrary, if they choose the yellow or the red one, it will probably mean they have followed their own likes, subjective reasons.

Finally, a sweet will be extracted to compare the answers with reality as it has been done previously. If needed, we will also have a final debate to conclude. Moreover, as there are three extractions, there will be at least one flavor that will not come out.

10.2 SPINNING SWEETS

The ‘spinning sweets’ correspond to the experimental part of the previous activity, and it is the shortest part. Below it will be explained how it is organized, the materials needed, the theoretical basis and the different stages of the activity.

10.2.1 Organization

This section consists of a debate where students must contrast the results obtained in the real experiment with the virtual one. As well as comparing them with the theoretical percentage of probability. This part will be done orally with the whole group class as the main goal is for them to interact and share their reasonings.

10.2.2 Materials

The only needed materials are the projector and the computer in order to show the students the different pictures with the spinning. The adjustable spinner is an online resource obtained from the national council of teacher of mathematics (<https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Adjustable-Spinner/>). This resource allows putting into practice an experiment and comparing it with the theoretical probability. The spin is adjustable which means the data can be changed in accordance with the data from our specific activity (Annex). Moreover, in this task it is not needed a sheet as there is not a written part, it is done just orally.

10.2.3 Theoretical basis

This second task is related with the frequentist and classical meaning of probability. The use of a virtual spinning allows students to observe the outcomes after several trials in a small period. In our specific case we can repeat the experience as many times as we want. Moreover, they can compare the frequentist meaning with the classical meaning as the table recollects data according to both meanings of probability. This shows students the *Law of Large Numbers*, where the experimental data tends to get equal with the theoretical probability calculated through Laplace’s formula (Batanero & Díaz, 2007).

In addition, this activity develops three of the five different types of language related with probability (Vásquez & Alsina, 2017). On the one hand it develops tabular language by using tables to represent data (relative frequency, absolute frequency...). During this section, students must analyze the data from the table, comparing the different frequencies. On the other hand, graphic language refers to the variety of ways in which probabilistic data can be represented graphically. A circular graphic is used to represent the relative frequencies of each variable. The data is represented through sectors in a circle. The angles or areas of those sectors are proportional to the frequencies of the category they represent (Alsina *et al.*, 2020). In this case, students must work with two pie charts, one of them represent the data from the problem while the second one changes

after each spin as it represents the experimental pie chart by considering only the frequencies. Finally, it also develops their numerical language which is related with the numerical number of probability and the language used to compare probabilities.

The use of technologies when learning any concept increases the motivation of the students and their wish to participate and solve the mathematical problems. Godino *et al.* (2006) defends that virtual didactical resources can work as the base to promote the activity and critical thinking of mathematics. The use of virtual resources follows a constructivist pedagogy which encourages a deeper learning as it considers the previous knowledge of the students (Contreras *et al.*, 2019).

10.2.4 Stages of the activity

There is a virtual spinning in which the four colors (orange, green, yellow, and red) represent the flavors of the sweets. The area of each sector is proportional to the frequency of the flavor it represents. Therefore, the color with most area is the green whereas yellow has the smallest area. Students will carry out two independent experiments. In each experiment, students will obtain the results after five spins, twenty spins and fifty spins. The main goal of this phase is to compare if the experimental probability is always the same, or on the contrary, if it varies.

The first time they will just do one spin in order to understand the different data given on the website. On the one hand they have an experimental graph and a table with results obtained with the spins. On the other hand, they have a table that recollects the percentage obtained experimentally and the theoretical percentage obtained by using Laplace formula. The data is always changing after each spin.

Once they have understood the different resources available, they will start to spin five times, twenty and fifty. As the website allows doing the five spins at once the debate will take place just after each block of spins. After having done the first experiment they will do the second one and compare them.

10.3 CHESS AND PROBABILITY

The last activity is ‘chess and probability’ and it lasts for around thirty minutes. In the following sections it will be exposed how it is organized, the materials needed, the theoretical basis and the different phases of the activity.

10.3.1 Organization

As the first activity, this must be done individually due to the health situation. Each student will be given a sheet with the wording of a problem based on a chess game between two players from the elite. Students can ask any doubt they have about the wording or perhaps about chess. Those doubts will be solved in a group way by the rest of the students. This problem can be solved following subjective probability as it depends on the previous knowledge of the students and their opinion.

10.3.2 Materials

It is needed the sheet with the wording and the questions. To make it easier to understand for the students, it is also needed a projector and a presentation with the data in a more summarize way (Annex).

10.3.3 Theoretical basis

This last activity is based on subjective probability. The probability cannot be calculated mathematically with Laplace formula as there are way too many factors that take place during a chess match (pieces color, opening preparation, level of concentration *etc.*). We cannot base our predictions in previous matches because each match has its own characteristics and cannot be perfectly imitate. Therefore, this section is based on the previous knowledge of the students and their beliefs. Depending on the amount of knowledge they have about the chess world their answers will change. There is not a correct or wrong answer. Moreover, their intrinsic motivation to answer correctly and think critically about the questions is also affected by that fact.

Regarding the type of language used, this activity develops mainly verbal language related with probability. Students must express with specific terms their own beliefs of the occurrence of a success. In this activity the three categories distinguished by Shuard and Rothery (1984) can appear at some point.

10.3.4 Stages of the activity

In all the sections, the main data from the wording will be written on the whiteboard and will be visible in the projector in order to make it more visual for the students.

In the main wording, is essential to clarify that the date regarding each player refers to different amount of time. In the case of Anish Giri (among all the games played by Anish Giri (908) almost half (456) were draw) the information recollects all the games he played since he started to play chess. While in the case of Magnus Carlsen, (Magnus Carlsen lost only three games in the last two years and a half) the data is only about the last two years and a half. Therefore, the information about Anish Giri recollects the results of more games. Moreover, as this information has been taken from the official rating site of the *International Chess Federation* (FIDE, French acronym) (<https://ratings.fide.com/>), students who play federated chess might already know the information.

In the first phase, in order to make it easier for those who do not understand chess, we can compare the situation with other sports. For instance, in tennis it is very difficult to beat Rafael Nadal in clay court as he wins most of the matches. Something similar happens in chess with Carlsen who lost very few games during the last years. On the other hand, if there is any debate regarding the last question (Would it be any more probability if Anish had played with black pieces?), the teacher can do a comparison with the three-in-a-row. The answer justification of this section will vary depending on their previous knowledge about chess. In chess it is usually believed that the player with white pieces has a light advantage as it starts the game. In three-in-a-row starting the game also means

to be one turn ahead and even at the end of the game if all the spaces are occupied, the first player who has started the game has an extra figure on the board.

The second phase allows to debate about the importance of practicing and the different attitudes when facing a specific situation. In the debate they can also talk about how the pandemic situations has affected them, if they felt more tired, disheartened; or on the contrary, if they felt happier. A similar situation to understand better the question is the example of the summer between two scholar periods. After the summer, students usually have a slower rhythm or more difficulties to write, read or calculate due to the lack of practice during the summer. The third phase has similar aspects so the topics can be the same as the previous section. In both phases, we add new information about the match that can lead to changes in respect with the previous sections. Regarding only the quantitative part Magnus Carlsen might have more probability to win. However, subjectively students can argue that Anish Giri has better results after the quarantine and therefore, has more probability to win.

In conclusion, students must realize that sometimes an experiment or a match can give different results depending on when or where it takes place. Although circumstances might seem the same, in many cases it is impossible to repeat an experiment with all its variables like the case of football matches or exams.

11.Results

In this part it will be exposed the different results from an objective perspective. This section is divided in two parts. On the one hand, the results from the debate which has been written down during the lesson. On the other hand, the results from the written sheet that have been written by the students during the activities.

11.1 DEBATE

As the activities have been done in two classes, there are two debates. First, we should start with the first activity known as ‘Sweet box’ where students must answer several questions. In the case of group A, some students declared with the first pair of questions before any extraction that it was impossible to guess which flavor would come out as they did not have enough data. Therefore, most of the class suggested the answer according to their likes, but they still reclaimed the need to be given more quantitative information. Just after the first extraction, which was an orange sweet, the students were given de number of green sweets that were on the box. As there were five green sweets one student justified that the next sweet should be green as they were almost half of the total. However, another student contraposed that perhaps another flavor, for instance, orange, could also have five sweets. In the next extraction the sweet that got out was the red flavor which caused disbelief but also celebration by those who had chosen red due to their likes. After this second extraction, all the quantitative data was given which led to more students predicting that the green sweet would come out. Nevertheless, the same student who had disagreed previously said that it should be yellow. Many students contraposed describing the situation as impossible since there was only one yellow sweet. The student kept defending its choice justifying that the yellow sweet had not come out before and it was only matter of time it should come out. In the last extraction the sweet which came out

was the red one which caused again disbelief from many of the students who reclaimed it should have been the green sweet the one to come out.

In the case of the group B, before the first extraction the immense majority chose the flavor according to their own likes. There was only one student who said that there were not enough data to predict which flavor was going to come out. The green sweet was the first flavor to come out. After giving the number of green sweets students still justified their answers according to their likes. Only the student, who had reclaimed more data, justified that it would come out the green sweet. His reasoning was that as there were more green sweets, they occupied more space which means it was more probable to take a green one from the box. Other students said during the debate that he had changed one of his previous answers due to the result of the first extraction. The question was: “What sweet do you think is NOT going to come out?” He defended that the sweet he had written on the answer would be in fact the one which would come out. In conclusion, he believed that if he did not think a sweet would come out, that flavor would in fact come out. Therefore, he wrote the flavor he wanted to come out which was red. After the second extraction, a red sweet, students had all the data. However, the majority still based their answers on their likes. During the debate, only two students took account of the quantitative data. Finally, the last flavor was orange.

The second activity, the ‘spinning sweets’, only created debate in the class A. They confirmed that theoretically the flavor that had more probability to come out was indeed the green sweet. However, some students still were against, justifying that the green sweet had not come out even once so perhaps the theory was not good enough.

Finally in the third activity, ‘chess and probability’, in the class A they had previous knowledge and experience of chess as many students had played chess at least one time. In the first phase the question “Would he have more chances if he played with black pieces?”, created a debate between the students. Many of them defended that although black pieces are one move behind, they have advantage. Their reasoning was because it was what their own experience had taught them as they had had better result when they had played with black pieces. Nevertheless, the few students who did not know how to play, preferred white due to the example of the game three in a row. And one student justified that the color did not matter as the important fact was the players’ level and not who starts the game. Whether you play with white or black pieces did not have value if you are better than your opponent. On many occasions some students forgot that in chess sometimes you draw, which means you neither win nor lose. Therefore, in the second phase as Anish Giri had not lost any game some of them understood that Anish Giri had won all the games. After those reasonings many students who had played chess explained them that perhaps Anish Giri had drawn all games. Others with no previous knowledge justified that the winner should be Magnus Carlsen as it is the current world champion. If he has that title, it only means he is the best player on the world and consequently, he should win everyone.

In the case of the class B, only one student had previous knowledge about chess. Therefore, during the debate students would generally justify their answer according to

the specific wording or the fact that Carlsen is the world champion which meant he should be the winner. Regarding the question of whether Anish Giri had more chances to win with black pieces, after the example of three in a row, the majority concluded it would be worse for him. Only the student who played chess opposed with the justification that his experience had taught him otherwise as he had won more times with black pieces.

11.2 WRITTEN ANSWERS

The written answers will be organized in relation to each group. First, the data corresponding to class A and then, class B. The reason to divide the data in two groups is because the written answers might have been affected by the debate. In consequence, the data must be separated for the subsequent analysis.

11.2.1 Class A

- Sweet box

The first set of questions had been written before any extraction and just knowing the total number of sweets there were in the box (Table 1). We will count the answers to each question independently, as on many occasions students changed their reasoning from one question to another.

Table 1: Number of students from Group A according to their answers in the questions from the first stage.

Question	Data	Likes/Dislikes	Beliefs	Without justification	Other reason
1	2	4	8	2	2
2	0	5	5	6	2
3	0	1	14	3	0

In the first question within those from beliefs, four believed there were more sweets of the chosen flavor. Nevertheless, the other four started the answers with “I believe...” but they assure the reason was because there were in fact more sweets of that specific flavor. For instance, some answers were: “because there are more green sweets than the rest” or “because it is the more abundant flavor”. Four students followed their likes (favorite color or flavor), and someone wrote: “It will be orange because I like it and I think I am going to be lucky”. Two students justified that those questions could not be answered. And within the remaining two, one wrote “because the probability of sweet is low and those that are tasty have more chances to be extracted”; and the other “green because there are some possibilities”. As in the previous question, those in the column “belief”, think there are few sweets of the one they had chosen. Within those who have another reason, one chose the green sweet because he/she had never seen a green sweet, while the other wrote: “a sweet with wings because there are not any”. In the third question, the majority of the students used expressions like “I think”, “I believe”... Some of them had the feeling it would come out the sweet they wanted, while others thought it would not come.

The next group of questions had been answered after the first extraction which was orange flavor and after being given the number of green sweets (5). (Table 2).

Table 2: Number of students from Group A according to their answers in the questions from the second stage.

Question	Data		Likes/ Dislikes		Beliefs/ Luck		Previous experience		No reason		Other reason	
	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No
1	8		4		3		0		2		1	
2	0		2		4		5		6		6	
3	6	0	3	0	2	1	4	0	1	1	0	0

In the first question, eight students based their answer on numerical data. While seven chose green for having five sweets, one student wrote “the red one because it is the flavor with more sweets”. Within those who were guided by their likes, one student chose the red one because of the opposite as he/she did not like that flavor. The student who gave another reason was: “the red sweet because there are few possibilities and few students had chosen it”. In the second question some students based their answer on the previous experiences. While four wrote that orange would not come out as it had already come out, another student chose the yellow one because it had not come out. One student believed the yellow sweet would not come out because there were not any. And as in the previous stage, one student wrote: “other sweet with wings because they do not exist”. In the last question, those who based their answer on the data had changed their answer due to the information given about the number of green sweets. Two students had changed their choice in order to “try another luck”. Then four students had changed their answer because of the previous experience where their chosen sweet had not come out. One of them wrote: “I have changed it because my answer was wrong”.

Before the last set of questions, the sweet that came out was a red flavor. Then the students were given the number of sweets of all flavors except red ones as they could calculate it (Table 3).

Table 3 Number of students from Group A according to their answers in the questions from the third stage.

Question	Data		Likes/ Dislikes		Beliefs/ Luck		Previous experience		No reason		Other reason	
	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No
1	8		2		1		2		4		1	
2	9		2		0		2		3		2	
3	2	4	2	0	4	1	3	0	0	2	0	0

In the first question, it happened the same as in the previous stage. Within the eight students who based their answer on a numerical data, one chose the red sweet. One student wrote: “the yellow sweet because third time is a charm”. Two students chose the yellow sweet as it had not come out in the previous experiences. Another chose the orange sweet just to change his/her answer. In the second question eight students chose yellow for being the flavor with less sweets while one student chose the red one for having two sweets. Within those who based their answer on the previous experience, one wrote: “those which

had already come out will not come out again”. As in the previous stages one student wrote: “a blue sweet because there are not any in the box”. Other student justified his /her answer as: “because I say so”.

- Chess and probability

Now, we will count the results from the third activity (Table 4). Within those who thought Anish Giri would win, one of them wrote: “because hope is the last thing ever lost”, while another based his/her answer on luck. Other student believed Anish Giri would win because he is a good chess player. A student who based his/her answer on the data given wrote: “Anish Giri would win because although Carlsen had lost few games in the last two years and a half, it does not mean he had not lost more games before”. Those who defended Magnus Carlsen would win was because he is the world champions and therefore, must win. One student based his/her answer on his/her previous knowledge about chess players writing: “Magnus would win because of three motives: the elo difference between the players, he is the world champion and Magnus has a solid defense against e4 which is what Anish plays most”.

Table 4 Number of students from Group A according to their answers in the first question.

Which will be the result of the match?	Anish wins			Draw			Carlsen wins		
	Data	No reason	Other reason	Data	No reason	Other reason	Data	No reason	Other reason
	3	1	4	0	1	0	5	2	2

In the next question, only one student based his/her answer on luck while the majority answered that it does not matter the color of the pieces. Within those, one student wrote: “The color of the pieces does not influence, it is luck and if you play better than your opponent”. Three based their answer on previous experience, as white begins the game and can establish their own strategy. Moreover, one student writes about a well-known checkmate played with white pieces (“Scholar mate”). Finally, one student does not give an answer as he/she has not any experience playing chess.

Table 5: Number of students from Group A according to their answers in the second question.

Would Anish have less chances if he played with black pieces?	Beliefs/Luck		Previous knowledge		Without justification		There is no difference between black or white pieces	Other reason
	Yes	No	Yes	No	Yes	No		
	0	1	3	0	0	4	9	1

In the next question (Table 6), within those who believed that lockdown was better for Anish, the majority justified that Anish might have trained harder than Carlsen. One of them wrote: “Anish was luckier and trained and now might be better than Carlsen”. Those who justified lockdown had affected them both, argued that both were not able to practice so they were in the same conditions.

Table 6: Number of students from Group A according to their answers in the third question.

Do you think the lockdown was better for one of them?	It was better for Anish			It was better for Carlsen	It is the same	It was better for one of them (without justification)
	Data	Without justification	Other reason			
	1	3	5	0	6	3

Finally, within those who used numerical language, some used a scale between 0 to 10, other from 0 to 100 and others used percentages. In the case of percentages some were: 35.7, 81 or 38.

Table 7: Number of students from Group A according to their answers in the last questions.

Would you change your previous answer?	Yes	No
		2

If this match happened before the lockdown, which would be Anish's chances to win?	Verbal language	Numerical language	Both	Without answer
		4	12	1

11.2.2 Class B

- 'Sweet box'

The first set of questions had been written before any extraction. Some students followed different logics depending on the questions. Given this circumstance the questions will be counted independently (Table 8). In the first question, there is one student who justified the impossibility to answer the questions as there was not enough information to answer properly. Within those who answered by following their likes, there are some that justified their answer because it is their favorite flavor, while the others justified it for being their favorite color. Moreover, one student wrote: "Green because they are the green flavor". In the second question, within those with a different reason, one justified his/her answer because green gives bad luck, while the other wrote: "yellow because it never gets out". In the third question one student would like the green sweet to come out although he still emphasized the lack of quantitative data.

Table 8: Number of students from Group B according to their answers in the questions from the first stage.

Question	Data	Likes/Dislikes	Beliefs	Without justification	Other reason
1	1	8	0	3	1
2	1	6	3	1	2
3	1	9	3	0	0

The next group of questions had been answered after the first extraction which was green flavor and after being given the number of green sweets (Table 9). In the first question, one who based his/her answer on quantitative data wrote: “green because there is more quantity which means more volume”. Within those who justified their answers in relation with their likes, four was because it was their favorite color and two because it was their favorite flavor. In the second question, one student did not choose any sweet as there was not enough data to answer properly. The two students who gave other reasons, one chose orange because he/she had not written that sweet before while the other student wrote “blue will not come out because there are not any sweets of this color”.

Table 9: Number of students from Group B according to their answers in the questions from the second stage.

Question	Data		Likes/Dislikes		Beliefs		Without justification		Other reason	
	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No
1	2		6		1		4		0	
2	1		5		1		4		2	
Question	Data		Likes/Dislikes		Beliefs/ Luck		Without justification		Other reason	
	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No
3	1	0	5	2	1	1	3	0	0	0

Before the last set of questions, the sweet that came out was a red flavor. Then the students were given the number of sweets of all flavors except red ones as they could calculate it. In the first question, two students answered green because there were more green sweets but one of them is the first time, he/she gives this reasoning. In the second question, there were again five students who did not justify their answers, but they are not all the same students than in the previous question. Two students chose the yellow flavor because it is the one which has a smaller number of sweets. In this case, they are the same students who follow this reasoning in the previous question. One student wrote “yellow, why not?”. In the last question, the student within beliefs wrote: “Yes, I have changed the answer because I trust green color”.

Table 10: Number of students from Group B according to their answers in the questions from the third stage.

Question	Data		Likes/Dislikes		Previous experience		Without justification		Other reason	
	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No
1	2		5		1		5		0	
2	2		4		1		5		1	
Question	Data		Likes/Dislikes		Beliefs/ Luck		Without justification		Previous experience	
	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No
3	2	0	2	2	1	0	2	2	2	0

- ‘Chess and probability’

Now, we will observe the results from the first questions of the third activity (Table 11). Within those who defended Anish Giri would win, one wrote: “Anish Giri would win because he has not lost a game while Carlsen lost three”. One who said it would be a draw was because he said: “both of them are great players”.

Table 11 Number of students from Group B according to their answers in the first question

Which will be the result of the match?	Anish wins			Draw			Carlsen wins		
	Data	No reason	Other reason	Data	No reason	Other reason	Data	No reason	Other reason
	2	3	1	0	1	1	2	1	2

In the next question, a student based his/her answer on the previous knowledge of chess as white pieces make the first move. One student wrote: “Yes it matters the piece color because there are higher probabilities”. As in the other class, most of the students believed there is no difference whether they play with white or black pieces.

Table 12: Number of students from Group B according to their answers in the second question.

Would Anish have less chances if he played with black pieces?	Previous knowledge		Without justification		There is no difference between black or white pieces	Other reason	No answer
	Yes	No	Yes	No			
	2	0	1	1	7	1	1

In the following question (Table 13), within those who thought lockdown was better for Anish, three of them justified that Anish might have trained harder than Carlsen during that period. While one student defend lockdown was not better for anyone as it was mere luck, other wrote: “no, because Anish always wins”. Finally, one student did not understand the question and wrote: “yes they could play but not in November”.

Table 13: Number of students from Group B according to their answers in the third question.

Do you think the lockdown was better for one of them?	It was better for Anish			It was better for Carlsen	It is the same	It was better for one of them (without justification)
	Data	Without justification	Other reason			
	2	1	3	0	3	3

In the last question (Table 14), there were two confusing answers. On the one hand, one student wrote: “he could not win because he could lose”, while another wrote: “he has to be stronger to work his chess abilities”.

Table 14: Number of students from Group B according to their answers in the last questions.

	Yes	No	No answer	
Would you change your previous answer?	2	10	1	
	Verbal language	Numerical language	Confusing answer	Without answer
If this match happened before the lockdown, which would be Anish's chances to win?	7	2	2	2

12. Analysis of the results

Now, we will analysis the results exposed previously. Both classes will be analyzed together but when there is any reference to the debate it will be specified. First, we are going to analyze the results from the activity sweet box.

- 'Sweet box'

Most students in the first questions based their answers on intuitive and subjective reasons (likes, beliefs, luck...) by using verbal language. As they did not have any data this would be the only meanings of probability possible. However, some students argued they had not enough data. These students have a classical meaning of probability as they reclaimed the need of numerical data and they believed mathematical problems can only be solved with mathematical language.

From this point there are more possible meanings of probability. Even though they had been given some data, they would not be able to calculate the probability by following the Laplace formula yet. In the case of the debate of class A where one student had said that perhaps another flavor could also have five sweets, this shows he/she has not really thought about this idea. As we know there are a total of 11 sweets and four flavors which at least have one sweet. If two flavors had five sweets, then one flavor would not have any. However, no one else followed this reasoning during the debate. In the case of class B, a student changed one of his/her answers as he/she believed the one he/she would not want to come out, would be in fact the one that did. This would be a subjective meaning as he/she justified his/her answer on luck. Nevertheless, he/she also has a frequentist meaning as he/she considered the previous experience. Some students also based their answers on previous experience; nevertheless, they had a misconception of chance. As the orange flavor had come out (class A), some students guaranteed it would not come out again, when they had seen how the sweet was put in the box again after the extraction. They are not aware that each experiment is independent from the previous one. Even after having all the data, some students still had this reasoning.

Lastly, two students answered the question "Which flavor sweet will NOT come out?", with a sweet that was not in the box. For instance, a sweet with wings or a blue sweet. They follow a divergent reasoning as they solve the question from a different point of view which is not delimited by the data from the problem.

- **‘Spinning sweets’**

This activity showed that students have a lower knowledge about graphic and tabular language. It was difficult for them to understand the frequentist meaning appeared on the tables. On the contrary, they understood graphics faster as they are more visual. Although they should have studied how to elaborate and interpret double entry tables during fourth grade, in the last course there was the quarantine where students would not be able to advance on contents since April. Therefore, as probability is taught at the end of the course, they had not studied it for two years.

- **‘Chess and probability’**

In this case the debate will have more influence in the written answers of one class than in the other. This is because in class A there were more students with previous experience in chess; even one of them had been federated. Consequently, those who had not previous ideas, were more influenced by their classmates’ reasoning.

In the first questions the majority answered following an intuitive and subjective meaning of probability (beliefs, luck...). Some students had difficulties to correctly understand the data. When it says that Carlsen has only lost three games it does not imply that he has won the rest of the games as in chess there is also the possibilities of drawing. Consequently, it is not a lack of mathematical knowledge, but reading comprehension which is essential to solve mathematical problems. In the following questions a lot of students based their answers on their previous knowledge and experiences lived in chess. It is a subjective meaning of probability as depending on your previous experiences the answer will vary.

Regarding the last question we can observe the different language students used most and how they used it. For instance, in class A the majority used a numerical language by using percentages. They used whole numbers, fractions, and decimal numbers. However, some numbers seem a bit random like 35.7, 81, 38 or even 5/30. They know what a percentage is, but they do not know how to calculate it. Therefore, they just write numbers as they feel that is what they are supposed to do. The contrary happened in class B where the majority used a verbal language while a few used words such as “impossible”, “very low” or “none”, other just wrote the result they thought it would happen and the reason. We can observe that sometimes students have a wrong understanding of the probabilistic vocabulary. It is the case of the word “impossible” which means that an event can never happen, its probability will be zero. Nevertheless, students used it for those events whose probability was closed to zero, but there is a minimum chance that it takes place. Some words they could have used are: unlikely or improbable.

13. Conclusion

With this work we can appreciate the need for probability teaching in primary education. The use of games as a teaching resource to explain the different meanings of probability increases the motivation and participation of students (Muñiz-Rodríguez *et al.*, 2014, 2021). In addition, it allows them to appreciate in a more dynamic way the

presence of probability in numerous situations of daily life, which is a need in probability literacy (Gal, 2005; Alsina, 2019).

However, this work also shows the many difficulties that students have to correctly use some probabilistic terms (Batanero *et al.*, 2013). As well as a lack of ability to argue the reasoning on which they are based to provide answers. Moreover, we can perceive the absence of subjective and intuitive probability in the classroom. Being the two meanings of probability more present in everyday life. It can therefore be concluded that it is essential to present probability not only as formulas and probabilities calculations but also through contextualized situations that are realistic and relevant to students (Alsina *et al.*, 2020), for instance, sports. This fact led us to underline the need of working with these meanings of probability since early stages (Alsina *et al.*, 2021; Vásquez & Alsina, 2019) because, as this work shows, students are able to deal with them without major difficulties (Kazak & Leavy, 2018). Therefore, more activities involving these meanings of probability are needed (Alsina *et al.*, 2020; Blanco-Fernández *et al.*, 2016; Muñiz-Rodríguez *et al.*, 2020a). Moreover, it is necessary to increase the training in argumentation in terms of probability both among students and pre-service teachers (Alonso-Castaño *et al.*, 2021; Vásquez & Alsina, 2017; Hourigan & Leavy, 2020; Muñiz-Rodríguez *et al.*, 2020b; Muñiz-Rodríguez & Rodríguez-Muñiz, 2021).

Finally, it is necessary to remark some limitations arising from the didactic intervention. On the one hand, the sample size and the sampling method do not allow generalizing the results. On the other hand, we are in an exceptional course where students have some lack of probabilistic knowledge due to confinement, that prevented the study of certain related contents in the previous academic year.

Conclusión

Con este trabajo podemos apreciar la necesidad de la enseñanza de la probabilidad en la etapa de Educación primaria. Gracias al uso del juego como recurso didáctico para explicar los distintos significados de la probabilidad se incrementa la motivación y la participación del alumnado (Muñiz-Rodríguez *et al.*, 2021). Además, se les permite apreciar de una forma más dinámica la presencia de la probabilidad en numerosas situaciones de la vida cotidiana, algo que es necesario en la alfabetización probabilística (Gal, 2005; Alsina, 2019).

No obstante, este trabajo también deja entrever las numerosas dificultades que el alumnado tiene para utilizar correctamente algunos términos probabilísticos (Batanero *et al.*, 2013). Además de una falta de capacidad de argumentación para explicar los razonamientos en los que se basan a la hora de aportar unas respuestas, se puede percibir también la ausencia de la probabilidad subjetiva e intuitiva en las aulas. Siendo dos significados de probabilidad más presentes en la vida cotidiana. Por ello, se puede concluir que es fundamental presentar la probabilidad no solo como fórmulas y cálculos de probabilidades sino también a través de situaciones contextualizadas que sean realistas y relevantes para el alumnado (Alsina *et al.*, 2020), como por ejemplo en los deportes. Ello nos lleva a subrayar la importancia de trabajar con estos significados de la probabilidad desde edades tempranas (Alsina *et al.*, 2021; Vásquez & Alsina, 2019) ya

que, como se constata en este trabajo, el alumnado es capaz de manejarlos sin mayores dificultades (Kazak & Leavy, 2018). Por ello, son necesarias más actividades que involucren este tipo de probabilidad (Alsina *et al.*, 2020; Blanco-Fernández *et al.*, 2016; Muñiz-Rodríguez *et al.*, 2020a). Además, es preciso incrementar la formación en argumentación probabilística tanto del alumnado como del profesorado en formación (Alonso-Castaño *et al.*, 2021; Vásquez & Alsina, 2017; Hourigan & Leavy, 2020; Muñiz-Rodríguez *et al.*, 2020b; Muñiz-Rodríguez & Rodríguez-Muñiz, 2021).

Finalmente, es necesario señalar algunas limitaciones derivadas de la intervención didáctica. Por un lado, el tamaño de la muestra y el método de muestreo no permiten una generalización de los resultados. Por otro lado, nos encontramos en un curso excepcional donde el alumnado tiene cierta falta de conocimientos probabilísticos debido al confinamiento, que impidió el estudio de ciertos contenidos relacionados en el curso anterior.

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Verdes	
Naranjas	
Amarillos	
Rojos	
TOTAL	11

LA CAJA DE CARAMELOS

1- ANTES DE LA EXTRACCIÓN

- ¿Qué caramelo crees que va a salir? ¿Por qué?

- ¿Qué caramelo crees que NO va a salir? ¿Por qué?

- ¿Qué caramelo te gustaría que saliese? ¿Crees que va a salir?

2- AHORA QUE YA VISTE EL CAMELO QUE SALIÓ, RESPONDE A LAS PREGUNTAS:

- ¿Qué caramelo crees que va a salir? ¿Por qué?

- ¿Qué caramelo crees que NO va a salir? ¿Por qué?

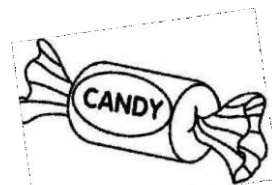
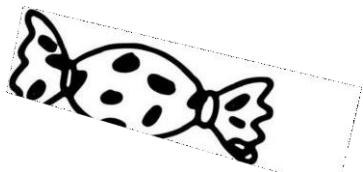
- ¿Has cambiado tu respuesta respecto al apartado anterior? ¿Por qué?

3- UNA VEZ SEPAS EL NÚMERO DE CAMELOS DE CADA SABOR, RESPONDE OTRA VEZ:

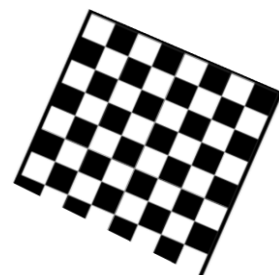
- ¿Qué caramelo crees que va a salir? ¿Por qué?

- ¿Qué caramelo crees que NO va a salir? ¿Por qué?

- ¿Has cambiado tu respuesta respecto al apartado anterior? ¿Por qué?



AJEDREZ Y PROBABILIDAD



En el torneo de ajedrez de Grenke se va a enfrentar Anish Giri de blancas, contra el actual campeón del mundo, Magnus Carlsen, de negras. De todas las partidas que Anish Giri ha jugado (908) prácticamente la mitad (456) fueron tablas. Por el contrario, Magnus Carlsen perdió solo tres partidas en los últimos dos años y medio.

- a. ¿Cuál crees que va a ser el resultado? ¿Conseguirá Anish Giri ganar con blancas al campeón del mundo? Justifica tu respuesta.

¿Tendría menos probabilidad de ganar si jugara de negras?

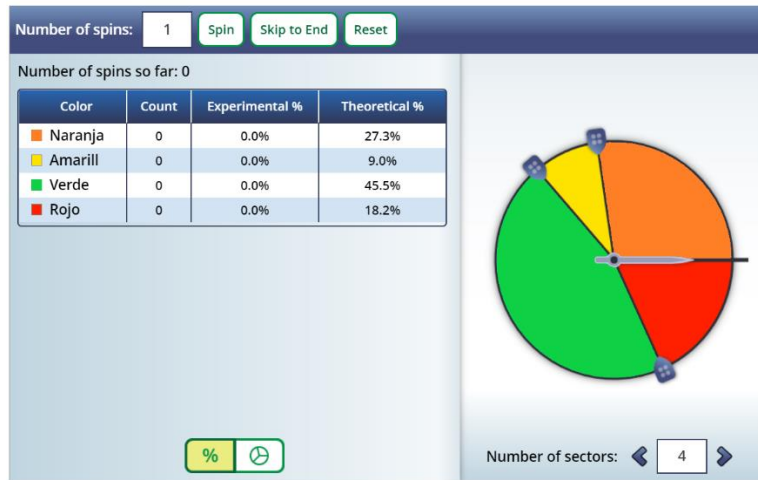
- b. Debido a la pandemia del Covi-19 ambos jugadores estuvieron varios meses sin jugar partidas lentas. Las tres partidas que perdió Carlsen en los dos últimos años y medio fueron después del confinamiento. Mientras tanto, Anish Giri no perdió ninguna partida después del confinamiento. ¿Crees que el confinamiento fue más beneficioso para uno que para el otro?

Tras conocer estos datos, ¿cambiarías la respuesta del apartado anterior?

- c. Si esta misma partida hubiera ocurrido antes de la pandemia cuando Carlsen llevaba 120 partidas sin perder. ¿Cuál crees que sería la probabilidad de que Anish Giri ganara?

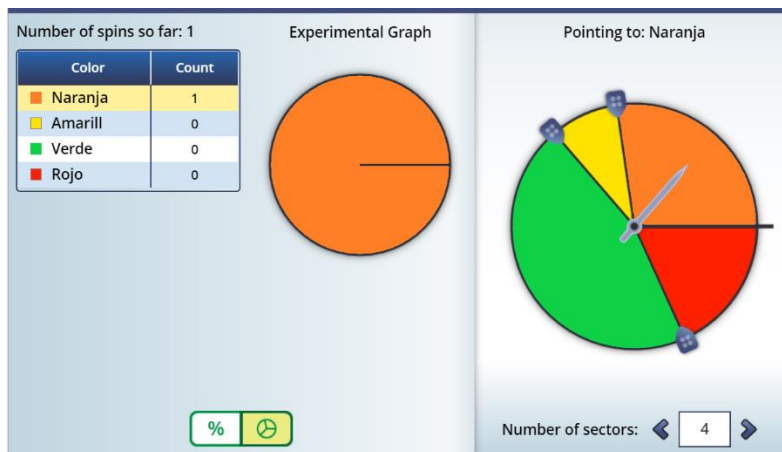
Spinning sweets' images:

Initial situation

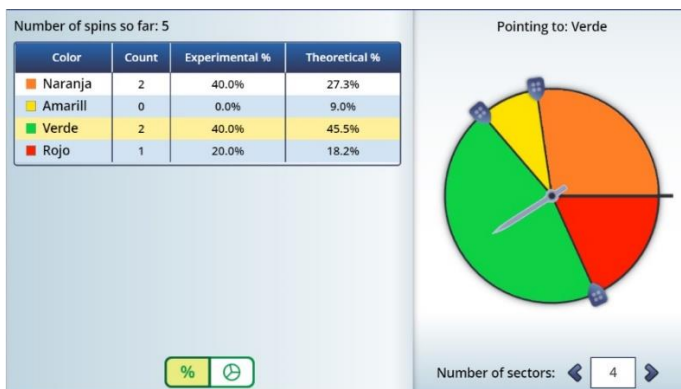


First experiment

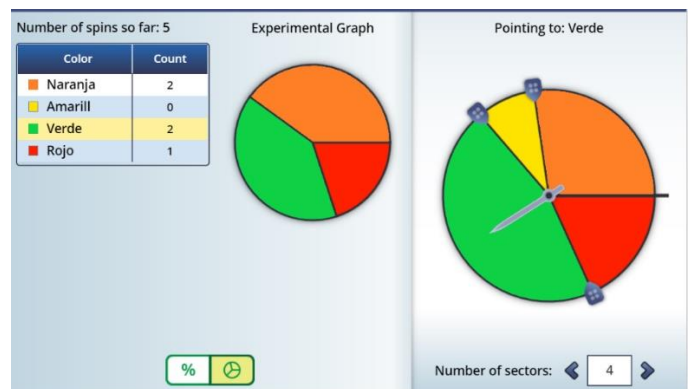
One spin



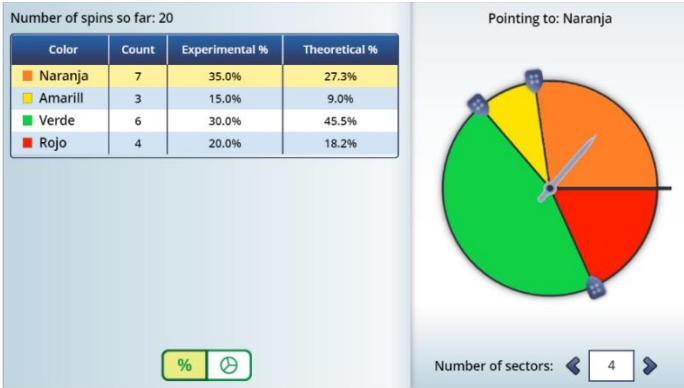
Five spins (table)



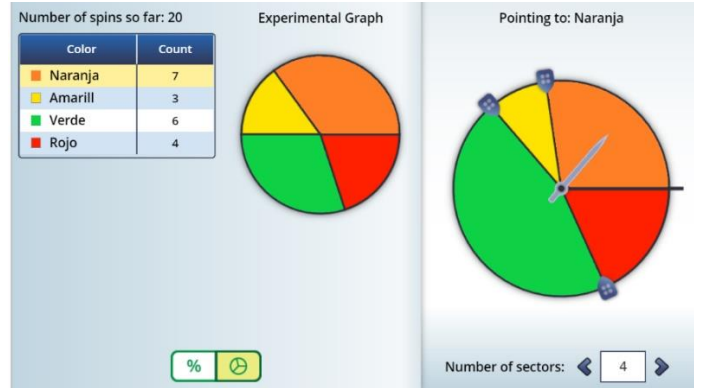
Five spins (graphic)



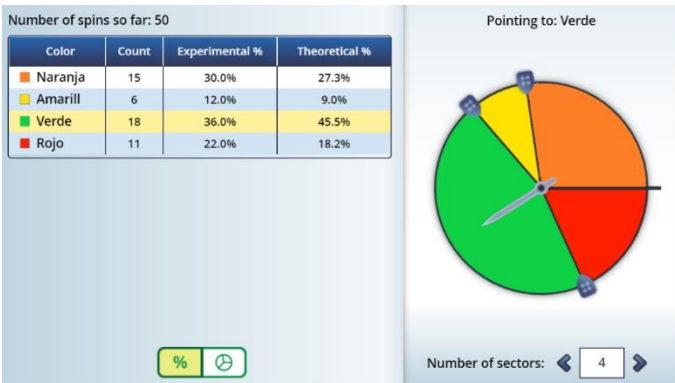
Twenty spins (table)



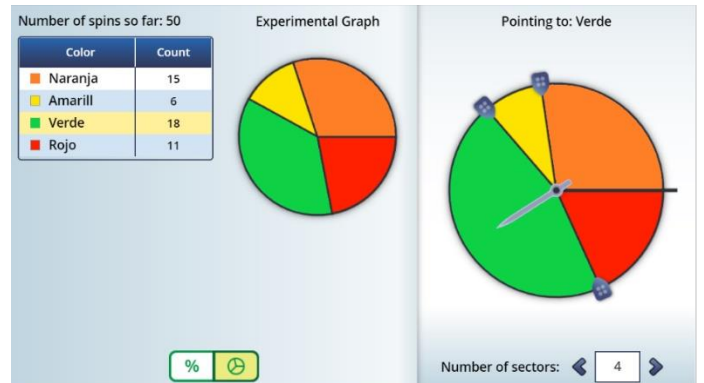
Twenty spins (graphic)



Fifty spins (table)

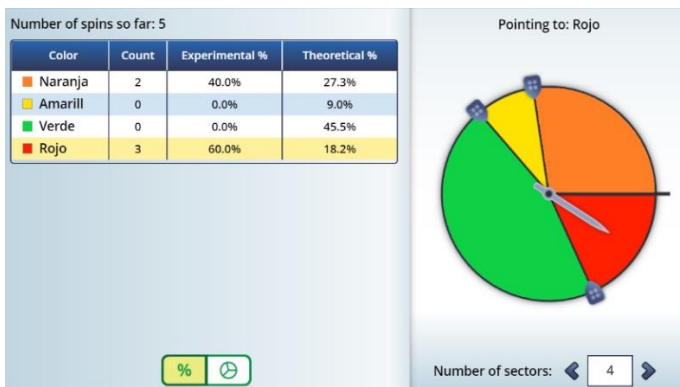


Fifty spins (graphic)

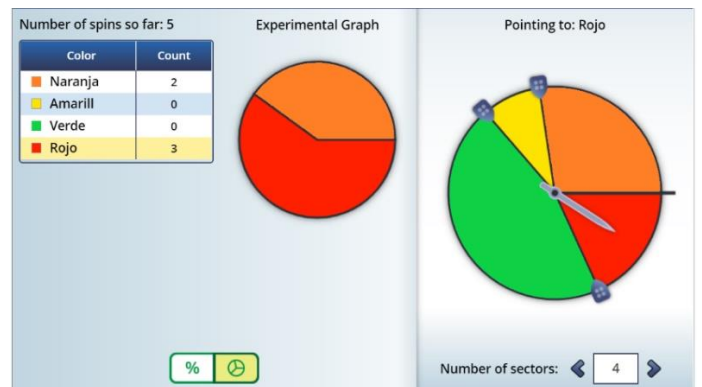


Second experiment

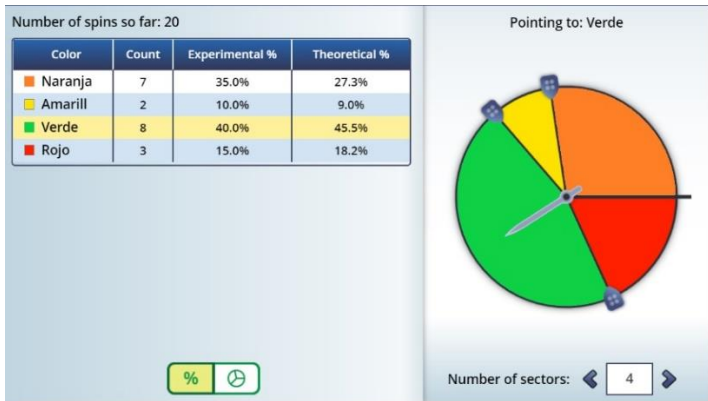
Five spins (table)



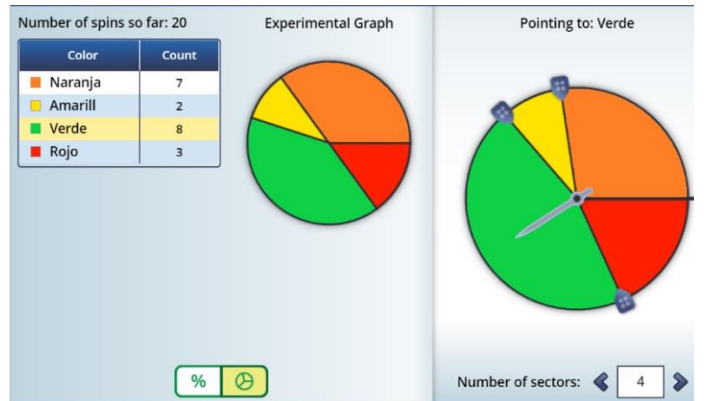
Five spins (graphic)



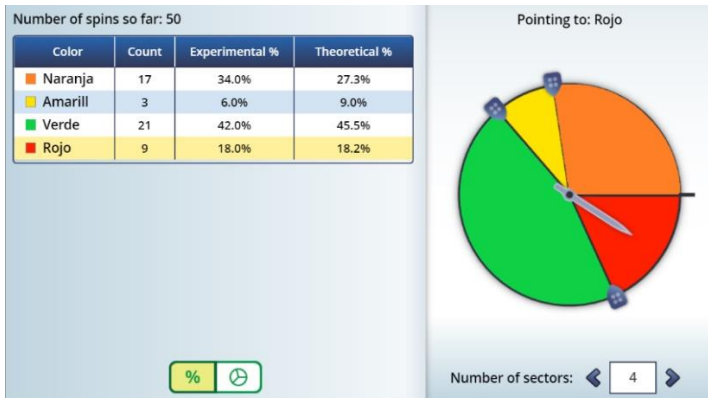
Twenty spins (table)



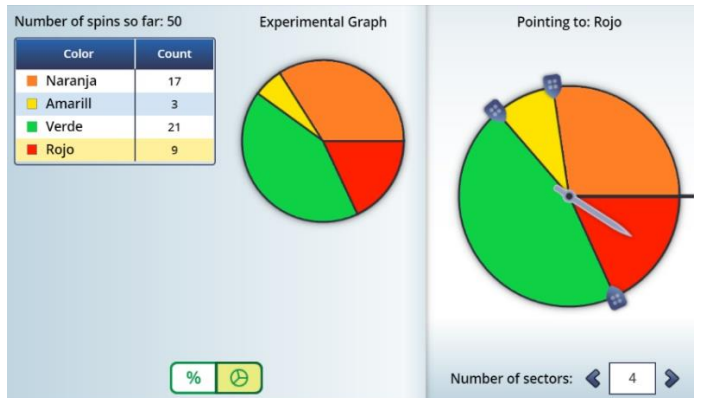
Twenty spins (graphic)



Fifty spins (table)



Fifty spins (graphic)



Presentation to explain the activities.



La caja de caramelos



1- ANTES DE LA EXTRACCIÓN

•¿Qué caramelo crees que va a salir? ¿Por qué?

•¿Qué caramelo crees que NO va a salir? ¿Por qué?

•¿Qué caramelo te gustaría que saliese?
¿Crees que va a salir o no?

Verdes	
Naranjas	
Amarillos	
Rojos	
TOTAL	11



2- AHORA QUE YA VISTE EL CARAMELO QUE SALIÓ

•¿Qué caramelo crees que va a salir? ¿Por qué?

•¿Qué caramelo crees que NO va a salir? ¿Por qué?

•¿Has cambiado tu respuesta respecto al apartado anterior? ¿Por qué?

Verdes	5
Naranjas	
Amarillos	
Rojos	
TOTAL	11



3- UNA VEZ SEPAS EL NÚMERO DE CARAMELOS DE CADA SABOR:

•¿Qué caramelo crees que va a salir? ¿Por qué?

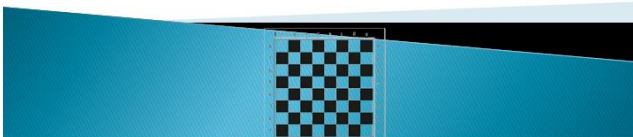
•¿Qué caramelo crees que NO va a salir? ¿Por qué?

•¿Has cambiado tu respuesta respecto al apartado anterior? ¿Por qué?

Verdes	5
Naranjas	3
Amarillos	1
Rojos	
TOTAL	11



Ajedrez y probabilidad



Anish Giri:

- 456 tablas de 908 partidas (la mitad)

Magnus Carlsen

-Dos años y medio: tres partidas perdidas



Después del confinamiento

Anish Giri:

- No perdió ninguna partida

Magnus Carlsen

- Tres partidas perdidas.



Antes del covid-19

Magnus Carlsen

-120 partidas sin perder

