SL(2,Z) symmetries, supermembranes and symplectic torus bundles

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Abstract: We give the explicit formulation of the 11D supermembrane as a symplectic torus bundle with non trivial monodromy and non vanishing Euler class. This construction allows a classification of all supermembranes showing explicitly the discrete $SL(2, Z)$ symmetries associated to dualities. It hints as the origin in M-theory of the gauging of the effective theories associated to string theories.

Contents

1 Introduction

Nonperturbative effects like monopoles, instantons in conventional gauge theories, or dualities in the context of M/string theories rely on global aspects of those theories. Properties like confinement may well be due to non trivial topological aspects as well. Non trivial fibrations have also been used in the context of noncommutative theories, like the noncommutative formulation of the torus [1] as well to characterize compactification spaces useful for string phenomenology, see for example [2, 3].

There is evidence that string theory can be consistently defined in non-geometric backgrounds in which the transition functions between coordinate patches involve not only diffeomorphisms and gauge transformations but also duality transformations [4, 5]. Such backgrounds can arise from compactifications with duality twists [6] or from acting on geometric backgrounds with fluxes with T-duality [5, 7, 8] or mirror symmetry. In special cases, the compactifications with duality twists are equivalent to asymmetric orbifolds which can give consistent string backgrounds [9–11]. In this type of compactifications, T-folds are constructed by using strings formulated on a doubled torus T^{2n} with n coordinates conjugate to the momenta and the other n coordinates conjugate to the winding modes [7].

In [5, 7] it was argued that a fundamental formulation of string/M-theory should exist in which the T- and U-duality symmetries are manifest from the start. The natural framework for M-Theory would generalize ten- or eleven-dimensional space-time into a higher-dimensional geometry in which auxiliary dimensions would be related to non-metric degrees of freedom. The duality symmetries of string- and M-theory would be discrete geometric symmetries of this generalized space. In particular, it was argued that many massive, gauged supergravities cannot be naturally embedded in string theory without such a framework [4, 8, 12, 13]. Examples of generalized T-folds can be obtained by constructing torus fibrations over base manifolds with non-contractible cycles. In particular, an example

is to consider S^1 as base manifold when the monodromy of the theory in the fibres includes a non-geometric element of O(Z) corresponding to a generalised T-duality [14]. For other interesting works emphasizing the relation with U-duality, M-theory and T-folds see [15]–[17]. However, in spite of advances, up to our knowledge, a full-fledged realization of these ideas in terms of the string worldsheet action for String Theory, or in terms of the supermembrane for M-theory is still lacking.

The aim of this paper is to prove that the action of the Supermembrane with nontrivial central charges, whose local structure was given in [18–21] may be globally defined in terms of sections of a symplectic torus bundle with nontrivial monodromy and Euler numbers. The monodromy is given as a representation of the fundamental group of the symplectic base manifold of the supermembrane in the homotopic π_0 group of symplectomorphisms of the fiber. In the case we will consider, the latest becomes the SL(2,Z) group acting on the fiber. It defines an automorphism on the fibers providing the global structure of the geometrical setting. The proof involves a nontrivial step in showing that the action of the Supermembrane with central charges, which explicitly depends on the moduli of the fiber manifold, is invariant under the Z-module associated to the monodromy.

We think that this global construction, which allows a classification of all supermembranes that can be formulated as symplectic torus bundles with nontrivial monodromy and Euler numbers, is the origin in M-theory of the gauging of the effective theories associated to String theories [22].

The Supermembrane with nontrivial central charges, motivated by the light cone gauge formulation of the Supermembrane in [23–26], introduces a topological restriction on the physical configurations. It defines an associated Chern number. From an algebraic point of view it can be interpreted as a nontrivial central charge in the Supersymmetric algebra. From a geometrical point of view it ensures the existence of a U(1) principle bundle and a monopole connection [27] on it whose curvature is in the Chern class associated to the topological restriction. In this sense, it is a natural way to introduce monopole configurations which stabilizes the supermembrane. In fact, the resulting regularized Hamiltonian has a discrete spectrum , that is the essential spectrum is empty [28–31]. The additional global structure we will consider involves in a manifest way the SL(2,Z) duality group of String theory. Its supermembrane origin was emphasized in [32] in relation with the (p,q) string solutions, see also [21]. The SL(2,Z) group acts on the fiber bundle structure as the cero homotopic group of symplectomorphisms preserving the symplectic form. This action induces a modular transformation on the basis of homology of the fiber, and correspondingly a Mobius transformation on the moduli of Teichmuller space. The final consistency in the construction arises when the global SL(2,Z) structure becomes compatible with the monopole (or central charge) topological restriction.

The paper is structured as follows. In section 2 we show some properties of a symplectic torus bundle. In section 3 we present the local structure of the supermembrane with central charges. In section 4 we show how the hamiltonian of the supermembrane with central charges is invariant under the group of monodromies of the symplectic torus bundle and consequently, it may be formulated in terms of sections of symplectic torus bundles. Section 5 is devoted to conclusions.

2   Supermembranes and symplectic torus bundles

A symplectic torus-bundle [33] is a smooth fiber bundle

$$\xi : F \to E \xrightarrow{p} \Sigma \qquad (2.1)$$

F is the fiber, which we take to be the 2-torus T^2. E is the total space and Σ the base manifold which we consider to be a closed, compact Riemann surface. The structure group is the group of symplectomorphisms preserving a given symplectic structure on T^2. So far, this symplectic fibration naturally fits in a Supermembrane formulation in the light cone gauge since this one is invariant under area preserving diffeomorphisms which are symplectomorphisms preserving the associated symplectic structure. The latest are symplectomorphisms on the base manifold , however in the supermembrane with nontrivial central charges [18] which we will describe in detail in the following sections , these ones correspond to the pull-back of the symplectomorphisms on the fiber. In order to describe global aspects of the supermembrane we introduce a monodromy and the associated Z-module. The monodromy is the natural extension of the monodromy in a torus bundle on a circle as considered by Thurston. We follow here the approach of [33]. Related work may be found in [34–36] and [37].

The action of the structure group on T^2 produces a $\pi_0(G)$ action on the homology and cohomology groups of T^2. The homomorphisms $\pi_1(\Sigma) \to \pi_0(G)$ give to each homology and cohomology group of T^2, the structure of $Z[\pi_1(\Sigma)]$-module. $\pi_0(G)$ in the case under consideration is known to be $\mathrm{SL}(2, Z)$. Moreover, the action of $\pi_0(G)$ on $H_1(T^2)$, the first homology group, may be identified with the natural action of $\mathrm{SL}(2, Z)$ on Z^2. Given any representation $\rho : \pi_1(\Sigma) \to \mathrm{SL}(2, Z)$ we denote Z_ρ^2 the corresponding $Z[\pi_1(\Sigma)]$-module.

A theorem, see [33],ensures the existence of a bijective correspondence between the equivalent classes of the symplectic torus bundle together with a representation ρ inducing the module structure Z_ρ^2 on $H_1(T^2)$ and the elements of $H^2(\Sigma, Z_\rho^2)$, the second cohomology group of Σ with local coefficients Z_ρ^2. This theorem classifies the symplectic torus bundle ξ with a representation ρ in terms of the characteristic class $C(\xi)$. In order to formulate the supermembrane with central charges in terms of sections of a symplectic torus bundle with a representation ρ inducing a $Z[\pi_1(\Sigma)]$-module, we have to consider the transformation of its hamiltonian under the action of $\mathrm{SL}(2, Z)$ on the homology basis since the moduli of the 2-torus T^2 appear explicitly in the hamiltonian.

3   The supermembrane in the light cone gauge

In this section we describe the Supermembrane with non trivial central charges in terms of maps from the base manifold to the target space. It corresponds to a formulation in terms of local sections of a symplectic torus bundle.

The hamiltonian of the $D = 11$ Supermembrane may be defined in terms of maps X^μ, $\mu = 0, \ldots, 10$, from a base manifold $\Sigma \times R$ onto a target manifold which we will assume to be 11D Minkowski. Σ is a Riemann surface of genus g. $\sigma^a, a = 1, 2$ are local spatial coordinates over Σ and $\tau \in R$ represents the worldvolume time. Decomposing X^μ and P_μ

accordingly to the standard Light Cone Gauge (LCG) ansatz and solving the constraints, the canonical reduced hamiltonian of the $D = 11$ supermembrane is given by

$$H = T^{-2/3} \int_\Sigma \sqrt{W} \left[\frac{1}{2} \left(\frac{P_M}{\sqrt{W}} \right)^2 + \frac{T^2}{4} \{X^M, X^N\}^2 + \sqrt{W} \overline{\theta} \Gamma_- \Gamma_m \{X^m, \theta\} \right] \qquad (3.1)$$

subject to the constraint

$$\phi \equiv d(P_M dX^M + \overline{\theta} \Gamma_- \theta) = 0 \qquad (3.2)$$

and to the global one

$$\phi_0 \equiv \int_{\mathcal{C}_f} P_M dX^M + \overline{\theta} \Gamma_- d\theta = 0 \qquad (3.3)$$

where $\mathcal{C}_s$ is a basis of homology on Σ, with $M = 1, \ldots, 9$, and P_M are the conjugate momenta to X^M. $\sqrt{W}$ is the scalar density introduced in the LCG , Σ is the base manifold which we take to be a Riemann surface, θ represents the 11D Majorana spinors and Γ_μ are the corresponding Dirac matrices. T is the tension of the supermembrane. ϕ and ϕ_0 are the generators of the area preserving diffeomorphims homotopic to the identity and they preserve the area element $\sqrt{W} \epsilon_{ab} d\sigma^a \wedge d\sigma^b$, a symplectic 2-form.

$$\{X^m, X^n\} = \frac{\epsilon^{ab}}{\sqrt{W}} \partial_a X^m \partial_b X^n \qquad (3.4)$$

is the associated symplectic bracket. We now consider the supermembrane wrapped on a compact sector of the target space,[1] restricted by a topological condition: the supermembrane with nontrivial central charges.

3.1 The supermembrane with nontrivial central charges

We consider the Supermembrane on a target space $M_9 \times T^2$ where T^2 is a flat torus defined in terms of a lattice $\mathcal{L}$ on the complex plane C:

$$\mathcal{L} : z \rightarrow z + 2\pi R(l + m\tau), \qquad (3.5)$$

where m, l are integers, R is a real moduli, $R > 0$, and τ a complex moduli $\tau = Re\tau + iIm\tau$, $Im\tau > 0$, T^2 is defined by $C/\mathcal{L}$. τ is the complex coordinate of the Teichmuller space for $g = 1$, that is the upper half plane. The Teichmuller space is a covering of the moduli space of Riemann surfaces, it is a $2g - 1$ complex analytic simply connected manifold for genus g Riemann surfaces.

The conformally equivalent tori are identified by the parameter τ modulo the Teichmuller modular group, which in the case $g = 1$ is $SL(2, Z)$. It acts on the Teichmuller space through a Mobius transformation and it has a natural action on the homology group $H_1(T^2)$. In order to define a supermembrane with nontrivial central charges we consider maps X^m, X^r from Σ to the target space , with $r = 1, 2; m = 3, \ldots, 9$ where X^m are single

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[1]The supermembrane with winding without any extra topological condition was analyzed in [39].

valued maps onto the Minkowski sector of the target space while X^r map onto the T^2 compact sector of the target. The necessary winding conditions, in order to define a map onto the T^2, are:

$$\oint_{\mathcal{C}_s} dX = 2\pi R(l_s + m_s\tau) \tag{3.6}$$

where $dX = dX^1 + idX^2$, l_s and m_s, $s = 1, 2$ are integers and $\mathcal{C}_s$ the above mentioned basis of homology for a genus $g = 1$ Riemann surface. From now on, Σ will be a Riemann surface of genus $g = 1$. We will denote $d\widehat{X}^r, r = 1, 2$ a normalized basis of harmonic one forms on Σ:

$$\oint_{\mathcal{C}_s} d\widehat{X}^r = \delta_r^s \tag{3.7}$$

We may decompose the closed one-forms dX^r in terms of harmonic one forms plus exact ones. We obtain from (3.6)

$$dX = 2\pi R(l_s + m_s\tau)d\widehat{X}^s + dA \tag{3.8}$$

where dA denotes the exact one-form. We now impose a topological restriction on the winding maps: the irreducible winding constraint,

$$\int_\Sigma dX^r \wedge dX^s = n\epsilon^{rs} Area(T^2) \quad r, s = 1, 2 \tag{3.9}$$

where the winding number n is assumed to be different from zero. ϵ^{rs} is the symplectic antisymmetric tensor associated to the symplectic 2-form on the flat torus T^2. In the case under consideration ϵ^{rs} is the Levi Civita antisymmetric symbol. An important point implied by the assumption $n \neq 0$ is that the cohomology class in $H^2(\Sigma, Z)$ is non-trivial.

It also implies that there is an U(1) nontrivial principle bundle over Σ and a connection on it whose curvature is given by $d\widehat{X}^r \wedge d\widehat{X}^s$. This U(1) nontrivial principal fiber bundle are associated to the presence of monopoles on the worldvolume of the supermembrane explicitly discussed in [27].

The natural scalar density $\sqrt{W}$ on the geometrical picture we are considering is obtained from the pullback of the symplectic 2-form on T^2 by the map $\widehat{X}^r, r = 1, 2$,

$$W = \epsilon_{rs}d\widehat{X}^r \wedge d\widehat{X}^s \equiv \sqrt{W}\epsilon_{ab}d\sigma^a \wedge d\sigma^b \tag{3.10}$$

where $\sqrt{W} = \frac{1}{2}\epsilon_{rs}\partial_a\widehat{X}^r\partial_b\widehat{X}^s\epsilon^{ab}$.

The symplectomorphisms preserving the canonical symplectic structure on T^2 are then pull-back to symplectomorphisms preserving W on Σ. This is relevant in the construction of the supermembrane with central charges as sections of a symplectic torus bundle. Using $Area(T^2) = (2\pi R)^2 Im\tau$, condition (3.9) implies

$$n = det\begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix} \tag{3.11}$$

That is, all integers $l_s, m_s, s = 1, 2$ are admissible provided they satisfy restriction (3.11).

The supermembrane with non-trivial central charges is invariant under area preserving diffeomorphims homotopic to the identity. In particular, under conformal maps which leave invariant the homology basis on Σ. In fact, $d\widehat{X}^r$ remain invariant and hence the symplectic 2-form in Σ. It is also invariant under diffeomorphisms not homotopic to the identity acting on the homology basis in Σ as $\text{SL}(2, Z)$ transformations. In fact if $d\widehat{X}^r(\sigma) \rightarrow S_r^s d\widehat{X}^r(\sigma)$ with $[S_s^r] \in \text{SL}(2, Z)$, then the symplectic 2-form W remains invariant. In this case

$$dX \rightarrow 2\pi R(l_s + m_s\tau)S_r^s d\widehat{X}^r + dA \tag{3.12}$$

where the exact part A transform as a scalar field. Consequently, if we also transform the winding integers by

$$\begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix} \rightarrow \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix} \begin{pmatrix} S_1^1 & S_2^1 \\ S_1^2 & S_2^2 \end{pmatrix} \tag{3.13}$$

then the harmonic part of dX remains invariant. The $\text{SL}(2, Z)$ acts from the right on the winding matrix.

4 The supermembrane maps as sections of a symplectic torus bundle

In this section we are going to prove the invariance of the Hamiltonian under the $Z[\pi_1(\Sigma)]$-module. The hamiltonian of the supermembrane with central charges is given by

$$H = \int_\Sigma \mathcal{H} = \int_\Sigma T^{-2/3}\sqrt{W}\left[\frac{1}{2}\left(\frac{P_m}{\sqrt{W}}\right)^2 + \frac{1}{2}\left(\frac{P_r}{\sqrt{W}}\right)^2 + \frac{T^2}{2}\{X^r, X^m\}^2\right. \tag{4.1}$$
$$\left. + \frac{T^2}{4}\{X^r, X^s\}^2 + \frac{T^2}{4}\{X^m, X^n\}^2\right] + \text{fermionic terms}$$

subject to (3.2), (3.3), and (3.6), (3.9), where X^r are sections on the symplectic torus bundle ξ with structure group G, the symplectomorphims preserving the symplectic 2-form on the fibre T^2 defined previously . P_r are the conjugate momenta to the exact part in the decomposition of X^r. The integrand depending in $X^r, r = 1, 2$ may be re-written in terms of

$$dX = dX^1 + idX^2 \tag{4.2}$$

as

$$\frac{1}{2}\{X, X^m\}\{\overline{X}, X^m\} + \frac{1}{8}\{X, \overline{X}\}\{\overline{X}, X\}, \tag{4.3}$$

where

$$dX = 2\pi R(l_s + m_s\tau)d\widehat{X}^s + dA \tag{4.4}$$

where R and τ are the moduli of the T^2, $d\widehat{X}^s, s = 1, 2$ as before are the harmonic basis of Σ and dA is the exact one-form in the Hodge decomposition. $A = A_1 + iA_2$ carries the physical degrees of freedom of the compact sector. The action of $\pi_0(G) \equiv \text{SL}(2, Z)$ in

$H_1(T^2)$ is the natural one inducing a Mobius transformation on the upper half plane with complex coordinate τ. We now prove that the hamiltonian (4.1) is a well defined functional on the symplectic torus bundle with monodromy ρ, where ρ is a representation of $\pi_1(\Sigma)$ in $\mathrm{SL}(2,Z)$ as defined in section 2. In fact, it is invariant under the following transformation on T^2:

$$\tau \to \frac{a\tau + b}{c\tau + d} \tag{4.5}$$

$$R \to R|c\tau + d|$$

$$A \to Ae^{i\varphi_\tau}$$

$$\begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix} \to \begin{pmatrix} a & -b \\ -c & d \end{pmatrix} \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix}$$

where $c\tau + d = |c\tau + d|e^{-i\varphi_\tau}$ and $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{Sp}(2,Z)$. this invariance was found in [21]. The hamiltonian (4.1) as well as (3.9), (3.6) and the $Area(T^2)$ are invariant under the above transformation.

Notice that the $\mathrm{SL}(2,Z)$ (4.5) acts from the left on $\begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix}$ while the $\mathrm{SL}(2,Z)$ invariance on the basis Σ, discussed in previous sections, acts on the right.

Under these transformations the $det\begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix}$ remains invariant. Given $\begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix}$ with determinant $\neq 0$ there always exist elements of $\mathrm{SL}(2,Z)$ whose action from the left and from the right yields

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} l_1 & l_2 \\ m_1 & m_2 \end{pmatrix} \begin{pmatrix} S_1^1 & S_2^1 \\ S_1^2 & S_2^2 \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \tag{4.6}$$

where $\lambda_1\lambda_2 = n$. Moreover, if λ_1 and λ_2 are relative primes there always exist elements belonging to $\mathrm{SL}(2,Z)$ whose action from the left and the right yield $\lambda_1 = n$ and $\lambda_2 = 1$. If λ_1 and λ_2 are not relative primes one may redefine the parameter R and reduce to the case where λ_1 and λ_2 are relative primes. We thus obtain a canonical expression for the hamiltonian (4.1) subject to (3.2), (3.3), and (3.6), (3.9), in terms of sections of the symplectic torus bundle with a monodromy ρ:

$$H = \int_\Sigma \mathcal{H} = \int_\Sigma T^{-2/3}\sqrt{W}\left[\frac{1}{2}\left(\frac{P_m}{\sqrt{W}}\right)^2 + \frac{1}{2}\left(\frac{P_r}{\sqrt{W}}\right)^2 + \frac{T^2}{2}\{X, X^m\}\{\overline{X}, X^m\}\right] \tag{4.7}$$

$$+ \left[\frac{T^2}{8}\{X, \overline{X}\}\{\overline{X}, X\} + \frac{T^2}{4}\{X^m, X^n\}^2\right] + \text{fermionic terms}$$

where $dX = 2\pi R(d\widehat{X}^1 + n\tau d\widehat{X}^2)$. Although we may have winding numbers l_1, l_2, m_1, m_2 the symmetries of the theory allow to reduce everything to the central charge integer n.

The final point is to determine which representations $\rho : \pi_1(\Sigma) \to \pi_0(G) \equiv \mathrm{SL}(2,Z)$ leave invariant the form of the hamiltonian density in (4.7). The representations ρ_n :

$\pi_1(\Sigma) \to \mathrm{SL}(2,Z)_n$, where $\mathrm{SL}(2,Z)_n$ is the subgroup of $\mathrm{SL}(2,Z)$ whose elements are of the form

$$\begin{pmatrix} a & nb \\ c & d \end{pmatrix} \tag{4.8}$$

leave invariant the hamiltonian density in (4.7). ρ_n characterizes the representations compatible with the topological restriction (3.9) For example, if we take the representation $\rho : \pi_1(\Sigma) \to \mathrm{SL}(2,Z)_n$ defined in the following way:

$$\pi_1(\Sigma) \ni \begin{pmatrix} M \\ N \end{pmatrix} \to \begin{pmatrix} 1 & nM \\ 0 & 1 \end{pmatrix} \tag{4.9}$$

The element of $H_1(T^2)$ may be given by $\begin{pmatrix} p \\ q \end{pmatrix}$ being p,q integers. Then the natural action of $\mathrm{SL}(2,Z)$ on it is given by

$$\begin{pmatrix} 1 & nM \\ 0 & 1 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} p + nMq \\ q \end{pmatrix} \tag{4.10}$$

The cohomology group $H^2(\Sigma, Z_\rho^2) \cong \mathbb{Z}$, also the central charge condition (3.9) states that we are in the characteristic class $C(\xi) = n \neq 0$, consequently, there exists a $D = 11$ supermembrane with nontrivial central charges formulated in terms of sections of a symplectic torus bundle ξ with representation (4.9) inducing a $Z[\pi_1(\Sigma)]$-module.

5 Conclusion

We showed that the Supermembrane with central charges may be formulated in terms of sections of symplectic torus bundles with a representation $\rho : \pi_1(\Sigma) \to \mathrm{SL}(2,Z)$ inducing a $Z[\pi_1(\Sigma)]$-module in terms of the $H_1(T^2)$ homology group of the fiber. The representation ρ may be interpreted as a monodromy on the bundle. The non trivial point in the construction was to prove that the hamiltonian together with the constrains are invariant under the action of $\mathrm{SL}(2,Z)$ on the homology group $H_1(T^2)$ of the fibre 2-torus T^2. An interesting aspect of this geometrical structure is the possible existence of an extension of the symplectic 2-form on the fiber to the full space of the symplectic torus bundle. A theorem of Khan [33] establishes that the extension exists if and only if the characteristic class is a torsion class in $H^2(\Sigma, Z_\rho^2)$. In the case of the example of section 4, we conclude that there is not such extension since $C(\xi) = n$ is not a torsion class. The only one is $C(\xi) = 0$ which is not compatible with the topological restriction (3.9) of the supermembrane with central charges. Locally we have the usual interpretation of the supermembrane in terms of maps from Σ to the target. Globally we have now a more interesting geometrical structure since the hamiltonian is defined on a non-trivial symplectic torus bundle. Locally the target is a product of $M_9 \times T^2$ but globally we cannot split the target from the base Σ since T^2 is the fiber of the non trivial symplectic torus bundle $T^2 \to \Sigma$. The formulation of the supermembrane in terms of sections of the symplectic torus bundle with a monodromy is

a nice geometrical structure to analyze global aspects of gauging procedures on effective theories arising from M-theory. We noticed the particular case in which the representation ρ is given by the matrix

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}^{M+N} \tag{5.1}$$

the subgroup reduces to $Z_2 \times Z_2$ and this case was considered in [19, 20] and [40]–[44].

Acknowledgments

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