Faculty of Computing,
Engineering and Sciences

Effect of different controller for active suspension using MATLAB/Simulink

ENGG71020 MSC PROJECT

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1. ABSTRACT

Active and semi-active suspension systems are very frequently used in the aerospace or civil industry, but they acquire even a greater importance if possible, in the automotive industry. An appropriate control design allows proper control of the vehicle's vibrations, and this is translated into greater driving comfort and superior manoeuvrability. For a controller to function effectively, a model that adequately reproduces the dynamics and kinematics of the suspension must be developed.

The purpose of this project is to study and compare various control strategies for active suspensions in commercial cars. In order to achieve this goal, first, the dynamic models that have traditionally been used to assess manoeuvrability and driving comfort are studied, from the quarter model to the full car model. Once the linear dynamic model is developed, complexities such as nonlinearities, or saturations, are added. In the next step, the results obtained for different road types are compared. Finally, using the most appropriate model, different types of control are implemented, and the results compared. It is expected that once the study is completed, a clear conclusion will be obtained on the type of model and controller for active suspensions.

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3. INTRODUCTION

One of the most important design factors for commercial vehicle nowadays is the drive comfort and the driving manoeuvrability. Among all the elements of a car, it could be said that there is great consensus that the most significant ones are the suspension system and the tires. While the tires are the point of contact of the vehicle with the road, the suspensions isolate the chassis from the irregularities of the terrain, thus reducing the force transmitted to the driver. In addition, suspensions regulate the vertical movement of the wheel and ensure contact between the tire and the ground to maintain steering manoeuvrability and, avoiding any damage to the vehicle.

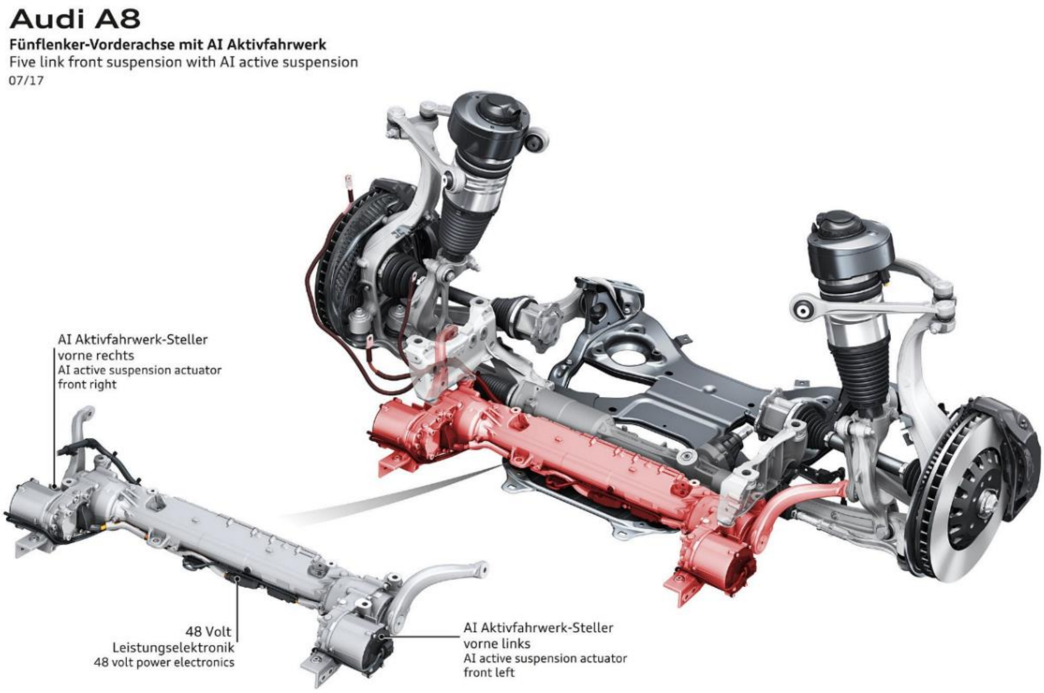
The design of a traditional passive suspension requires a commitment to meet both driving comfort and manoeuvrability. For example, a design aimed at driving comfort requires a smooth suspension, while good manoeuvrability is provided by a suspension with high rigidity. This translates in a problem for the versatility of a vehicle, since a passive suspension explicitly designed for one type of roadway will presents a severe decrease in its performance when it is operating on another type of roadway. In the same way, a suspension designed to maximize comfort on a road will not get good results when rolling on a bumpy road.

In order to face the design commitment required by passive suspensions are semi-active and active suspensions. Semi-active suspensions are those in which the damping coefficient is variable. In the case of active suspensions, it is a typical passive suspension construction that incorporates an actuator, generally of a hydraulic type, between the wheel and chassis. Said actuator generates thrust and attraction forces that help the suspension compensate for differences between actual and desired values. Both constructions are more complex than classic passive suspensions, both because of their construction and Because they need a feedback loop with sensors.

Traditionally, to study the behaviour of the vehicle for a given suspension and tires, the linear model of a quarter car has been used, since it allows a good representation of the dynamics of the vehicle for the design of the control system. In order to obtain better results, quarter car models can incorporate non-linearities corresponding to mechanical, hydraulic or control elements among others.

At present days, the modelling and control of semi-active and active suspension systems are subjects of intense research activity. This interest is expressed in the recent publication of reviews on specific aspects of these systems: the models and structures of vehicle suspensions integration between vehicle control subsystems suspension control systems strategies, adaptive suspension control systems (1), semi-active control strategies (2) semi-active control strategies and other non-vehicular applications (3).

With the recent increase in available computing power and the rise of the application of artificial intelligence and machine learning, the design of intelligent control systems for active suspensions is taking on a new dimension and some of the leading commercial car manufacturers are starting to apply this technology. This is the case, for example, of Audi, which incorporates active suspension systems in the AUDI A8 (4)



In this work, a review of the different dynamic models that can be used to evaluate the performance of an active suspension system will be carried out. The memory of this work will be structured sequentially as follows.

- The first chapter will review the different dynamic models available in literature and the different control strategies available to evaluate the behaviour of an active suspension.
- In the second chapter, quarter car, half car and full car model will be developed and compared using MATLAB Simulink.
- In the third chapter a multibody quarter car model of a McPherson suspension will be developed and evaluated.
- In the fourth chapter, using the lineal quarter car model and the McPherson quarter car model, two different control strategies will be implemented (PID controller and Neural

Network Controller) and its efficiency in terms of driving comfort for various types of roadway will be evaluated and compared.

- In the fifth chapter the conclusions of this work are obtained, and future developments are proposed.

4. SUSPENSION SYSTEMS AND CONTROL METHODOLOGIES (5)

Suspension systems of commercial vehicles can be classified according to the type of control applied on its parameters: the passive suspension has predetermined parameters and is not adjustable, the active suspension uses an actuator that inputs energy to modify the response to the system for different road disturbances and the semi-active suspension system regulates the stiffness and / or the damping coefficient without directly introduce energy to the system. A scheme corresponding to the classification of suspension systems is shown in the figure below.

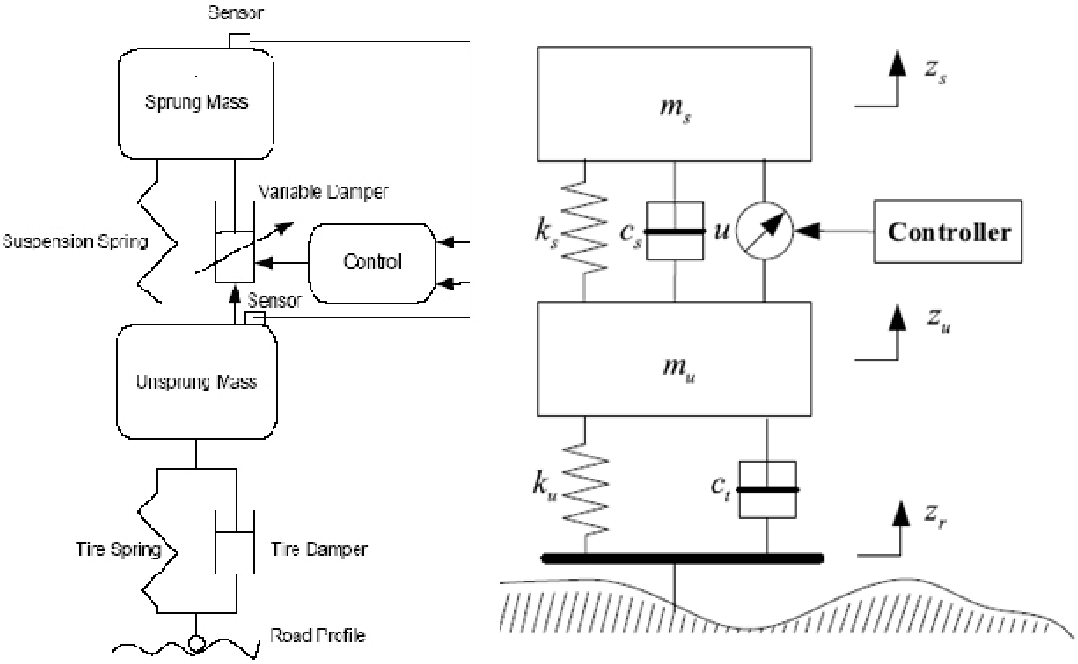


Figure 1. a) Semi-active suspension system, b) Active suspension system (6)

The criteria that is normally used to determine the performance of a suspension system is the acceleration or vertical displacement of the suspended mass, that is, the centre of gravity of the vehicle chassis, the tire deflection to analyse manoeuvrability and deflection of the suspension for the travel design requirements of the suspension.(7)

In certain applications, especially involving cars with commercial objectives, the degree of sensitivity of the human body to certain frequencies must be considered. In this way, an ideal suspension system should for example minimize the frequency response in the range of 0.2 to 10 Hz, to avoid dizziness and other discomfort to the passengers of the vehicle. (8)

In a suspension system two resonant frequencies are observed: one around 1 Hz associated with the vibration mode of the suspended masses and another around 10 Hz related to the vibration mode of the non-suspended mass. The active and semi-active suspensions allow to reduce the resonance peak and the amplitude of the movement of the suspended mass in almost the entire frequency range.

4.1. Passive Suspension Systems

Passive suspension systems are characterized by not directly receiving any external energy application. They store energy through springs and dissipate them by means of shock absorbers. The parameters of a passive suspension are fixed and correspond to a compromise design between the characteristics of the terrain, the load supported and the driving comfort. In general, in a passive suspension system there are problems of comfort or manoeuvrability when the stiffness or the damping coefficient is modified. This type of suspension has been the most used in commercial vehicles for years.(9)

4.2. Active suspension systems

An active suspension stores, dissipates and introduces energy to the system through actuators, usually hydraulic, pneumatic or electromagnetic. Its drive is regulated through sensors and controllers. The sensors are used to record the behaviour of the vehicle against disturbances and thus define the response according to the control objective. An active suspension system usually uses accelerometers at the corners of the chassis and on the wheels, whose integration provides the respective speeds, as well as linear transducers located between the wheel and the chassis to measure relative displacement. The use of force sensors for the control system has also been proposed. The active suspension incorporates comfort and manoeuvrability restrictions. In addition, it considers the irregularities of the terrain in the design of the model and allows controlling the transmitted forces. The development in microelectronics of the last decades has facilitated its application and is generally included in high-end vehicles.(10)(11)

4.3. Semiactive suspension systems

The semi-active suspension is characterized by having dampers whose damping coefficient is modified by an external control. Generally, these suspensions control the low frequencies with active elements and the high frequencies with passive elements. Different technologies for semi-visual shock absorbers are research objects, being the most representative, magnetorheological shock absorbers, whose response varies with the applied magnetic field, the pneumatic shock absorbers, generally using buses, trucks, dry friction dampers, highly non-linear and based in the frying between contact surfaces. (12)

Respecting to the variables for the control of the semi active suspension system, the position relationship, the drop-in pressure, the relative speed of the suspension, and the product between the relative speed of the suspension and the speed can be considered absolute mass suspended.

4.4. Modelling of the Dynamic of the vehicle

Currently, vehicle dynamics are studied through experiments with virtual prototypes, while real prototypes are used in the development phase to obtain simulation parameters, validate intermediate results and optimize. The simulation allows the researcher to analyse driving quality, road performance and comfort. Simulation also generates real-time animations of the dynamic response of the system. For the analysis of the suspension system and its interaction with the vehicle, general graphic techniques are used for modelling physical dynamic systems, which establish union relationships between its elements. The most representative are:

- Ligature graphics: this technique relates the components of the system through its energy exchange, which is useful for the complete modelling of a vehicle. This technique can be implemented in programming environments such as Dymola or Simulink. Bonding graphs can reveal the differences between the actuator power and its energy requirement in an active suspension, occupy different types of suspension in a quarter car model, or design the suspension structure with evolutionary algorithms. This is the methodology that is used in this work, using MATLAB / Simulink software.(13)
- Multi-body models: multi-body models relate their elements through kinematic pairs. Specific programming environments such as Adams, Nastran or Simscape can be used for this purpose. These models allow co-simulation with control strategies in other

environments such as MATLAB / Simulink. Thus, it can simulate the movement of the vehicle for the design of the control or evaluate the comfort of the suspension. This kind of implementation that will be followed to develop a multibody model of a McPherson suspension.

The number of degrees of freedom of the suspension system model depends on the type of analysis to be performed. Thus, the linear model of a quarter car has two degrees of freedom, which restricts it to the study of vertical dynamics. To consider pitching, the half-vehicle model with four degrees of freedom is used, and the complete model of the vehicle with seven degrees of freedom is required to evaluate the roll angle and incorporate the anti-tip and anti-skid features. In addition, a complete model with eight degrees of freedom can be used to include the vertical dynamics of the driver's seat.

4.5. Control methodologies for active suspension systems

The choice of the control strategy for a suspension system critical. The problem of control in a suspension is multi-objective and defined as improving the driving comfort, increasing the manoeuvrability and reducing the power needed by the actuator at the same time. Similarly, in a real suspension, nonlinearities such as friction, hysteresis, dead zones or saturations must be considered. For the solution of these problems, conventional control and intelligent control methodologies have been applied in the actual literature.

4.5.1. Conventional Control Methodologies

Conventional control methodologies use mathematical models of the system. The most representative are summarized below.

4.5.1.1 Proportional, Integrative and Derivative Control (PID)

The PID Control is a closed loop control system that uses the given error signal (that is, the difference between the desired value and the actual value obtained at the output of the system). It has the advantage that it gives a rapid response and compensation of the error signal against disturbances.

The adjustment of the three parameters of the PID control is critical to avoid system oscillations and can be carried out by theoretical approximations, by optimization considering the gains as design factors and by heuristics using diffuse control for an online self-regulation. The use of dual the adaptive control to be designed without requiring the dynamic model of the system. The output scale factor for a fuzzy control can also be self-regulated according to the trend of the process.

- *With reference model*, which allow to reduce the disturbance and the vibration to ideal levels. The model can be expressed in the form of a neural network. However, when the suspension system is composed of hydraulic actuators, its behaviour is not line and variable over time, being complicated the construction of a model-based controller.

4.5.2. Intelligent Control Methodologies

Intelligent control methodologies arise to solve the problem of dealing with complex and multivariable systems. This is the case of the suspension of a real vehicle, which is highly non-linear, with uncertainties and inaccuracies. The main artificial intelligence techniques applied to the modelling and control of these systems are fuzzy logic, neural networks and evolutionary algorithms.

4.5.2.5 Fuzzy Logic Control

The fuzzy logic controller (FLC) uses linguistic variables and the fuzzy set of ideas to form a control algorithm capable of emulating human logic, without the need for an exact description of the system or explicit programming. This technique allows an adaptive control by means of least squares of the actuator force, to carry out non-linear variants of the PID algorithm, to obtain the fuzzy rules using as input variables linear combinations of speeds and the displacement of the vehicle chassis, to determine the FLC by Information on the future road surface, generate the damping coefficient with a small number of fuzzy rules and improve the stability of the system through a hierarchy of fuzzy rules.(17)

4.5.2.6 Neural Network Control

The use of neural networks makes possible to use economic sensors in the design of the control system. From neuronal models, supervised and unsupervised training algorithms can be obtained. Its functionality is demonstrated by the application to semi-active suspension systems without requiring complete knowledge of the dynamics of the system.(15)

A neural network can be trained to control active suspension systems and manages to emulate an existing controller, perform adaptive control by combining a network for the regulator and another for the reference model, controlling a non-modelled system, through a performance function properly chosen and of an error signal, and adjust the learning parameters of the network with a correction and prediction method.

feedback PID controllers has also been proposed in the actual literature: an internal one for force control and an external one for the control of the suspension stroke.

4.5.1.2 Optimal Control

Optimal control is a methodology that seeks to optimize a controller with at least one cost function, which works as a performance index that satisfies the restrictions given by the dynamic system model. It's recommended when the behaviour of the system has uncertainties. It's classified into different types:

- *Quadratic linear controller (LQR)*(14), when a linear system uses a quadratic function in optimization. In the case of the suspension system, the uncertainty is inherent in the roughness of the road and the restrictions are characterized in the performance index using the quadratic means of acceleration of the suspended mass, deflection of the suspension and deflection of the tire.
- *Gaussian quadratic linear (LQG)*, when the linear quadratic controller is included with a Kalman filter that statistically characterizes noise and is a good state estimator. Different rates of performance and optimization have been proposed.
- *Predictive*,(15) applied to dynamics of special complexity, multivariable and / or unstable. It uses an optimized model of the system to predict its behaviour and the future control signal to be used. A model based on neural networks can also be used to predict the vertical acceleration of the vehicle.

4.5.1.3 Robust Control

Robust control approximates the dynamic model of the vehicle to one of constant coefficients. This type of control assumes that the model will present a modelling error, which is used in the controller design. Thus, robust control H_∞ allows incorporating human sensitivity to vibration in different frequency ranges. The robust H_2 controller ensures good noise rejection through random algorithms.(16)

4.5.1.4 Adaptative Control

Adaptative control allows to automatically adjust its characteristics to operate optimally in a changing environment, reducing vehicle disturbances and vibration to specified levels. The most representative are:

- *Self-adjusting*, when they emulate the dynamic behaviour optimizing comfort for the given state, considering the dynamic load and the deflection in the suspension. A fuzzy scheme with online learning ability to compensate for the error of functional approximation allows

4.5.2.7 Control based on evolutionary algorithms

Control based evolutionary algorithms are applied in solving multi-objective optimization problems for a wide range of applications. Generally, they are stochastic search processes and reduce the space of solutions efficiently. However, they are not recommended for online applications and critical systems. Most used evolutionary algorithms are:

- *Genetic algorithms* are based on natural evolution and genetics. Genetic algorithms allow to improve the performance by choosing the weight factors in a multi-objective strategy and to improve the weight matrices in an LQR quadratic linear regulator. The performance criteria can be optimized by different filters using suspension deflection, tire deflection and accelerations. In addition, the objective function of the maximum stroke restriction can be considered. The integration of genetic algorithms with a representation through ligature graphs has facilitated the synthesis of the control considering terrain and load disturbances. The combination with deterministic methods has been proposed for the optimization of the complete model of the suspension system of a vehicle.(18)
- *The particle swarm optimization (PSO)* is an evolutionary optimization algorithm based on the behaviour of flocks of birds and banks of fish. It has a rapid convergence with less calculation time than genetic algorithms. In combination with an FLC, the acceleration of the suspended mass decreases despite the irregularities of the terrain and in combination with an adaptive strategy allows the restriction of human sensitivity to frequency to be included.

4.5.3. Conclusion

In this chapter the state of art and knowledge in the modelling and control of active suspension systems has been reviewed.

The modelling and simulation of suspension systems is essential for the analysis of vehicle dynamics and the design of its controllers. According to this view, different approaches have been proposed for developing dynamic models of the vehicle, representing the nonlinearities of the dampers or considering the kinematics of the suspension system. However, the model of a quarter car is widely used in the literature.

Regarding control, it is a multi-objective and non-linear problem for which conventional control methodologies based on a mathematical model of the system are applied, as well as intelligent control strategies.

5. LINEAR DYNAMIC MODELS USING MATLAB/SIMULINK

As mentioned before in the introduction of this study, the use of dynamic models to represent the behaviour of the suspension is widely in the current literature for active suspension systems control. Within these models, the most used is the quarter car model, since it allows a good representation of the dynamics of the system (19). However, this model has certain inaccuracies that keep it from being an ideal system. Some of these problems are:

- The vehicle body has only two degrees of freedom, corresponding to the vertical displacement of the vehicle chassis and the wheel.
- The model does not represent nonlinearities corresponding, for example, to the cinematic of the suspension system or the nonlinear behaviour of the spring constant.
- Initially, the model does not represent another constrains such as the maximum deflection of the shock absorber.

Therefore, depending on the objective to be optimized, other models such as the quarter car model or the full-car model are being used, as well as multibody models that have a greater number of degrees of freedom, thus allowing the quantification of new parameters for a more thorough analysis. In addition, they allow a more reliable representation of vehicle dynamics.

The following sections will detail the construction process of the fourth, middle and complete car models using a block diagram, using Simulink software. They will be submitted to several inputs to verify their correct operation; its operation will be checked and the most appropriate will be evaluated.

5.1. Quarter Car Model

The quarter car model has two degrees of freedom, corresponding to the vertical displacement of the wheel and the vertical displacement of the chassis. A schematic diagram of the dynamic model of the quarter car with active suspension system can be seen in Figure 2.

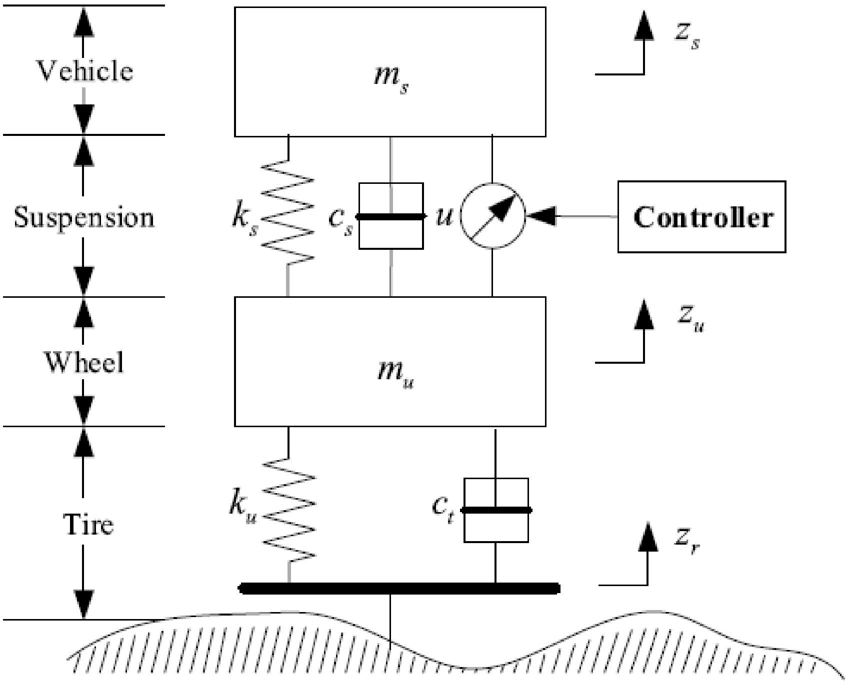


Figure 2. Squematic Diagram of an Active Quarter Car Model (11)

The model is defined by the following parameters:

- M_s= Sprung mass (mass of the vehicle).
- Mu= Unsprung mass (mass of the tire).
- Ks= Vertical stiffness constant of the suspension.
- Cs= Vertical damping constant of the suspension.
- u= Vertical force of the actuator.
- Ku=Vertical stiffness constant of the tire.
- Ct= Vertical damping constant of the tire.
- Zs= Vertical displacement of the Centre of Gravity (CDG) of the chassis of the vehicle.
- Zu= Vertical displacement of the CDG of the suspension.
- Zr= Vertical displacement of the point of contact between tire and road.

The dynamics equations corresponding to this model can be seen below:

$$\ddot{x}_s M_s = F_a + b_s(\dot{x}_u - \dot{x}_s) + k_s(x_u - x_s) + P_s$$

$$-x_u \ddot{M}_u = F_a + b_s(\dot{x}_u - \dot{x}_s) + k_s(x_u - x_s) + b_t(\dot{x}_u - \dot{x}_r) + P_u + k_t(x_u - x_r)$$

The system is developed using the ligature graphics in MATLAB/Simulink. As it can be seen in Figure 3, the system behaves like a black box with two inputs, the vertical profile of the road and the force exerted by the actuator (in the case of the active suspension), and one output, the vertical displacement of the CDG of the chassis of the vehicle.

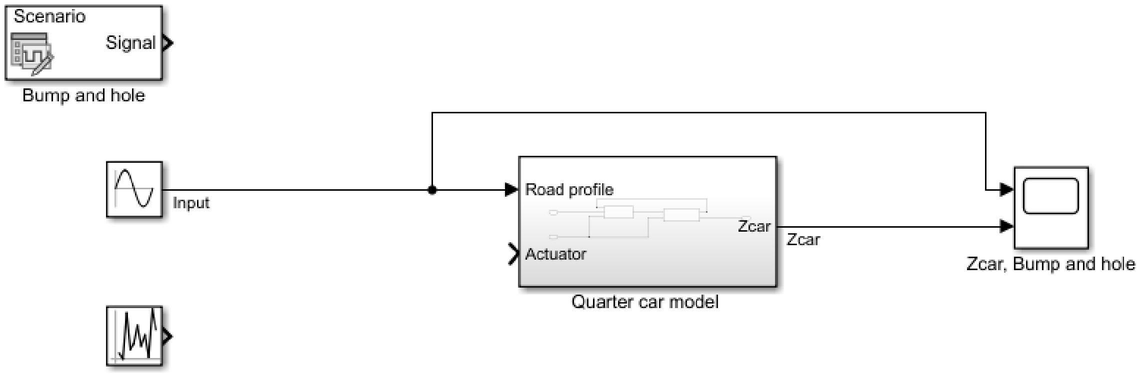


Figure 3. Block Diagram of Quarter Car Model

Visualizing the subsystem in detail, that there are two subsystems inside (Figure 4), corresponding to the equations of the dynamics of tire and suspension respectively.

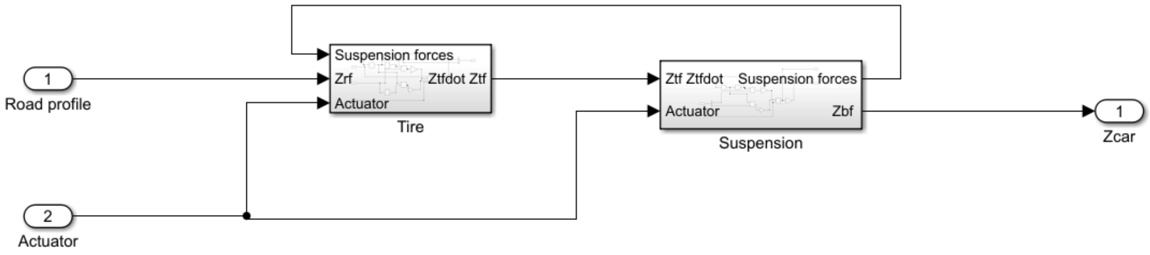


Figure 4. Subsystem Block of Quarter Car Model

Visualizing the subsystem in detail, that there are two subsystems inside (figure 1), corresponding to the equations of the dynamics of tire and suspension respectively. These subsystems can be seen in detail in Figure 5 and Figure 6. These subsystems will be reused in further developments of half-car and full-car model.

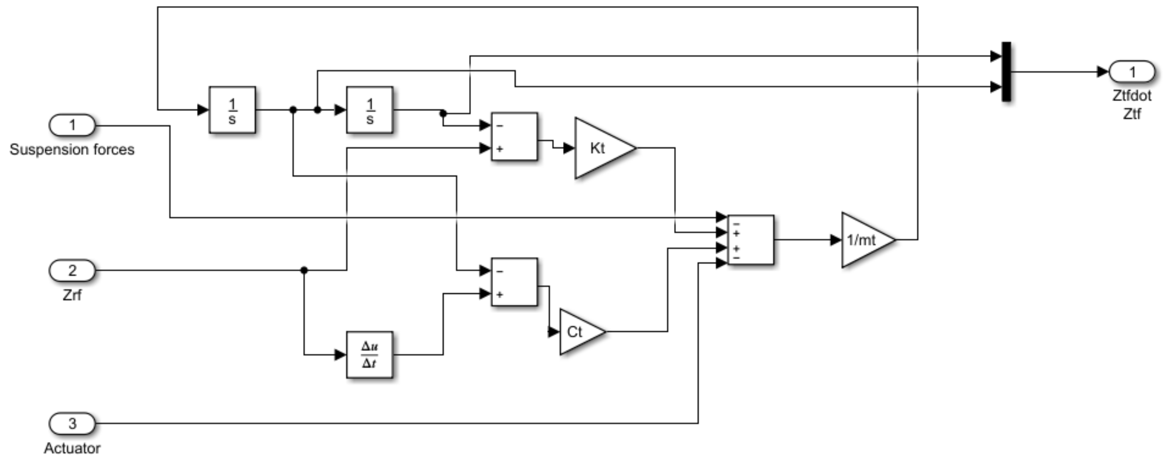


Figure 5. Subsystem Diagram of Tire Model

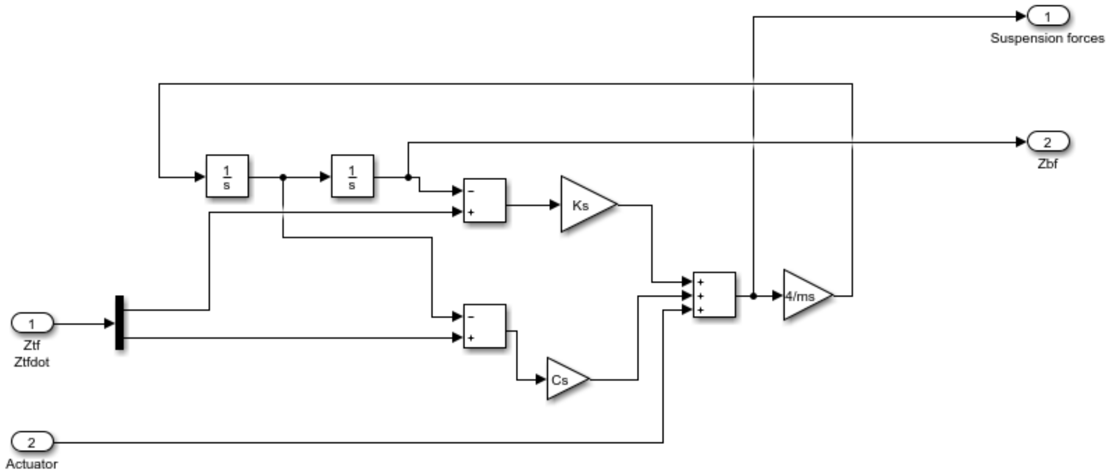


Figure 6. Subsystem Diagram of Suspension Model

5.2. Half-car Model

The half-car model has a total of 4 degrees of freedom, corresponding to the vertical displacement of both wheels, the vertical displacement of the chassis' CDG and the pitch angle. The schematic diagram of an active half-car model is the one shown in Figure 7:

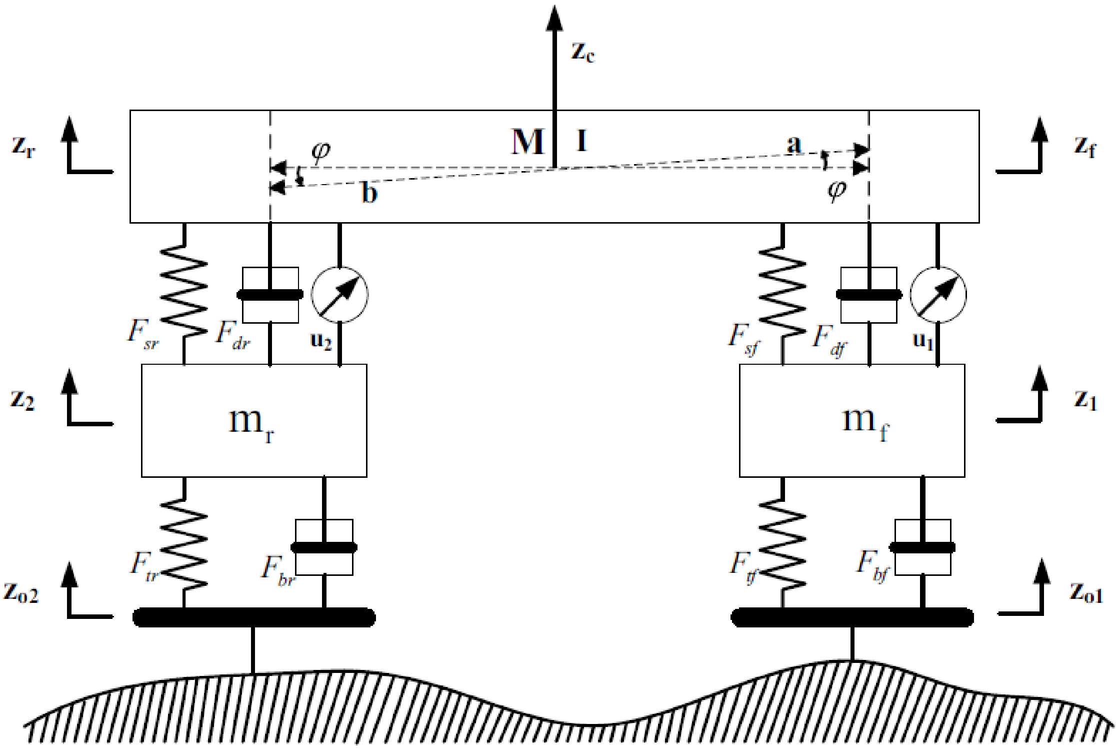


Figure 7. Squematic Diagram of an Active Half Car Model

The model is defined by the following parameters:

- M_S= Sprung mass (mass of the vehicle).
- I=Moment of inertia for pitch angle.
- $alfa$= Pitch angle.
- Mr, Mf= Unsprung mass (mass of front and rear tires).
- Fsr, Fdr, Fsf, Fdf= Vertical forces produced by front and rear tire constants.
- $u1, u2$= Forces exerted by the actuators.
- Fsr, Fdr, Fsf, Fdf= Vertical forces produced by front and rear suspension constants.
- $Z1, Zc, Z2$= Vertical displacement of the contact points between suspension and chassis and vertical displacement of the CDG of the vehicle.
- $Zo1, Zo2$= Vertical displacement of the front and rear tire.
- Zr, Zc, Zf= Vertical displacement of the front and rear contact point between road surface and tire.
- a, b= Distance from front and rear Wheel to the CDG of the vehicle.

Dynamic equations corresponding to the half car model are the following:

$$\ddot{x}_{uf} = \frac{F_{af} + b_{sf}\left(\dot{x}_{uf} - \dot{x}_{sf}\right) + k_{sf}\left(x_{uf} - x_{sf}\right) + b_{tf}\left(\dot{x}_{uf} - \dot{x}_{rf}\right) + P_{uf} + k_{tf}\left(x_{uf} - x_{rf}\right)}{-M_{uf}}$$

$$\ddot{x}_{ur} = \frac{F_{ar} + b_{sr}(\dot{x}_{ur} - \dot{x}_{sr}) + k_{sr}(x_{ur} - x_{sr}) + b_{tr}(\dot{x}_{ur} - \dot{x}_{rr}) + P_{ur} + k_{tr}(x_{ur} - x_{rr})}{-M_{ur}}$$

$$\ddot{x}_s = \frac{F_{ar} + F_t + k_{sr}(x_{ur} - x_{sr}) + b_{tr}(\dot{x}_{ur} - \dot{x}_{rr}) + P_{ur} + k_{tr}(x_{ur} - x_{rr})}{-M_s}$$

The model of the half-car developed in Simulink can be seen in Figure 8.

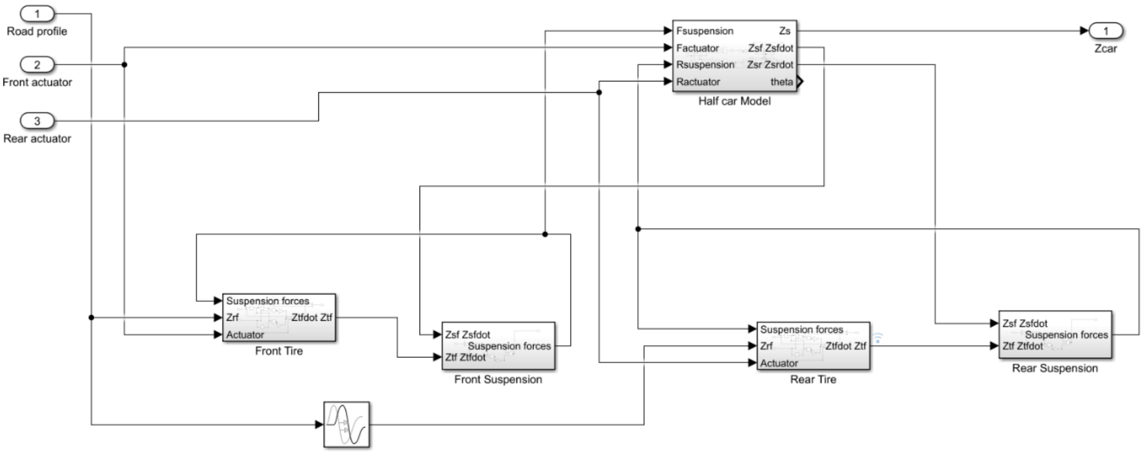


Figure 8. Half-car Model Subsystem

This model reuses the previously implemented blocks corresponding to tire and suspension. It also incorporates a delay block that represent the delay the rear tire meets due to the vehicle length. The diagram includes an additional block that models the dynamics equations corresponding to the two degrees of vehicle freedom (vertical displacement and pitch angle). This subsystem can be seen in detail in Figure 9.

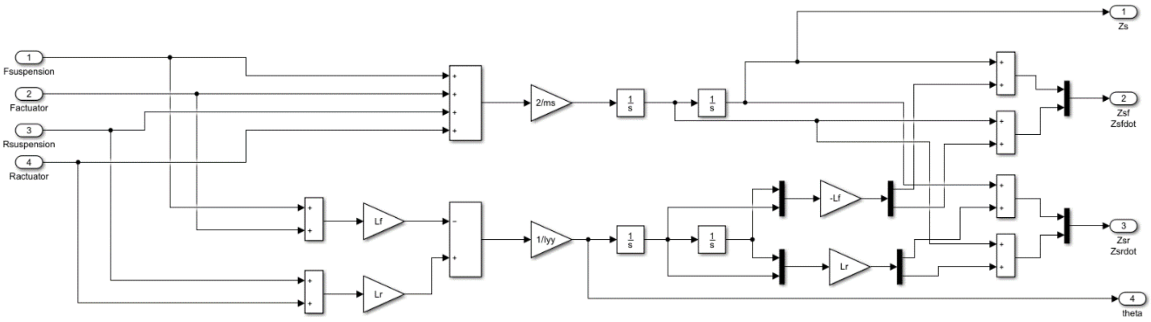


Figure 9. Subsystem of the dynamics equations of the half-car model

5.3. Full-car Model

Full-car model has a total of 7 degrees of freedom, corresponding to the vertical displacements of the four wheels, the vertical displacement of the vehicle chassis, the pitch angle and the roll angle. The schematic model of the complete car can be seen in the Figure 10.

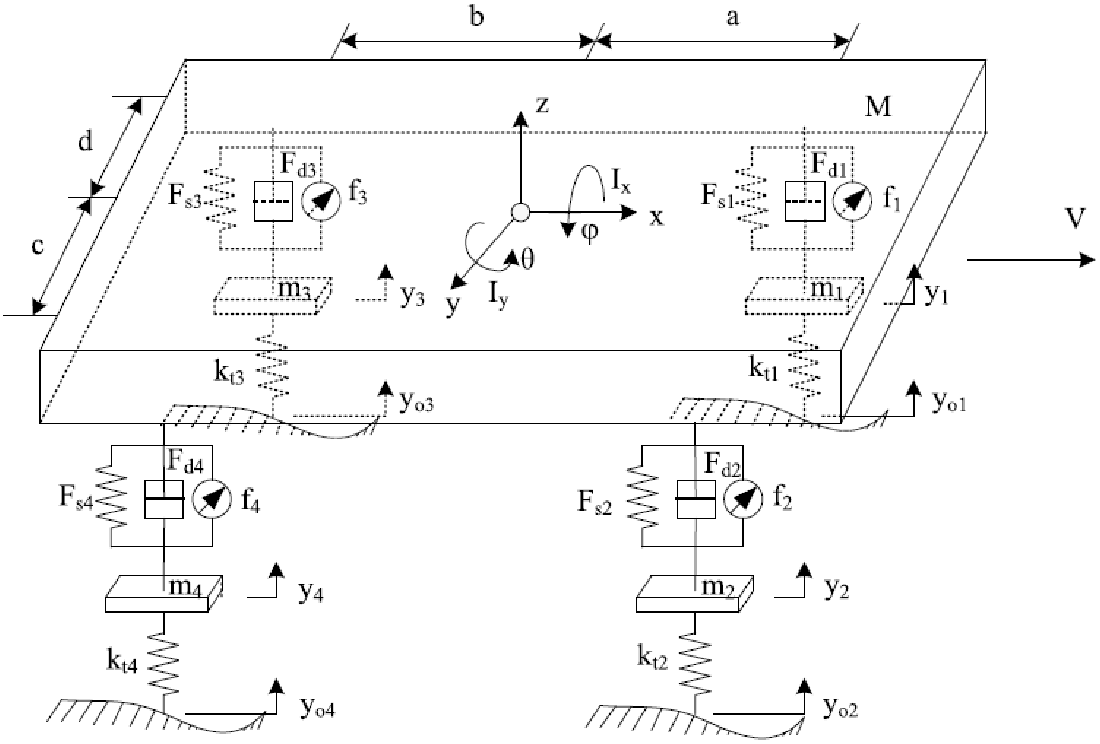


Figure 10. Squematic Diagram of an Active Full-Car Model(20)

The dynamics equations, on an assumption that pitch and roll angles are small, corresponding to the complete car model can be seen below.

$$\Sigma_I \begin{cases} \ddot{z} = -\frac{1}{M}\sum_{i=1}^{4}(F_{di}+F_{si}) + \frac{1}{M}\sum_{i=1}^{4}u_i(t), \\ \ddot{\theta} = -\frac{1}{I_y}(a\sum_{i=1}^{2}(F_{di}+F_{si}-u_i(t)) - b\sum_{i=3}^{4}(F_{di}+F_{si}-u_i(t))), \\ \ddot{\varphi} = -\frac{1}{I_x}(d\sum_{i=2,4}(F_{di}+F_{si}-u_i(t)) - c\sum_{i=1,3}(F_{di}+F_{si}-u_i(t))), \\ \ddot{y}_i = \frac{1}{m_i}\{F_{di}+F_{si}-k_{ti}(y_i-y_{oi})-u_i(t)\}, \end{cases}$$

The subsystem block for the complete car model can be seen in Figure 11.

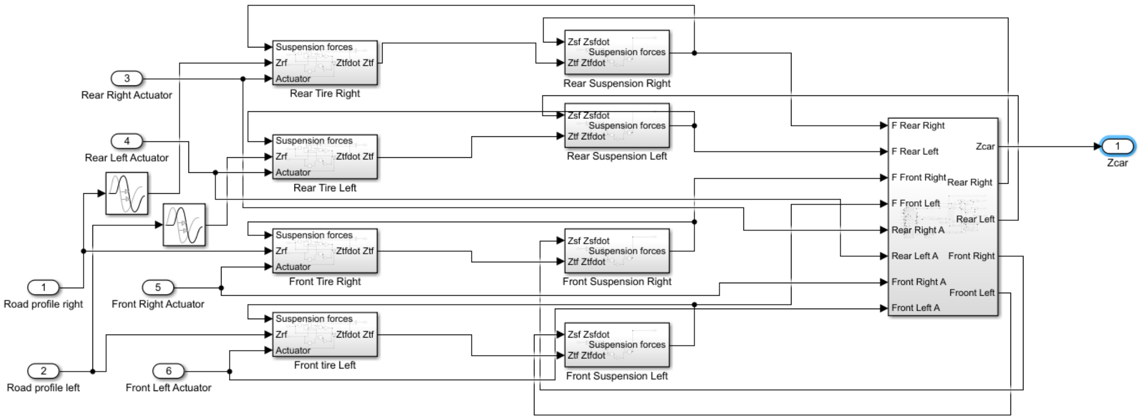


Figure 11. Full-car model subsystem

Full-car model incorporates eight blocks corresponding to the dynamics of springs and dampers. It incorporates two different input excitations corresponding to the right and left side of the car, followed by delays for the rear axle. Finally, it incorporates a block that contains the dynamics equations corresponding to the degrees of freedom of the full-car model.

The dynamics equations subsystem implemented in Simulink are as it can be seen in Figure 12.

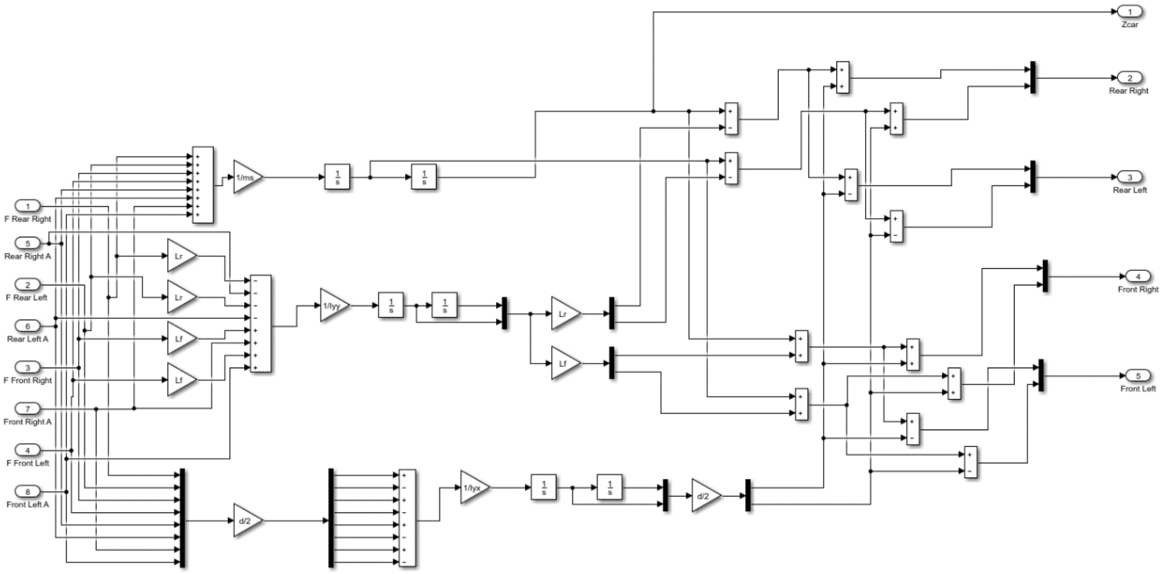


Figure 12. Dynamics equations subsystem for full-car model.

5.4. Road Inputs models

Road Inputs models refer to the signals that are introduced into a model at the point of contact between wheel and road and that represent form of the road. These signals allow to study the effect of various effects caused by the road on the performance of the suspension. The most used are the shock models. These models reproduce events of great intensity and short duration, usually produced by potholes, holes, or roughness on the road. The three most used excitation models are the following models:(19)

- Bump and hole: Represent the effect of a bump followed by a hole on the road.
- Sinusoidal: Represent the effect of a road with soft and cyclical irregularities.
- Random: Represent the effect of a road full of rough and no cyclical irregularities.

These disturbances created in Simulink to be introduced in dynamic models. In all cases, the maximum range of values assigned to these disturbances is from 0.1m to -0.1m. The graphs that represent the temporal evolution of the displacement of the contact point of the wheel with the road for each model can be seen in Figure 13, Figure 14 and Figure 15, respectively.

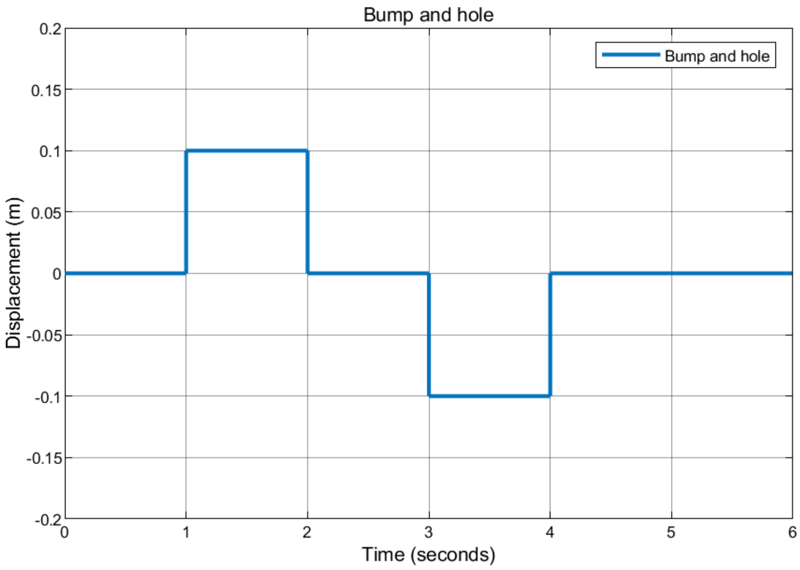


Figure 13. Bump and hole input

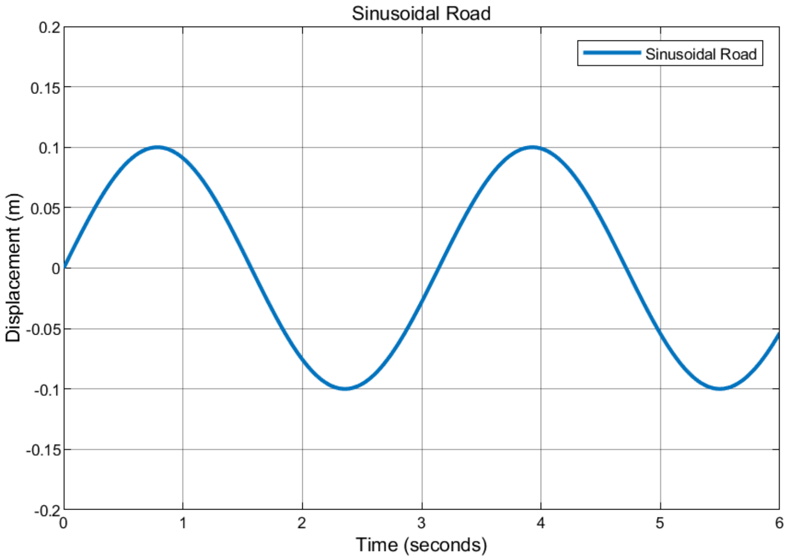


Figure 14. Sinusoidal Input

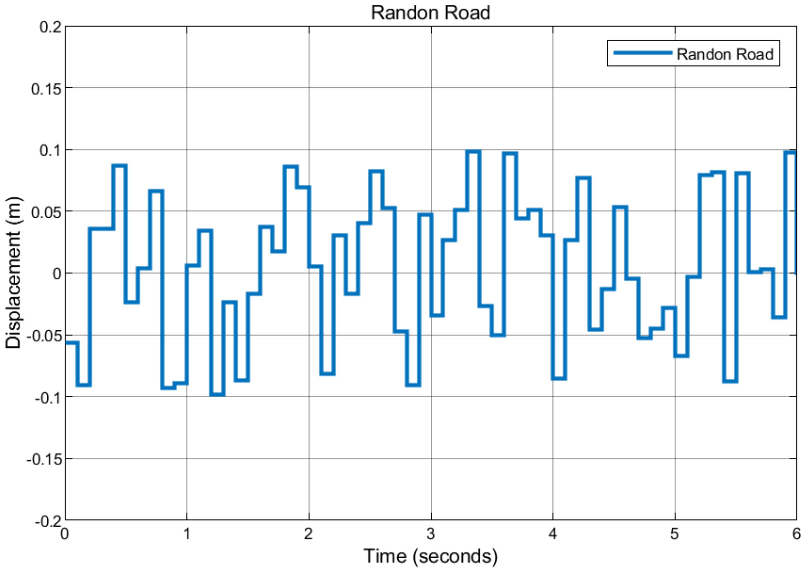


Figure 15. Random road input

As shown in the half-car and full-car developments, to introduce the disturbances on the rear axle it is necessary to introduce a delay that represents the time it takes for the rear wheel to reach the position of the disturbance. In the case of the complete car model, to introduce the disturbance on the left side it is necessary to clone the input disturbance in the case of a flat road (as in the bump and hole and sinusoidal excitations), enter another entry in the case of the random road. The introduction of this second input would aim to mimic the longitudinal section of the road profile, as shown in Figure 16.

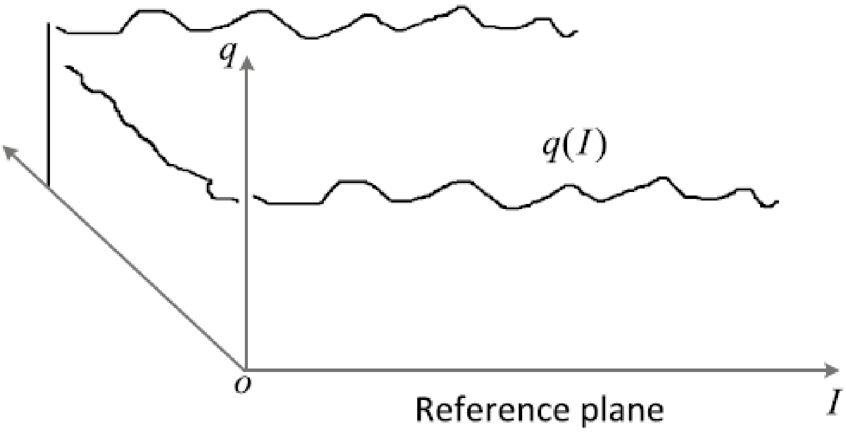


Figure 16. Rugosity reference plane of the road

The introduction of a reference plane with a certain roughness allows the effects of disturbances on the driving quality to be evaluated using the frequency aspect. However, such development is beyond the scope of this project.

5.5. Dynamic models evaluation and comparison

In this section the three dynamic models developed are subjected to the input disturbances specified in the previous section. The objective of this section is to analyse the behaviour of each model, compare the response of each model and determine which one is the most suitable for the present application. To perform this evaluation, similar number values are assigned to the three models. Also, some considerations are taken into account in order to simplify this analysis:

- The same stiffness and damping constants will be used for all suspension systems.
- The mass value is divided between 2 and 4 for the half-car and quarter-car models respectively.

The numerical values used in the models can be seen in Figure 17.

Lf (m)	1.1
Lr (m)	1.1
d (m)	1.5
Ms (kg)	1200
Mt (kg)	50

Iyy (kgm3)	210000
Iyx (kgm3)	18000
Ks (N/m)	18000
Kt (N/m)	180000
Cs (N/ms)	1200
Ct (N/ms)	120
Vx (m/s)	30

Figure 17. Numeric values used in comparison between linear models

In Figure 18, Figure 19 and Figure 20, the response of the three models to the three different types of disturbance can be compared. From these graphs the following conclusions can be drawn:

- The response of three models to Bump and hole and Sinusoidal Input is similar.
- In the half-car and full-car models, the slight influence of the pitch angle on the vertical displacement of the chassis can be observed.
- By introducing two different excitations for each side of the car in the case of the complete car model, it can be seen how this changes dramatically with respect to the quarter car and half car.

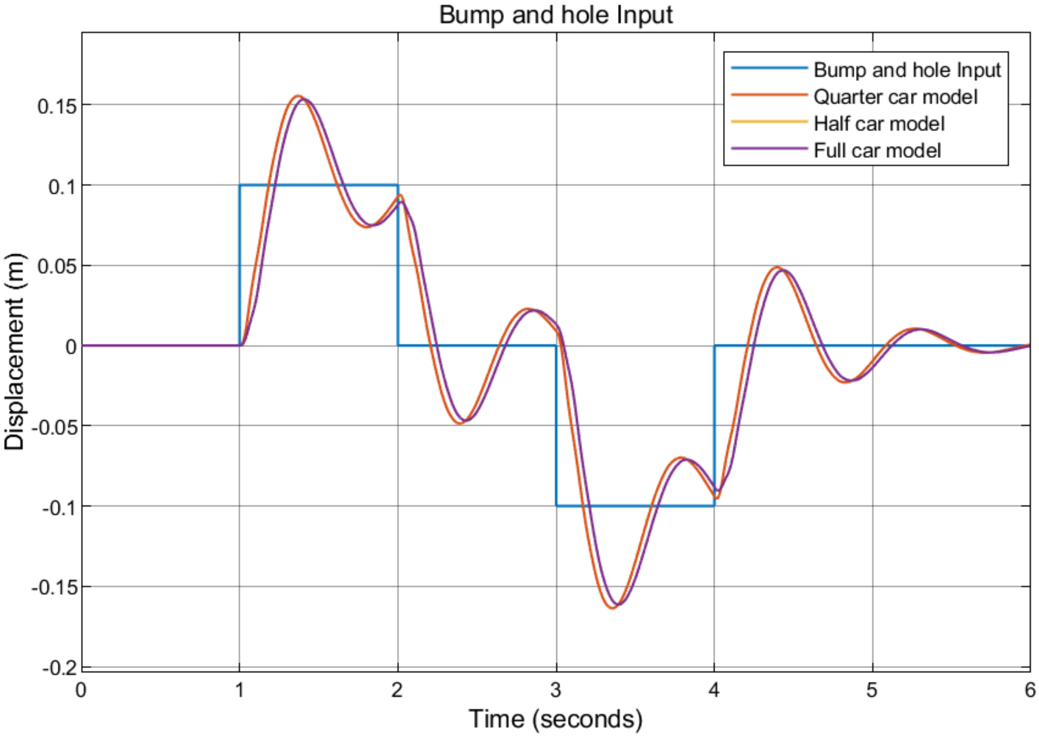


Figure 18. Displacement of the chasis for Bump and Hole Input

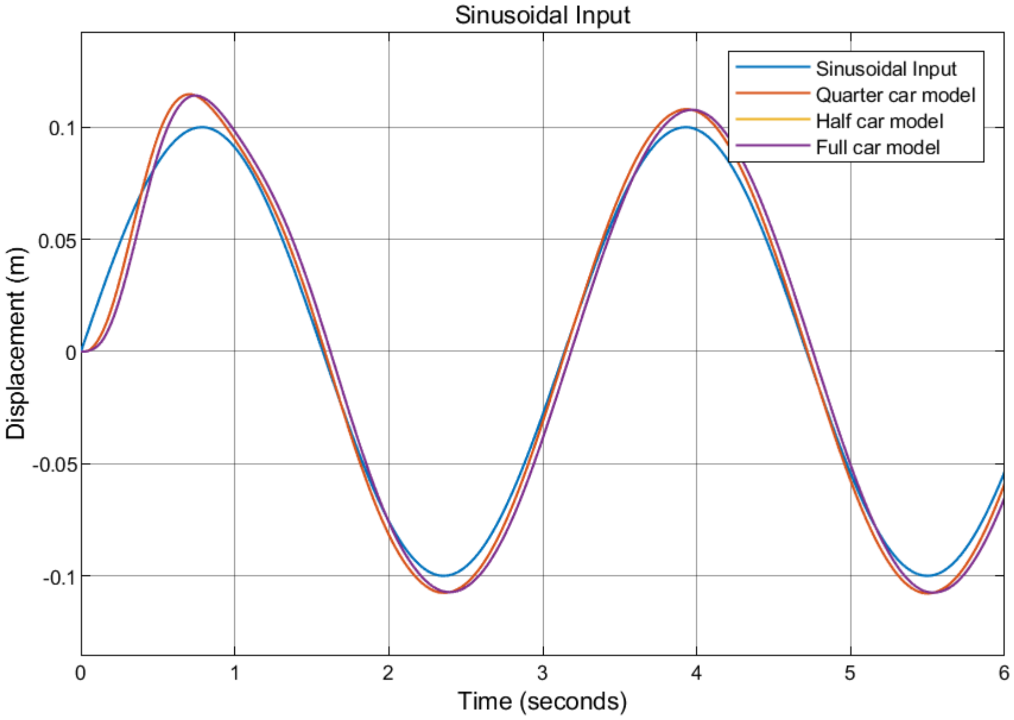


Figure 19. Vertical displacement of the chassis for sinusoidal Input

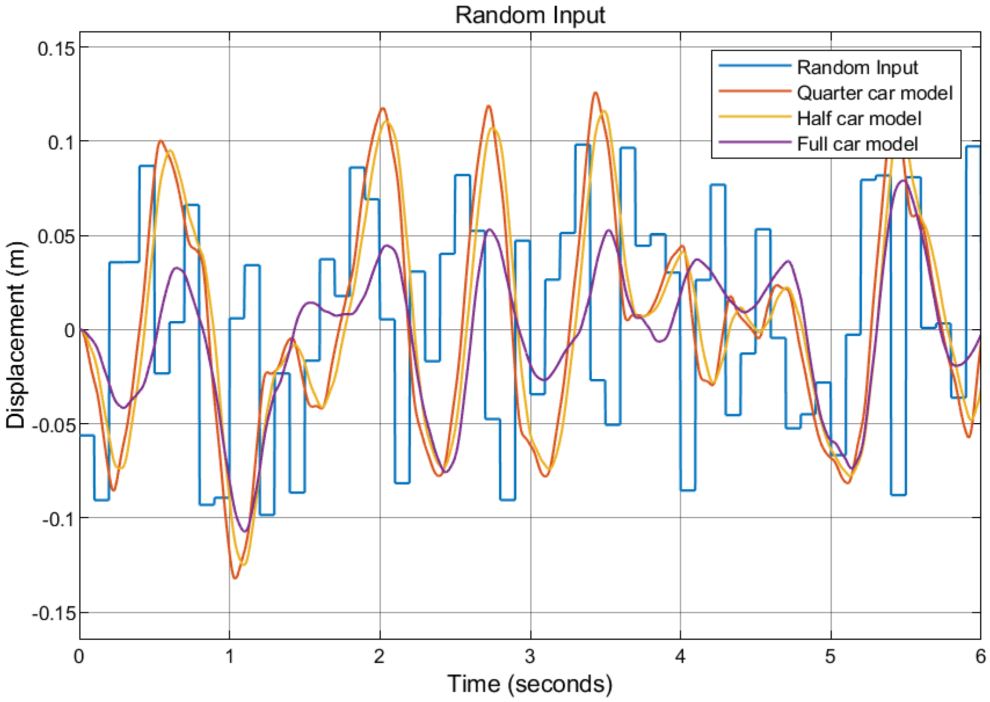


Figure 20. Vertical displacement of chasis for Random Input

An advantage of the implementation of models with more degrees of freedom is that it allows evaluating other important features for ride comfort, such as the pitch angle and roll angle.

Figure 21 shows the evolution of the pitch angle for the bump and hole input in half-car and full-car models. As it can be observed, the response of both systems is similar, although in the case of the full-car model it reaches higher values.

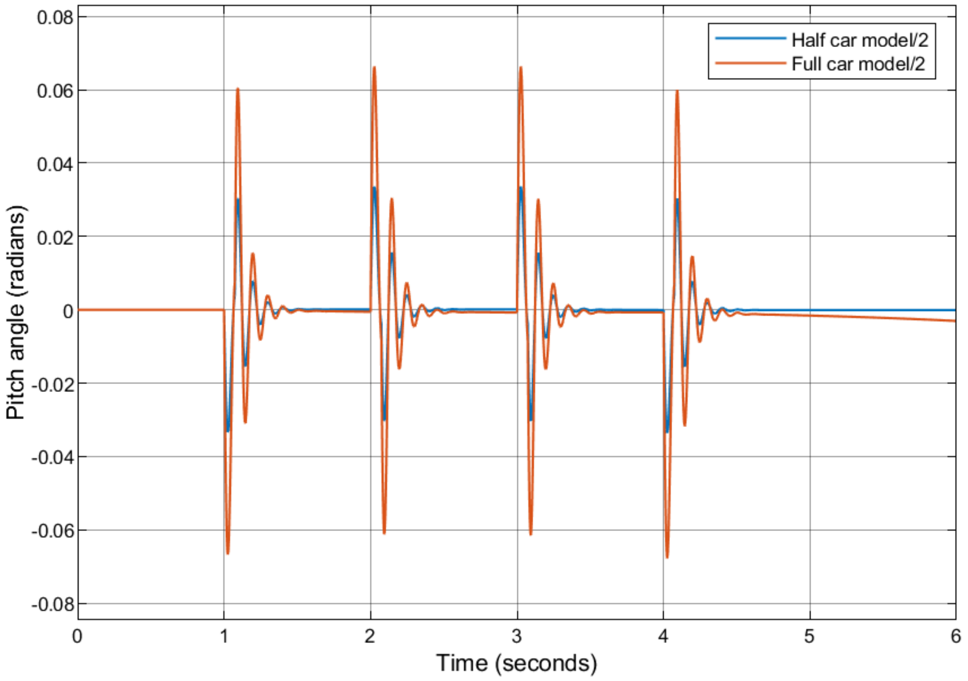


Figure 21. Pitch angle of half-car and full car for Bump and Hole Input.

Finally, the roll angle is evaluated, only available in the complete car model. To perform this evaluation, the random type input is used. This evaluation can be seen in Figure 22.

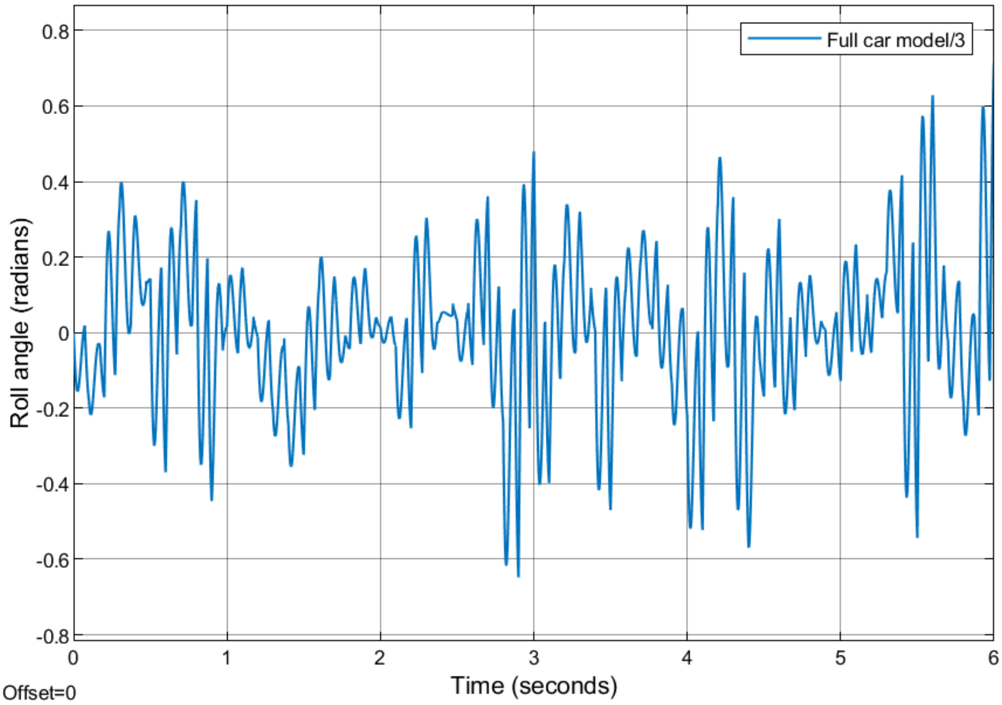


Figure 22. Roll angle of full car model for a Random Input

5.6. Parameter sensitivity analysis

In this section a sensitivity analysis is carried out to evaluate the influence of the parameters in the developed models. Since only the vertical displacement of the chassis is going to be evaluated, this sensitivity analysis will be carried out using the quarter car model. The procedure will consist in varying the order of magnitude of the stiffness and damping constants of suspension and tire respectively, and the sprung and unsprung mass.

5.6.1. Analysis of stiffness and damping constants of the tire

In Figure 23 and Figure 24 the response of the system for variations of the constant of stiffness and damping of the tire can be seen.

As can be seen in Figure 23, the stiffness constant only produces a variation of importance in the response when its value decreases by a certain order of magnitude. The value that has been given to it in the beginning is close to saturation and, therefore, because it is not the tire that is being studied in the present work, this behaviour is essentially correct and desired. Saturation is understood as the value above or below which a significant variation in the system response is no longer produced.

As can be seen in Figure 24, the tire's damping constant has little influence on the vertical displacement of the chassis, the original value being also 120, in saturation zone, which is essentially convenient and correct.

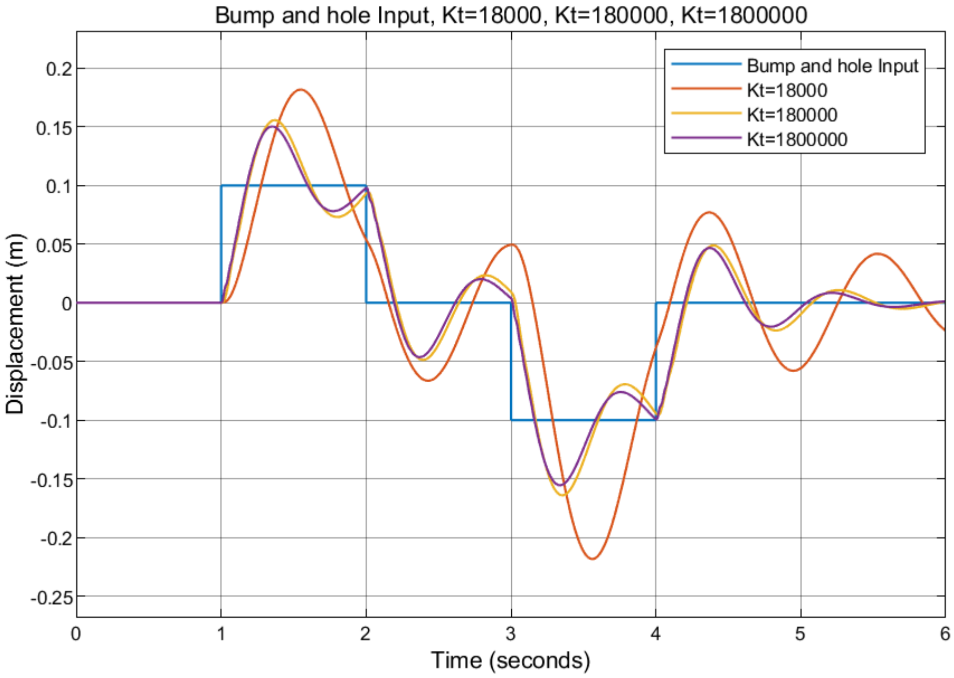


Figure 23. Vertical displacement for different tire stiffness constant

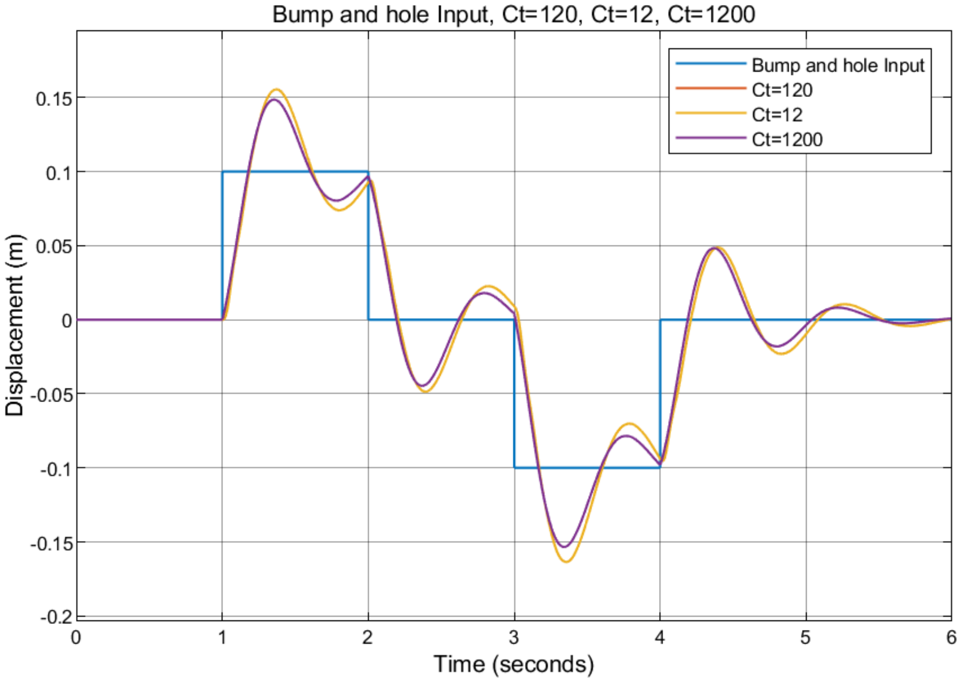


Figure 24. Vertical displacement for different tire damping coefficient

5.6.2. Analysis of stiffness and damping constants of the suspension

Figure 25 and Figure 26 show the response of the system for variations in the stiffness and damping coefficients.

As can be seen in Figure 25, the suspension stiffness coefficient has a very high influence on the frequency and amplitude of the response of the system to the disturbance. In the present work it is considered that the value Kt = 18000 grants a good commitment between the quality of analysis of the present work and reality.

As can be seen in Figure 26, the suspension damping constant has a very positive influence as its value increases. This justifies that in many applications traditionally it has been decided to implement semi-active suspension control. In the present study, it is considered that the given value of Cs = 1200, grants a good commitment that allows a better study of the control strategies in later sections.

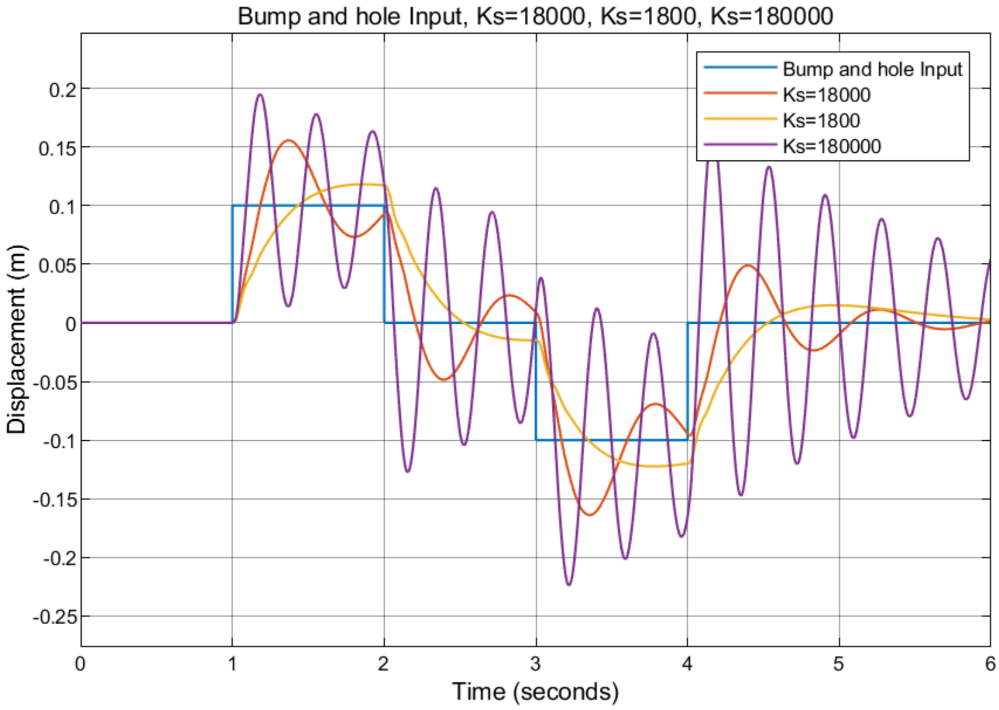


Figure 25. Vertical displacement for different suspension stiffness coefficient

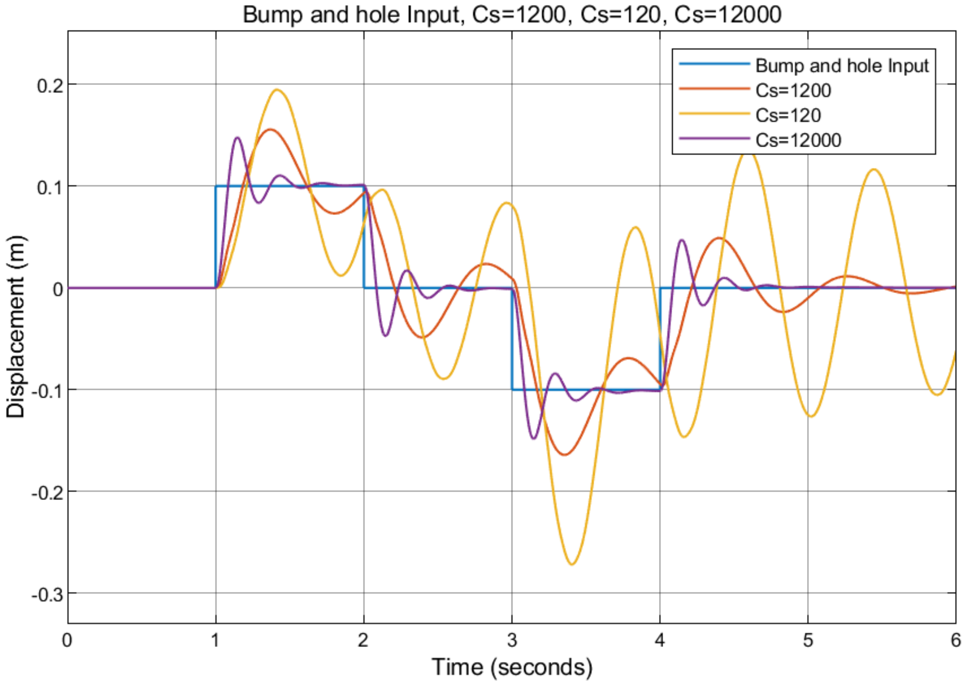


Figure 26. Vertical displacement for different suspension damping coefficient

5.6.3. Analysis of mass of tire and chassis

In Figure 27 and Figure 28, the response of the system to the variations in the mass of tires and chassis can be observed.

As can be seen in Figure 27, the influence of the mass of the tire is of relatively low. The original value, of high fidelity with the value of a real tire, reaches the saturation value, which is considered this appropriate value.

In Figure 28 influence of the chassis mass can be evaluated. The graph indicates that this value is of great importance in the transient response of the system, increasing the settling time and the amplitude of the response as the mass value increases.

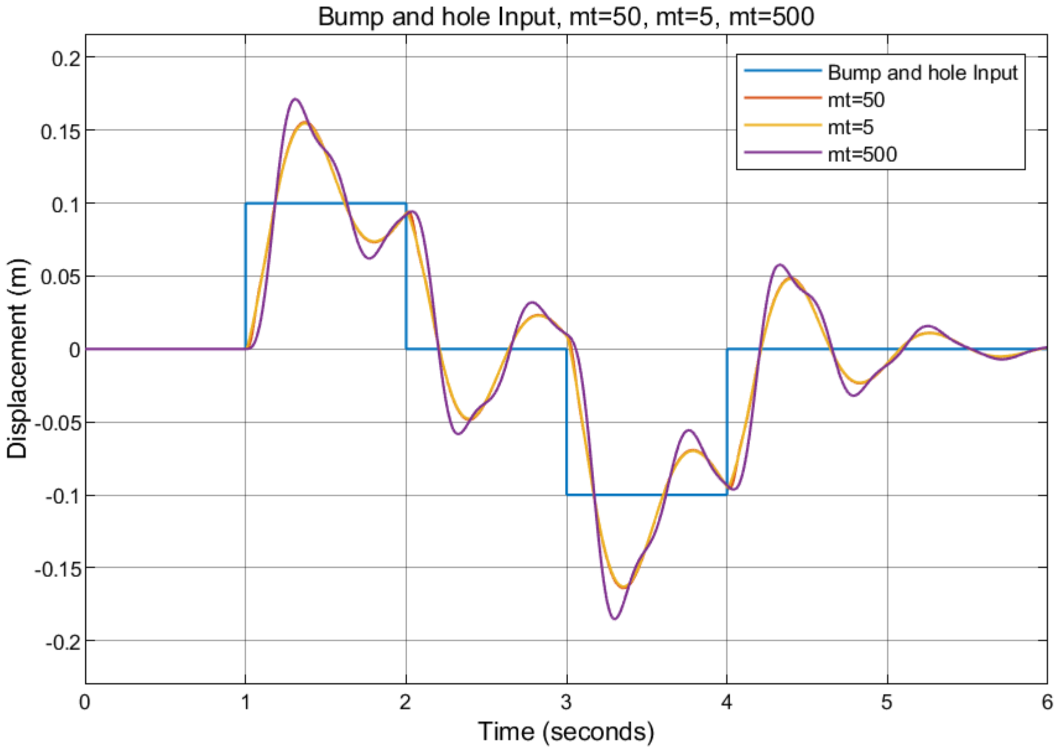


Figure 27. Vertical displacement for different tire mass

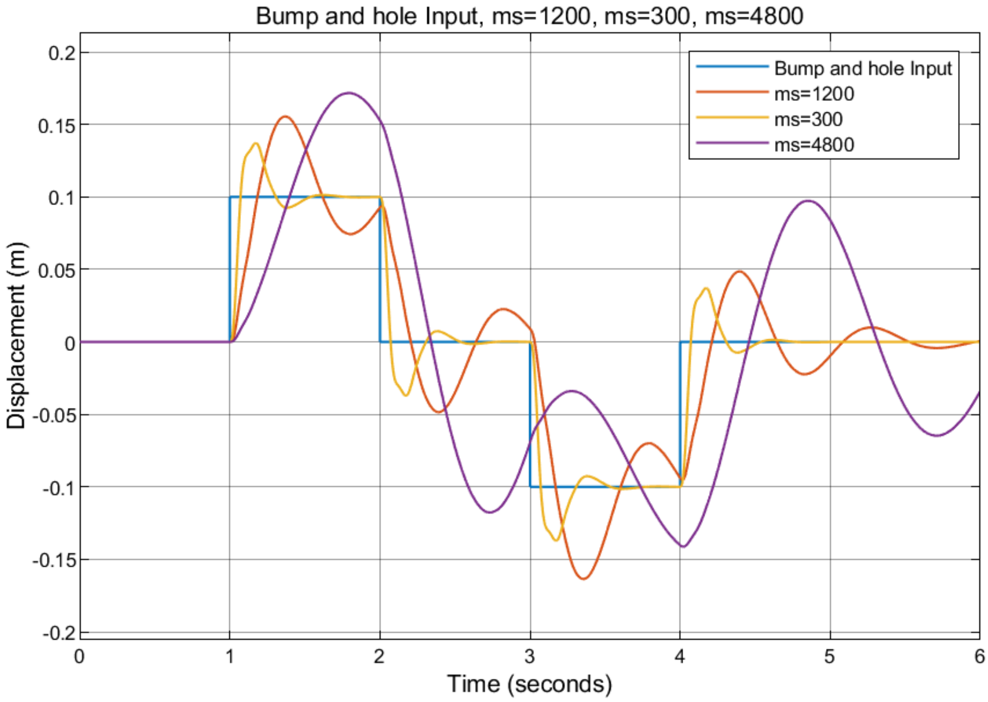


Figure 28. Vertical displacement for different vehicle mass

5.7. Conclusion

In this section, three different linear models of the dynamics of vehicle have been constructed to represent the behaviour of a car's suspension system. It has been found that, to evaluate the vertical displacement of the vehicle chassis, the quarter model provides a good approximation, requiring less work and computational power. Also, to evaluate the angle of pitch of the vehicle, the half-car model provides a good approximation. The usage of the full-car model might be relegated to applications in which the roll angle needs to be evaluated. It is based on these deductions that in the following sections the quarter car model will be considered.

Finally, a sensitivity analysis of the most significant variables of the models has been carried out, studying their influence on the behaviour of the car dynamics and justifying the choice of each value. Sensitivity analysis is a good starting point for the design of passive and semi-active suspensions. As for active suspensions, correctly chosen values facilitate a more concise study of control.

6. DESIGN OF A NO LINEAR MCPHERSON SUSPENSION

6.1. Introduction

The control of an active suspension system is a multi-objective and non-linear problem for which conventional control methodologies based on a mathematical model of the system are found, as well as intelligent control strategies. The modelling and simulation of suspension systems is essential for the analysis of vehicle dynamics and the design of its controller. In this sense, different approaches have been proposed to model the interaction with the vehicle. However, a model that represents the nonlinearities of the dampers or the kinematics of the suspension has not been developed. With all this, this section proposes the construction of a multi-body no linear model for a McPherson suspension.

Once the develop of the suspension is achieved, the model is tested and compared with the linear models previously developed. Once its correct operation has been verified, this model is used to perform intelligent control.

6.2. The McPherson Strut suspension

The McPherson suspension is a type of suspension widely used in small and medium-sized vehicles, due to its low weight, compact size and low cost. Although its simplicity and low manufacturing cost are the main advantages due to its kinematic, but that the vertical angle between road and tire varies a few degrees during its movement. It also the vibrations of the road directly to the chassis, causing noise and vibrations.

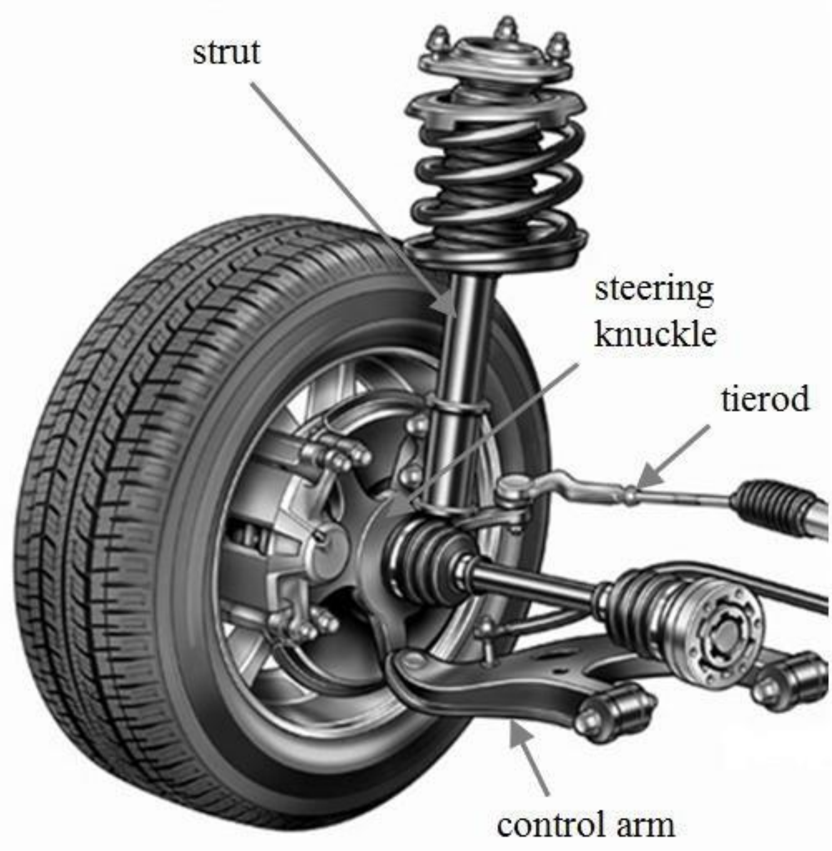


Figure 29. McPherson Suspension parts (21)

It is one of the most used systems on the front axle. This system only carries an oscillating arm, connected at one end to the frame by means of elastic bearings, and at the other end to the steering knuckle through the ball joint. The hose at the top is attached to the vertical shock absorber (Figure 29).

This arrangement, in addition to fulfilling its function as suspension and damping, also serves as a vertical axis of rotation of the wheels. Therefore, the set describes an angle proportional to that made with the steering wheel.

6.3. Planar Model of a McPherson Strut Suspension

The variable geometry of the McPherson suspension causes a non-linear behaviour in relation to its kinematics and dynamics. This behaviour cannot be directly analysed by the conventional model of a quarter car because it despises the effects due to the configuration of the suspension mechanism.

A two-dimensional planar model can capture the non-linear geometric effects of the McPherson suspension. In a McPherson suspension, small variations in suspension geometry greatly affect the kinematic and dynamic response. In this chapter the sections are organized as follows. First, the suspension scheme is developed, and its geometric values are determined. Secondly, a suspension design is carried out using the SolidWorks CAD package. This model is exported to Simulink using Simscape Link. Finally, the model is fixed, and its response to disturbances is tested.

Figure 30a) shows a representation of the proposed planar model, where the main elements of the McPherson suspension system are:

1. Chasis (Sprung mass)
2. Lower control arm.
3. McPherson Strut.
4. Tire and knucle.

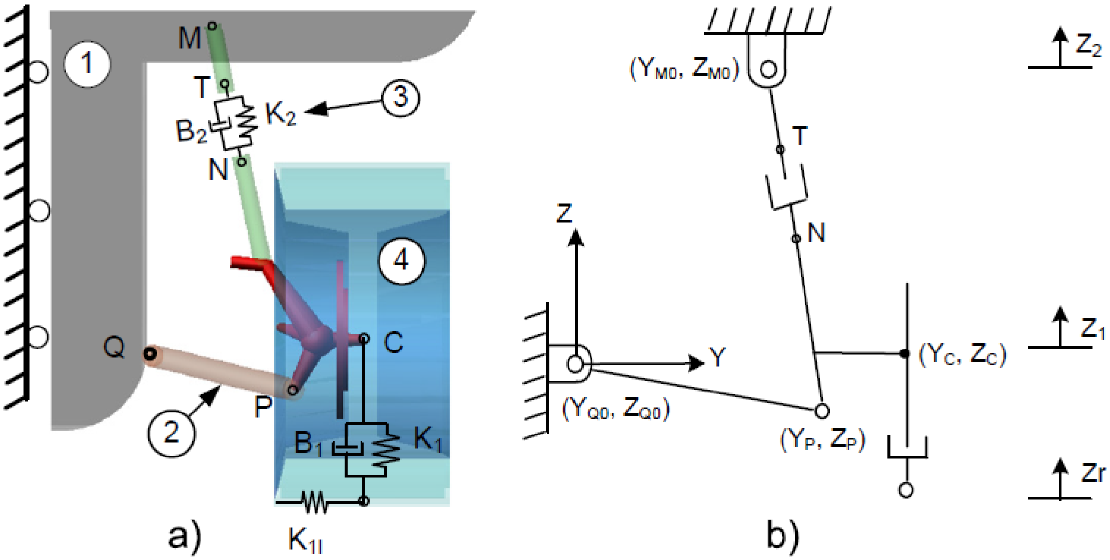


Figure 30. a) Planar McPherson model, b) kinematic McPherson model (22)

The kinematic model of the suspended mass can be seen in Figure 30b), where Z2 is the displacement of the suspended mass, Z1 is the displacement of the unsprung mass, and the disturbance on the road is Zr. The proposed model is valid under the following considerations:

- Chassis only have vertical movement. In a multibody model, that translates in the car boy having a vertical prismatic joint with the world frame.
- All elements are rigid, excepting the tire.
- Control arm and steering knuckle have neglectable masses.
- Tire only have vertical and rotational movements.
- All joints are ideal (no friction or internal forces)
- Spring and dampers coefficient of tire and suspension are linear.

The hypothetical movement of the kinematic model of the suspension is illustrated in Figure 31.

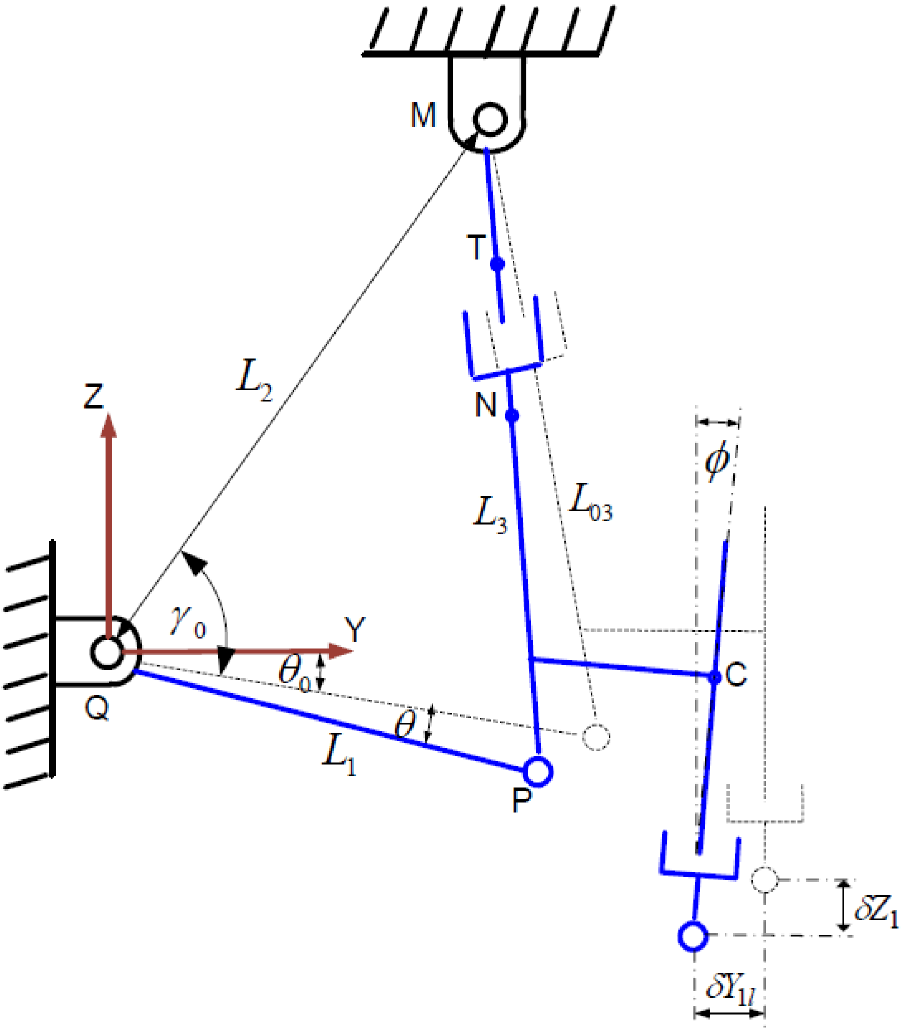


Figure 31. Kinematic McPherson model movement

The suspension is modelled using the SOLIDWORKS CAD package. Once the model is developed, the export to MATLAB Simulink is carried out using the Simscape Link application. Simscape Link is an application that allows to import assemblies from the main CAD platforms to Simscape today, (ie SOLIDWORKS, AUTODESK INVENTOR and PTC Creo). The 3D model resulting from the suspension already imported into SimMechanics can be seen in Figure 32. As it can be seen, it is a simplified model, but it serves to faithfully represent the kinematics of the suspension.

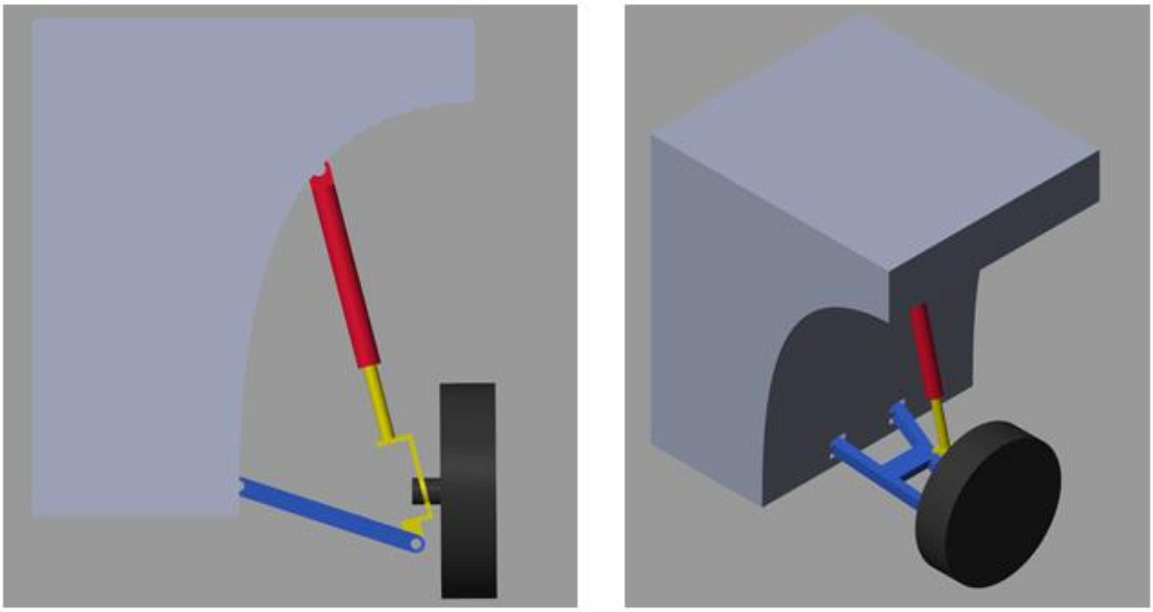


Figure 32. Quarter Car McPherson suspension 3D model in Simscape

The block diagram obtained in Simscape after import can be seen in Figure 33. In this figure some different types of block can be observed.

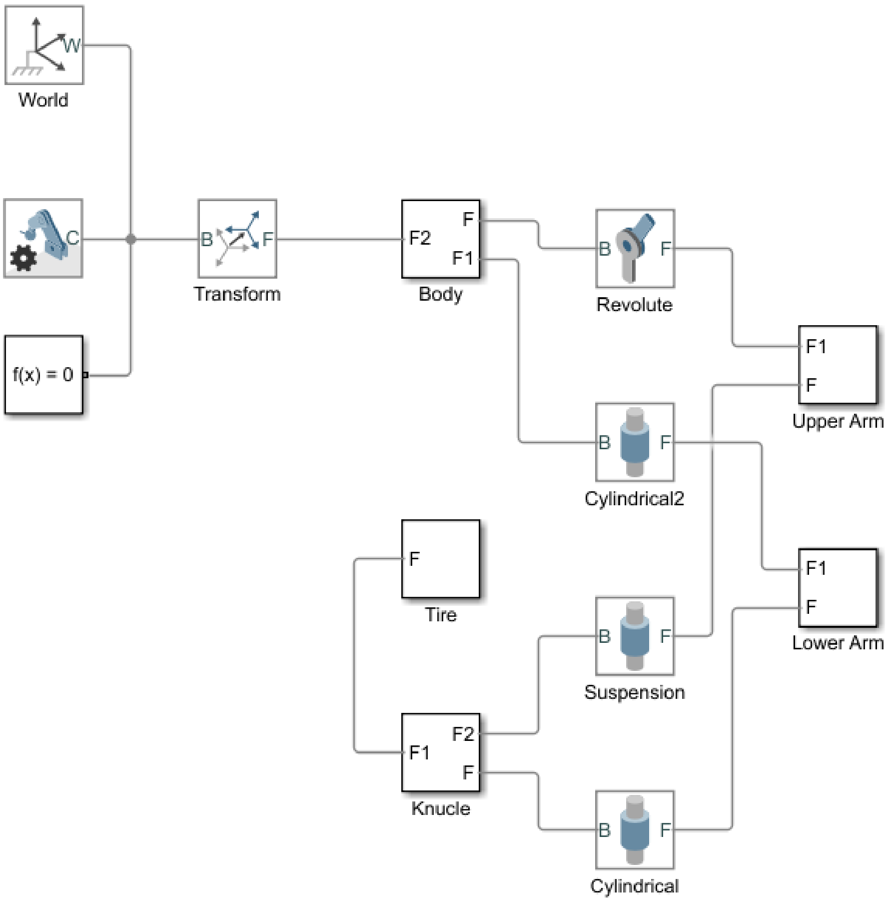
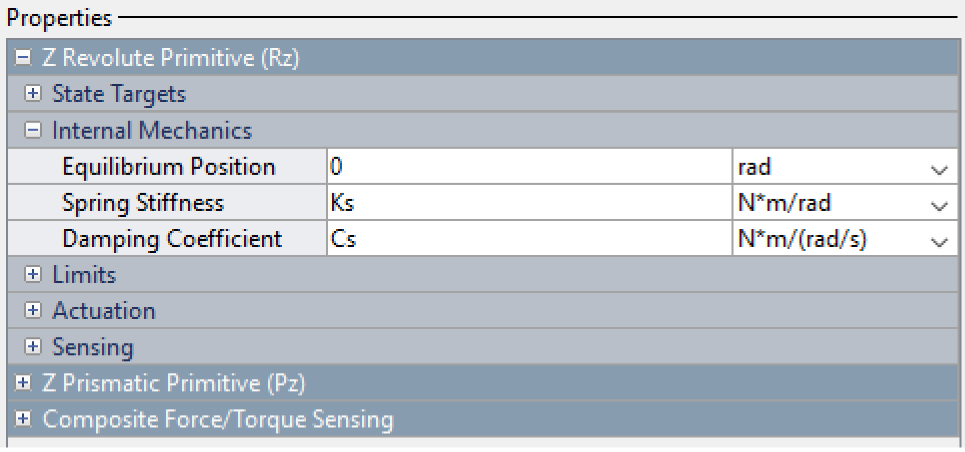


Figure 33. Quarter Car McPherson suspension block diagram in Simscape

Each physical element of the assembly corresponds to a block. Each of those blocks incorporates a "frame" corresponding to the assembly point of said element with another. Generally, assembly points are obtained from a geometric transform from the coordinate origin of that element.

In figure can also be seen another type of blocks that represents the type of joint between two elements. In these joints, in addition to internal mechanics, position restrictions or trajectories introduced by means of an input to the subsystem. As shown in Figure 34, it is the block used to enter the constants of the suspension.



Equilibrium Position	0	rad
Spring Stiffness	Ks	N*m/rad
Damping Coefficient	Cs	N*m/(rad/s)

Figure 34. McPherson Strut Prismatic Joint parameters

Finally, there are several blocks, associated with the "world" frame that establish the origin of coordinates, coordinate axes or direction of gravity. They also include parameters such as the type of solver to be used, the solution tolerance or the passage width between calculation and calculation.

6.4. Analysis of response to input disturbances

In this section, the suspension system is subjected to various disturbances, in order to check its correct behaviour and studying its performance. As in the analysis of linear systems, the Simscape model is subjected to three perturbations: Bump and hole, sinusoidal and random input.

The following figures show the response of the suspension system in terms of vertical displacement and vertical acceleration for the three inputs.

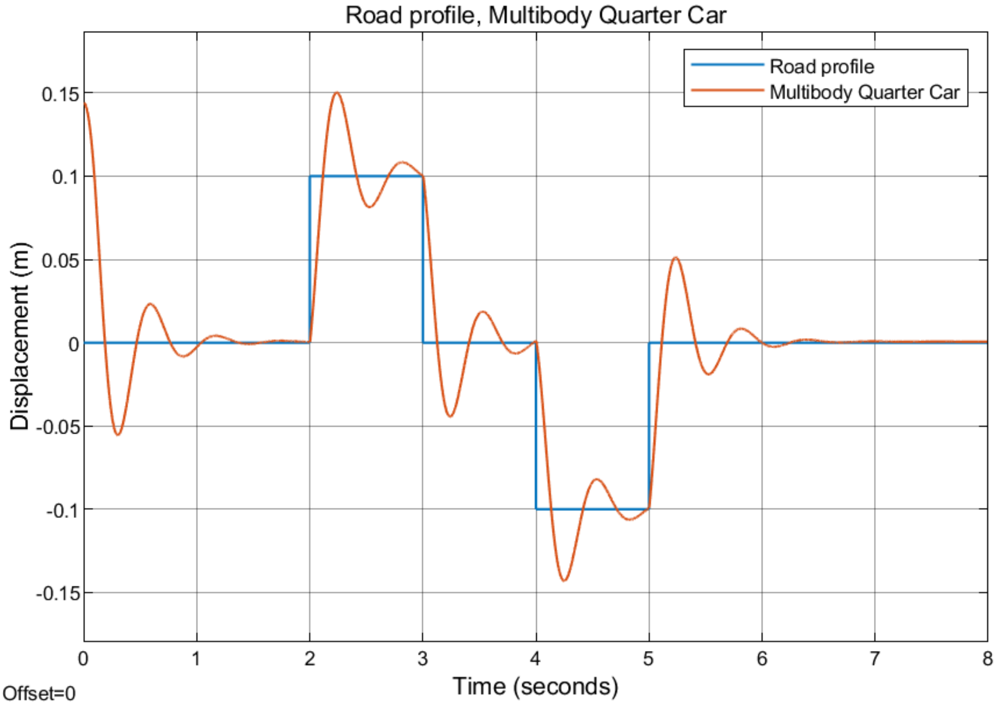


Figure 35. McPherson Model vertical displacement for Bump and Hole Input

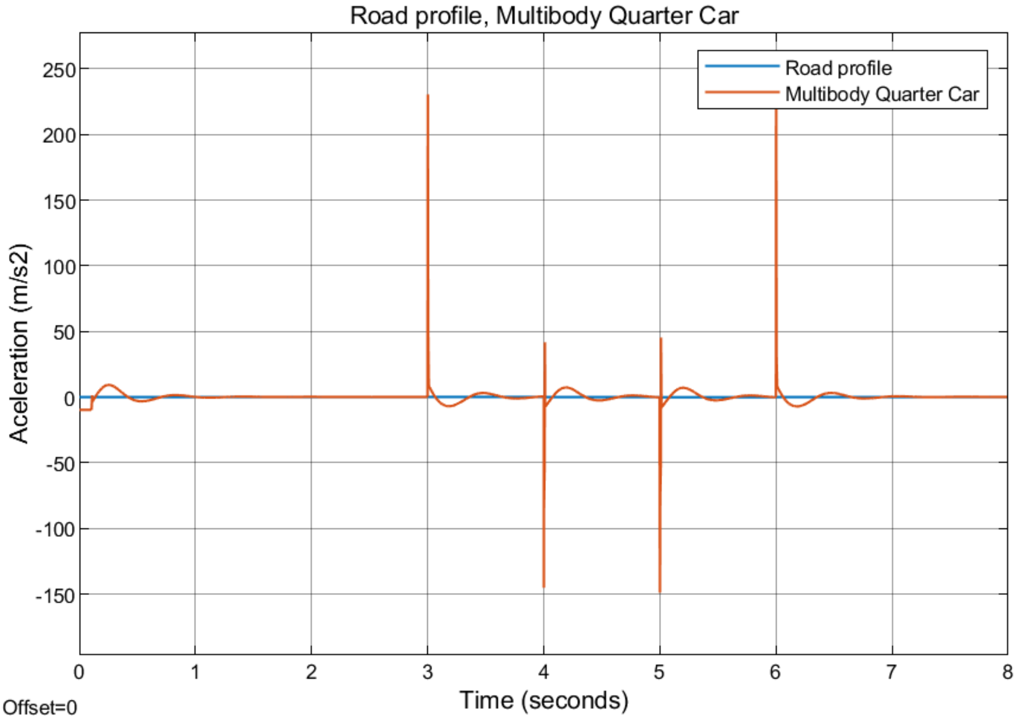


Figure 36. McPherson Model vertical acceleration for Bump and Hole Input

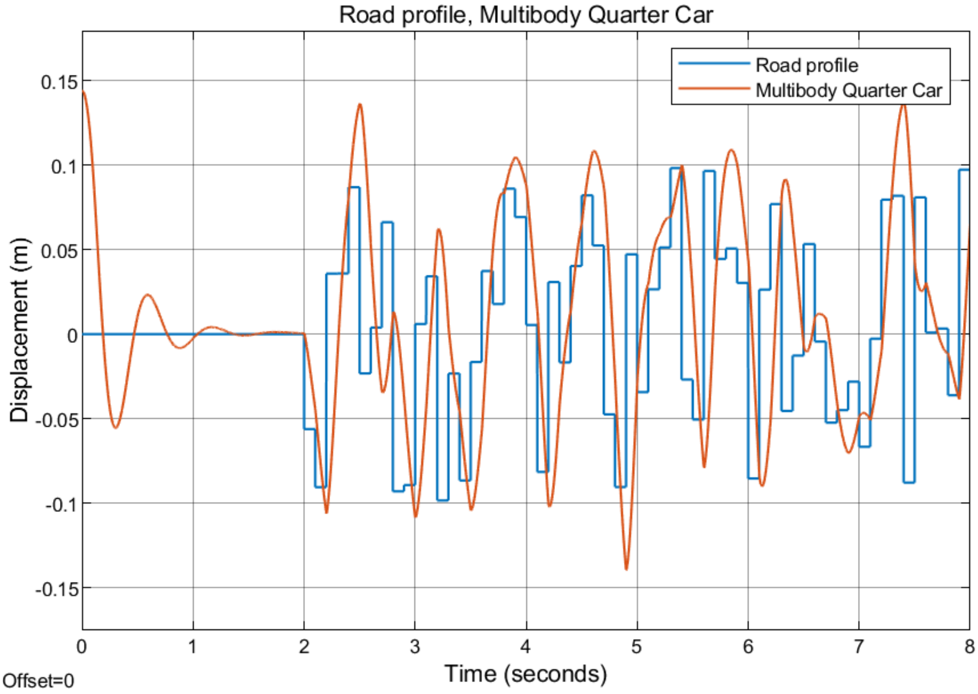


Figure 37. McPherson Model vertical displacement for Random Input

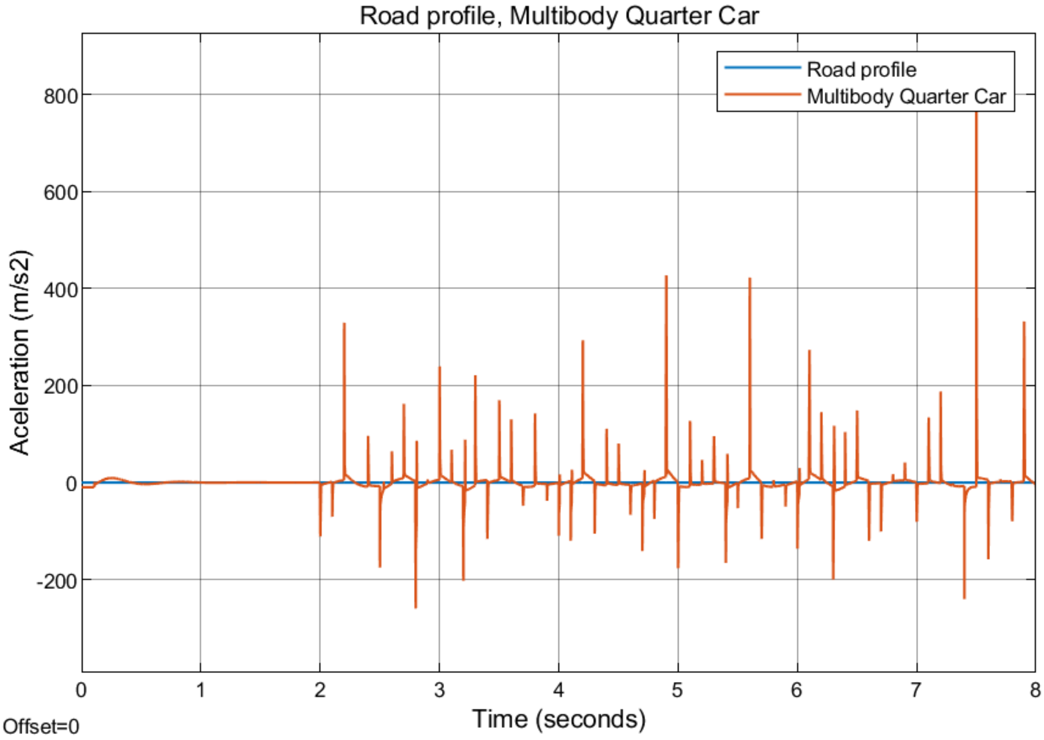


Figure 38. McPherson Model vertical acceleration for Random Input

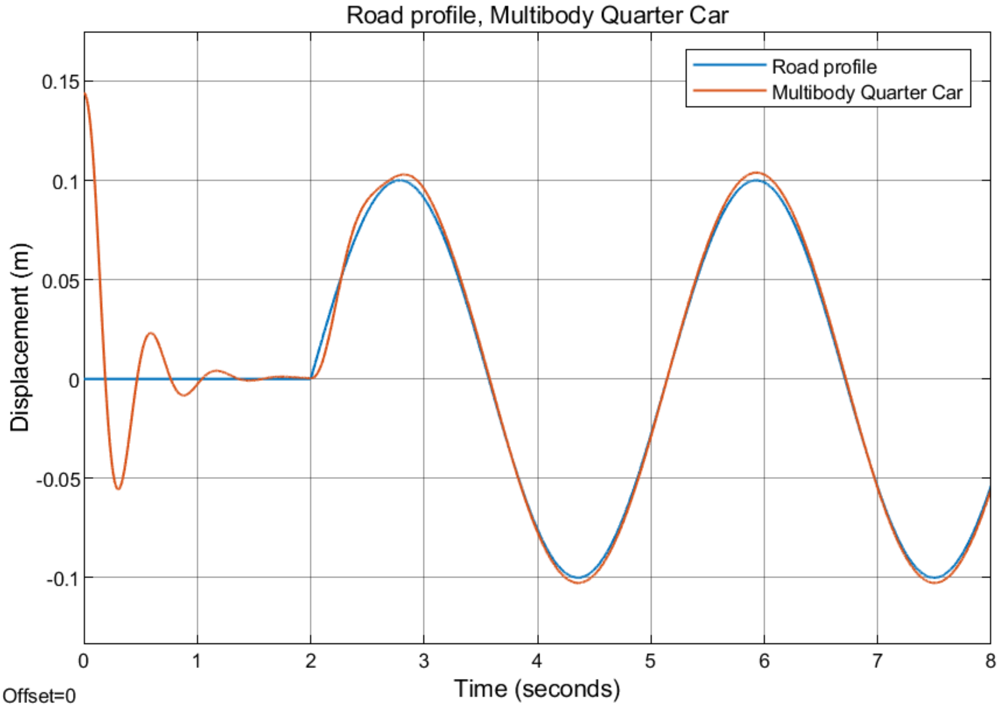


Figure 39. McPherson Model vertical displacement for Sinusoidal Input

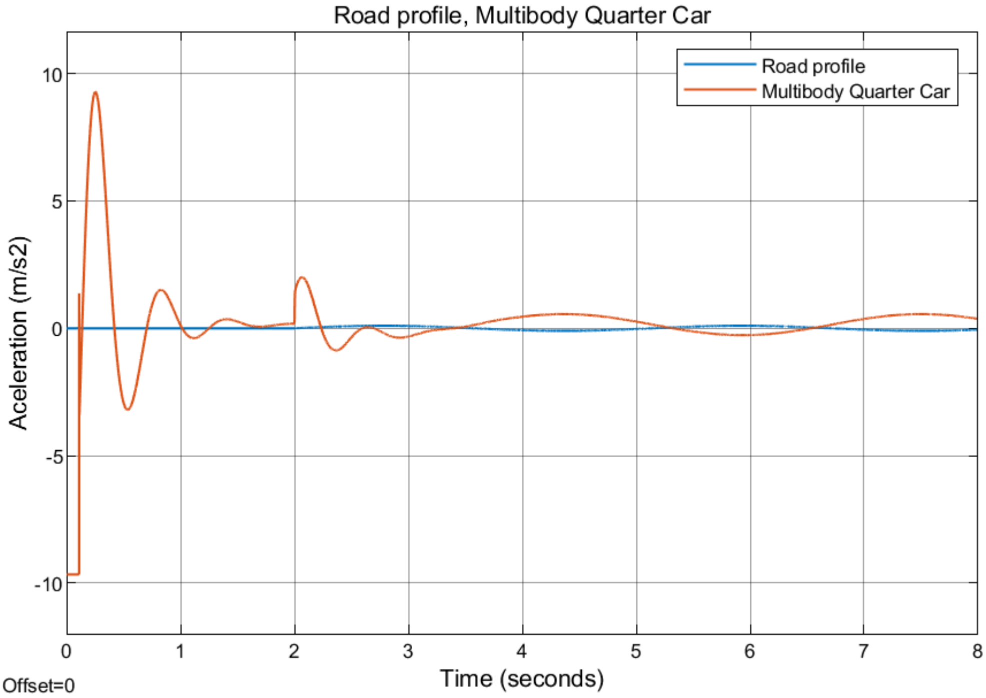


Figure 40. McPherson Model vertical acceleration for Sinusoidal Input

Regarding the kinematics of the system, the presented model allows to analyse the variations of the camber's angle that cannot be considered in the linear model. Figure 41 shows this variation of against when using a Bump and hole input.

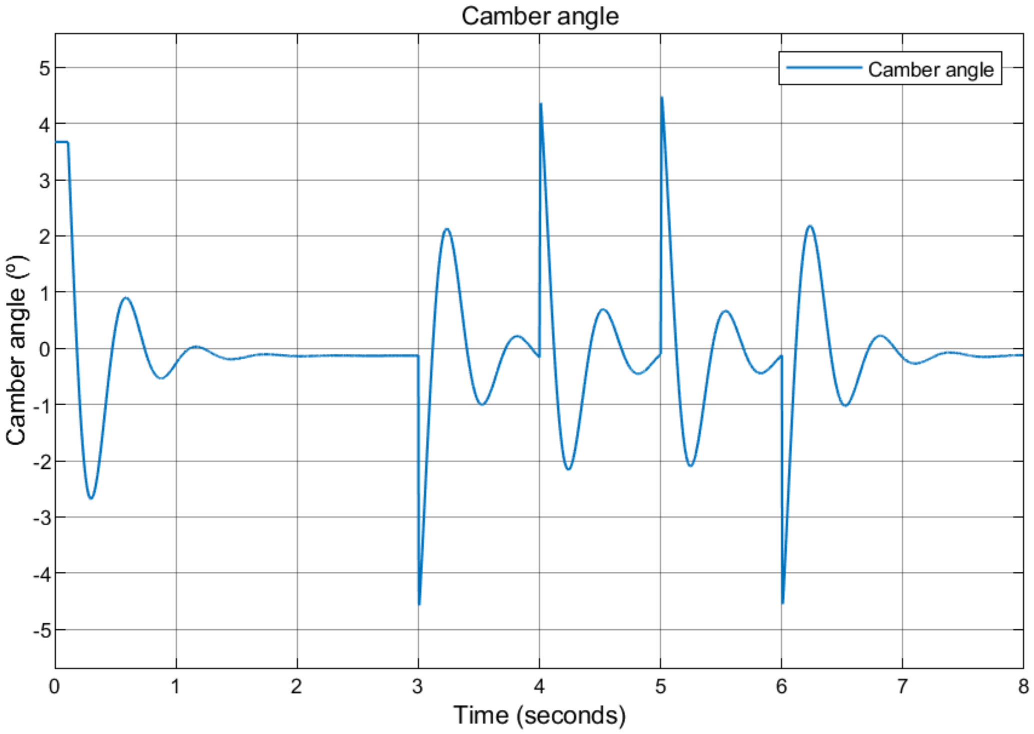


Figure 41. McPherson Model Camber Angle for Bump and Hole Input

6.5. Comparative Analysis of Linear Quarter Car and McPherson Quarter Car

The comparative analysis compares the responses of the linear quarter car and the McPherson multibody quarter car, that is, using the same stiffness and damping constants for suspension and wheel. The comparison is made using the Bump and Hole input and the results for position and acceleration can be seen in Figure 42 and Figure 43.

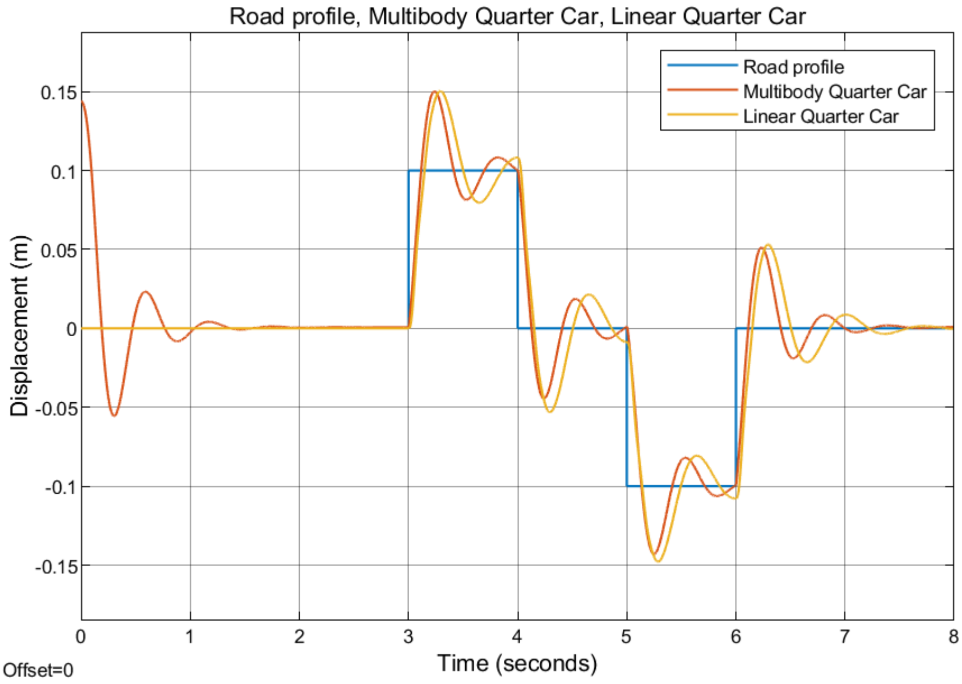


Figure 42. Vertical displacement McPherson Model vs Linear Quarter for Bump and Hole Input

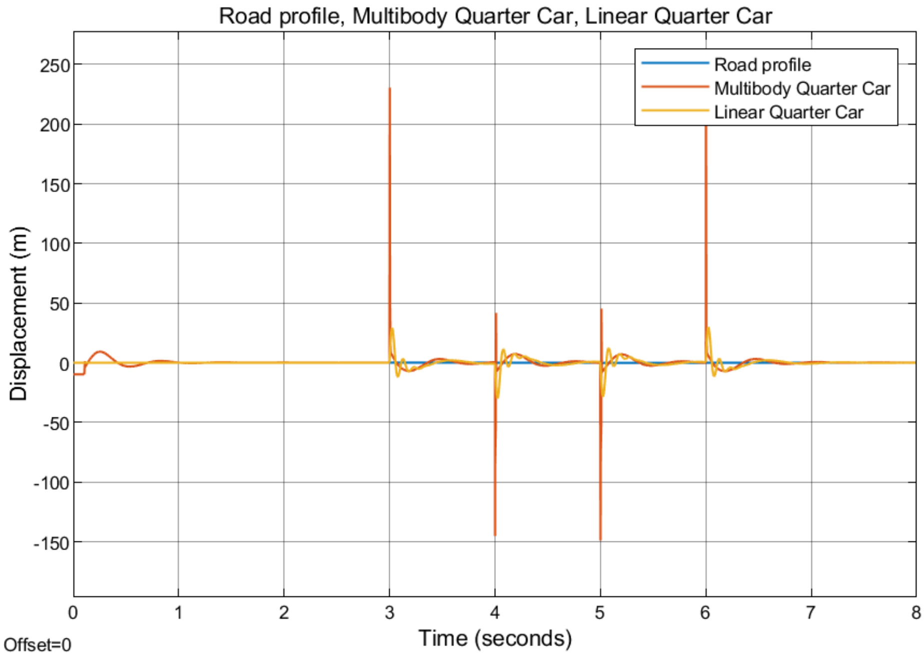


Figure 43. Vertical acceleration McPherson Model vs Linear Quarter for Bump and Hole Input

The linear model implemented in Simulink has a computation time of 0.75s, while for the multibody model in Simscape, the computation time is 38.4s. These results confirm that the linear analytical models are much faster and therefore more suitable for real-time simulation applications. The dynamic response of the two systems for the vertical displacement of the chassis with respect to the Bump and Hole are very similar, demonstrating both models are comparable. The frequency of oscillation is reproduced quite accurately by the planar model.

However, the dynamic response for vertical acceleration leads to much higher peaks in the multibody model. This is a consequence of the type of treatment that in each model is given to the first and second order derivative, given that the Bump and Hole input, at height changes, momentarily reaches infinite speed and acceleration values.

6.6. Conclusions

In this chapter a multi-body model of a quarter car for a McPherson suspension has been developed. This model not only considers the vertical movement of the suspended mass, but also:

- The rotation of the unsprung mass.
- The moment of inertia of the wheel with respect to the longitudinal axis.
- Non-linear suspension compression due to the system's own kinematics.

The proposed model has been compared with a typical linear quarter car model to analyse the dynamic behaviour against road disturbances, in addition to a dynamic parameter exclusive to the multibody model: the camber angle. The results have demonstrated a good approximation between both models and the ability of the planar model to approximately reproduce the behaviour of a McPherson suspension.

In summary, the Simscape Software allows to analyse the vehicle suspension from a physical modelling performed in CAD. This approach is convenient in high-precision studies, although analytical-linear models are more suitable for real-time simulation applications.

7. DEVELOP AND APLICATION OF CONTROL METHODOLOGIES

The controller of an active suspension system manages to properly balance the quality requirements of the vehicle in comfort, manoeuvrability and handling. Which are impossible to satisfy simultaneously in different types of road with a passive suspension. However, conventional controllers such as the PI and PID show limitations in the response to changes in the system's operating points, with poor handling of the nonlinearities of the suspension.

These limitations can be overcome by using intelligent controllers, which allow considering the nonlinearities and uncertainties of complex systems. In the introduction of this work we have talked about various intelligent control methodologies. Among them, the ones that present the greatest research activity today are controllers with diffuse logic and controllers with neural networks.

Diffuse logic controllers allow considering the nonlinearities and uncertainties of complex systems. Consequently, several investigations have applied fuzzy strategies to control vehicle suspension systems with the advantage that control rules can be developed from heuristic knowledge without needing for an accurate description of the system.

However, in a fuzzy logic controller the adjustment of parameters such as scale factors are complicated. Additionally, the determination of the factors by trial and error methods is very expensive in time.

Unlike diffuse control, which is a mathematical method for implementing control strategies based on natural language, neuronal control is another control method used when data is available in the form of observed measurements / numerical data of the behaviour of the suspension. This is the case, for example, of this work, thanks to the previously developed analytical models, data can be obtained that will help the network to fulfil its purpose.

This chapter proposes the development of two different types of control:

1. Conventional control by PID controller: PID control is implemented using the default Simulink block and calibrating the control parameters using the tuning tool.

2. Neural Network control: This section implements the control block for neural networks belonging to the Deep Learning Toolbox of MATLAB Simulink. The model is trained with a data set obtained from the suspension subjected to randomized standard inputs.

7.1. PID Control Implementation

PID control is a closed loop control strategy that allows you to control a system to achieve the desired output status. The PID controller is composed of three elements that provide a Proportionate action; Integral and Derivative. These three actions are what give name to the PID controller. (Figure 44)

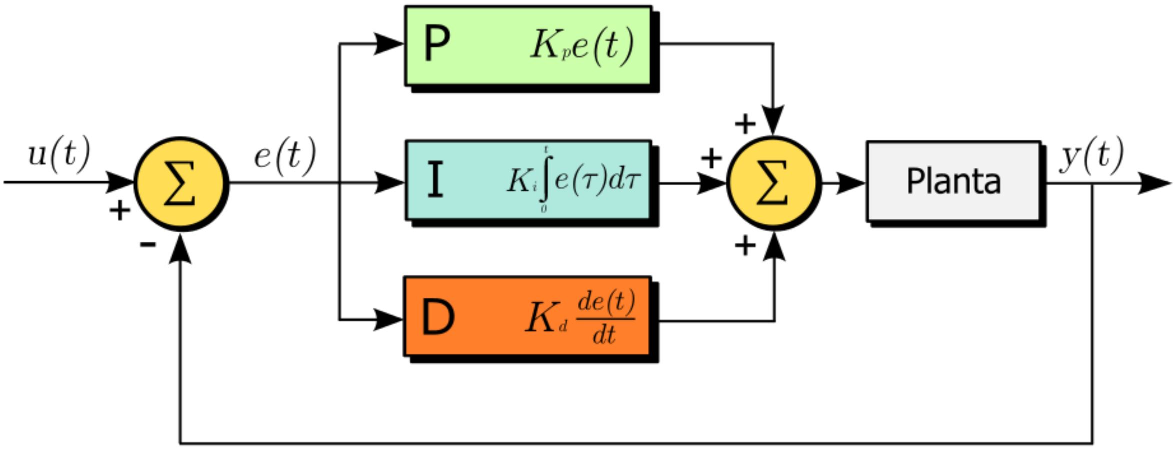


Figure 44. Layout of a PID controller

7.1.1. Reference signal and error signal

The signal r(t) is called a reference and indicates the state that is desired to be achieved at the system output and (t). In a temperature control system, the reference r (t) will be the desired temperature and the output and (t) will be the actual temperature of the controlled system.

As can be seen in the previous scheme, the input to the PID controller is the error signal e (t). This signal indicates to the controller the difference between the state to be achieved or reference r (t) and the actual state of the system measured by the sensor, signal h (t).

If the error signal is large, it means that the system state is far from the desired reference state. If on the contrary the error is small, it means that the system has reached the desired state.

7.1.2. Proportional Control Action

This control action is proportional to the error signal. Internally the proportional action multiplies the error signal by a constant, Kp.

This control action attempts to minimize the system error. When the error is large, the control action is large and tends to minimize this error. Increasing the proportional action Kp has the following effects:

1. Increase system response velocity.
2. It reduces the system error in stationary.
3. Increase system instability.

By increasing the proportional action there is an equilibrium point at which sufficient system response speed and error reduction are achieved, without the system being too unstable.

7.1.3. Integral Control Action

This control action calculates the integral of the error signal. The integral can also be seen as the sum or accumulation of an error signal. As time passes, small errors add up to make the integral action increase This reduces the system error in a permanent regime. The disadvantage of using integral action is that it adds inertia to the system, and therefore makes it more unstable.

The integral Ki action has the following effects:

1. It reduces the system error in permanent regime.
2. Increase system instability.
3. Slightly increase the response speed of the system.

7.1.4. Derivative Control Action

The derivative action, in analogy with the previous actions, is proportional to that derived from the error signal. When the system moves at a high speed towards the reference point, the system will pass by due to its inertia. This produces an overshoot and oscillations around the reference. To avoid this problem, the controller must recognize the speed at which the system approaches the reference to be able to break it in advance as it approaches the desired reference and prevent it from exceeding it.

Increasing the derivative control constant Kd has the following effects:

1. Increase the stability of the controlled system.
2. Decrease the system speed a bit.
3. The error in stationary decrease.

7.1.5. Implementation in Matlab Simulink

The PID controller block is a default block in MATLAB, very useful tool to quickly and sequentially implement a PID controller in a plant.

In Figure 45 it can be seen the layout of the PID controller, and the feedback loop closed and built for a quarter car model:

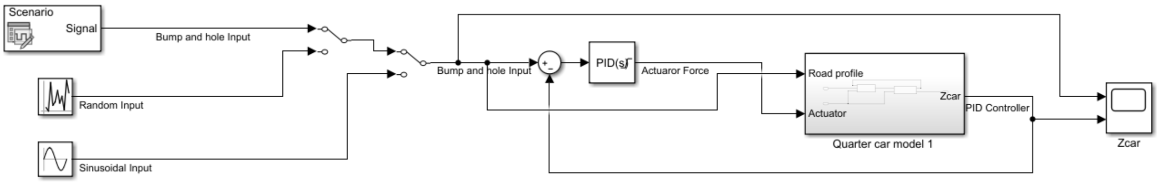


Figure 45. Closed Loop in Simulink for Quarter Car Model with PID Controller

MATLAB Simulink incorporates the PID Tuner function, which automatically linearizes the plant to obtain the transfer function of the feedback system. Once the system is linearized, it allows adjusting the response of the system to a step type input. The parameters in which it can be configured are response time and the robustness of the control.

It is also possible to assign saturation values corresponding to a real actuator to the PID controller. Although this implementation is considered an ideal actuator (that is, without dynamic behaviour), the saturation limits are attributed to it. In this case, an upper limit of 10000N and a lower limit of -10000N are attributed. In Figure 46 the Step response of the system in the PID tuner can be observed:

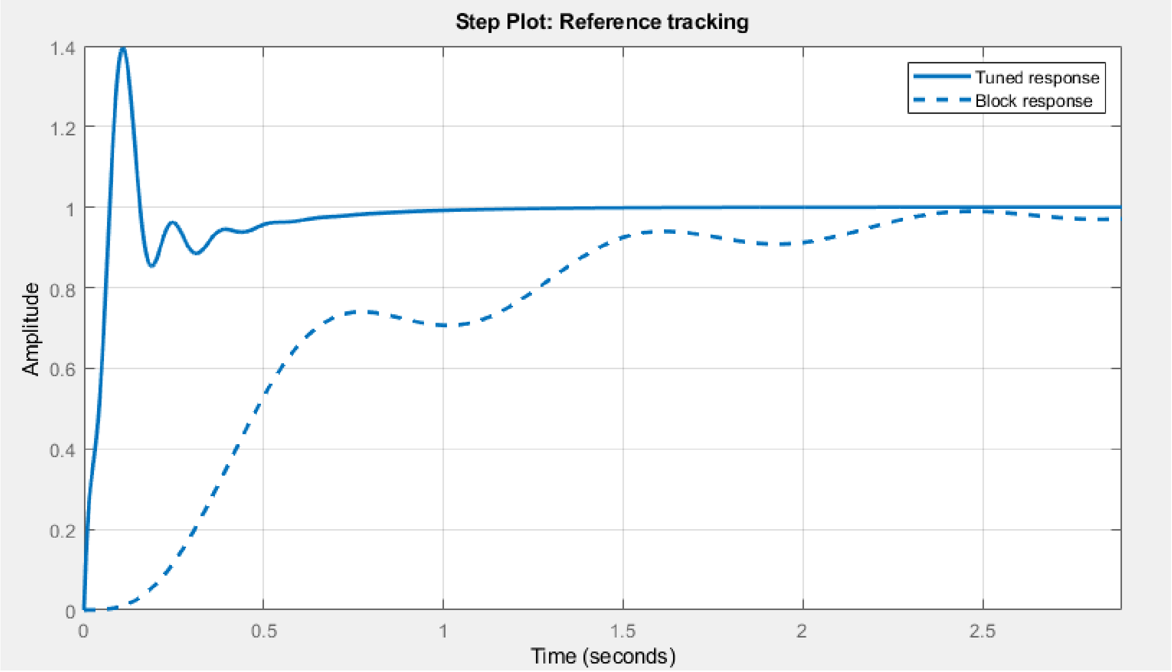


Figure 46. Step Response of the system in PID Tuner

Finally, the parameters obtained for the PID controller are those shown in Figure 47.

Proportional (P)	65448.55
Integral (I)	164792.1
Derivative (D)	5332.03

Figure 47. PID Controller design parameters

Figure 48, Figure 49 and Figure 50 show the response of the system with PID Controller compared to the response in open loop. A notable improvement can be observed in the tracking of vertical displacement for Bump and Hole and Sinusoidal Input. There is also an improvement, although less control for the Random Input.

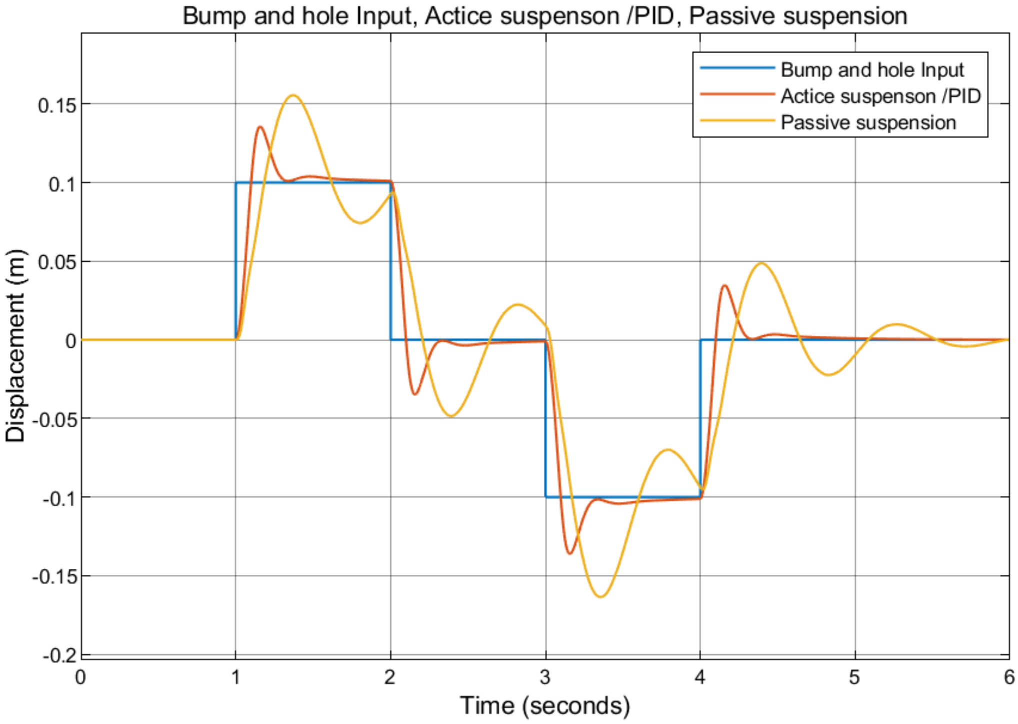


Figure 48. Passive vs PID Controller response for Bump and Hole Input

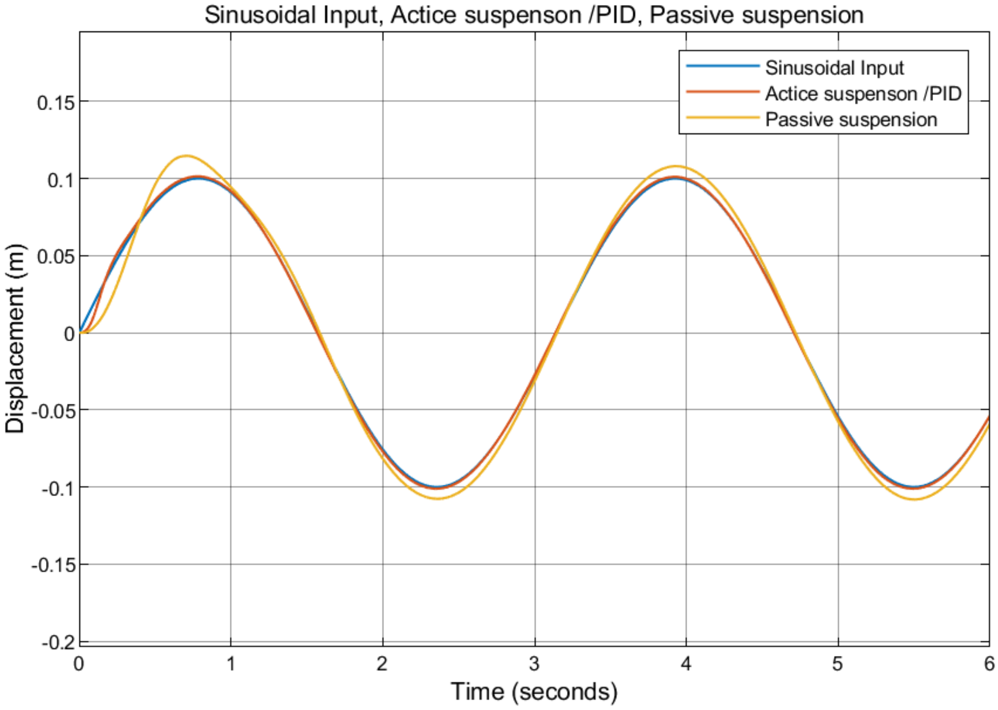


Figure 49. Passive vs PID Controller response for Sinusoidal Input

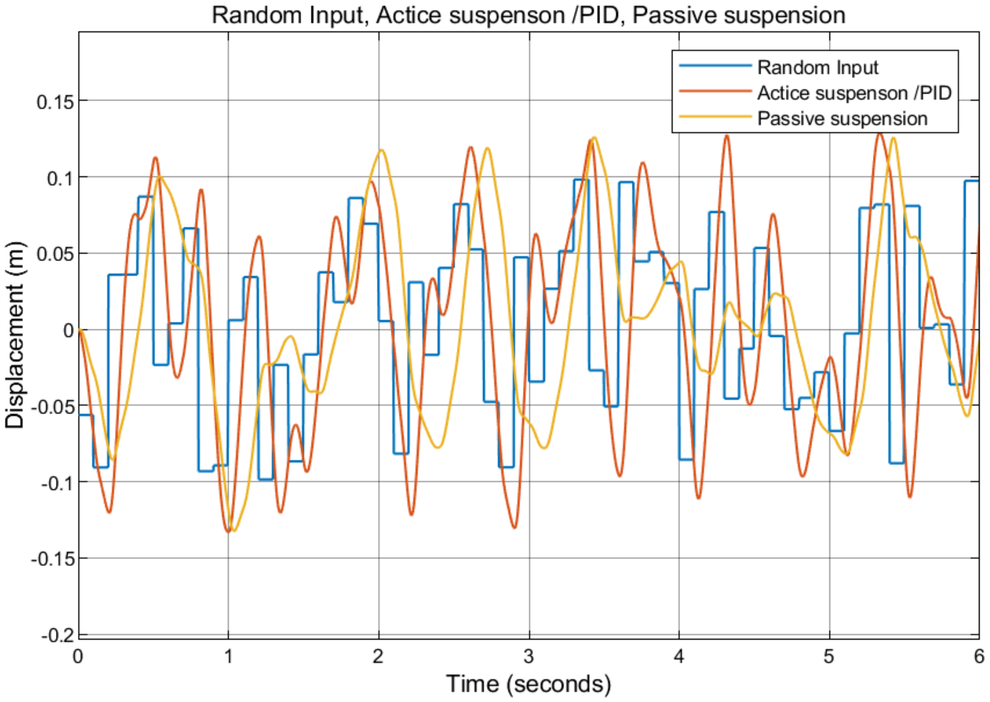


Figure 50. Passive vs PID Controller response for Random Input

By measuring the mean square error (RMS) between the reference signal (input) and the output signal, the improvement for the three cases can be quantified. This quantification can be seen in Figure 51.

Signal	**RMS**
Passive (Bump and hole)	0.03212
Passive (Sinusoidal)	0.007339
Passive (Random)	0.08001
PID Controller (Bump and Hole)	0.0195
PID Controller (Sinusoidal)	0.001524
PID Controller (Random)	0.0708

Figure 51. RMS for Passive and PID Controller

% Improvement (Bump and Hole)	64%
% Improvement (Sinusoidal)	381%

% Improvement (Random)	13%

Figure 52. % of Improvement PID Controller vs Passive

Regarding the percentage of improvement, Figure 52 shows that the behaviour of the linear suspension system for all types of road has been improved. This suggests PID control as a robust and easily implemented control method for active suspension systems.

7.2. Predictive Control by Neural Networks

Artificial neural networks are networks that have many characteristics like those of the brain. They can learn from experience, to generalize from previous cases to new cases or to abstract essential characteristics from entries that a priori present irrelevant information. Neural networks are especially useful in the field of adaptive learning. Adaptive learning refers to the ability of a system to learn to carry out certain tasks through training and examples. The construction of a neural network is completely beyond the scope of this project and the capabilities of its author. However, MATLAB's Deep Learning Toolbox provides a quick and easy way to implement adaptive predictive control based on neural networks.

7.2.1. Neural Network Training and Implementation

First, the Predictive Control block is placed between the reference value, the input signal and the system output signal, as shown in Figure 53.

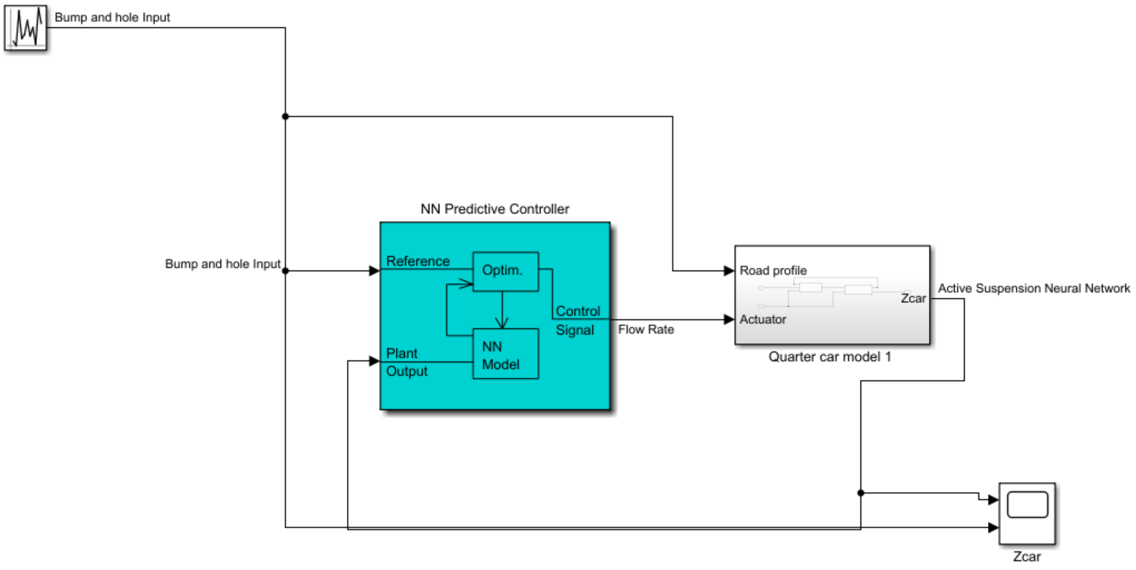


Figure 53. Predictive Controller Layout

By cloning the model in which the control is to be carried out, the entry assembly is obtained. Figure 54 graphically shows the values for system input and output for the training set.

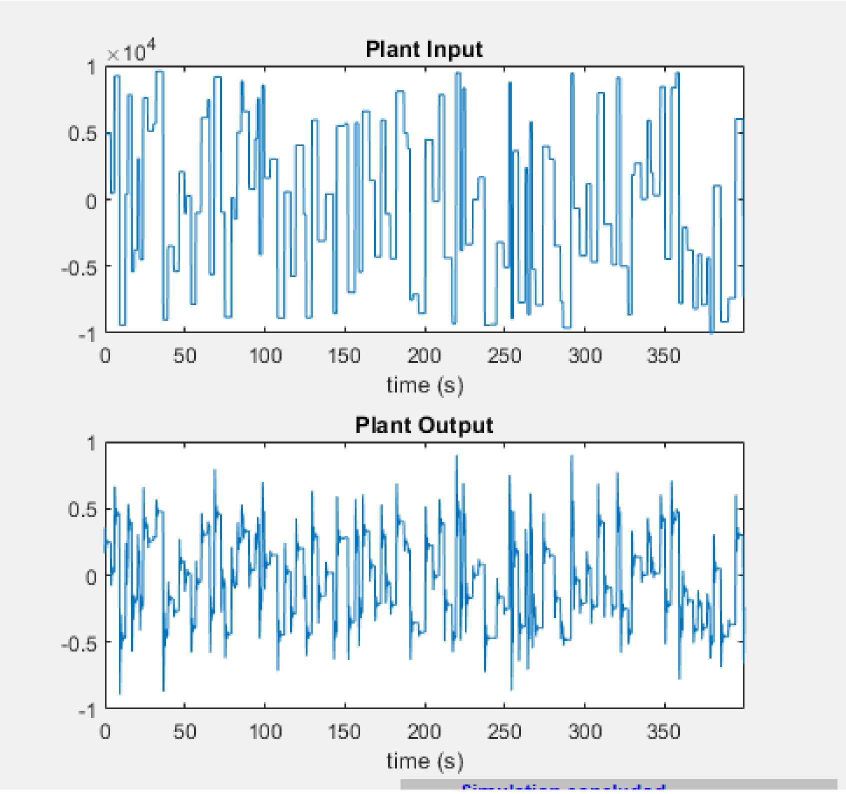


Figure 54. Training Set

Once the training set is obtained, the model is trained, which basically extracts features using the data set. However, in the present work, despite performing several trainings and testing with several parameters, an effective control has not been achieved. In Figure 55 it can be seen how the system, although stable, oscillates around a wrong value.

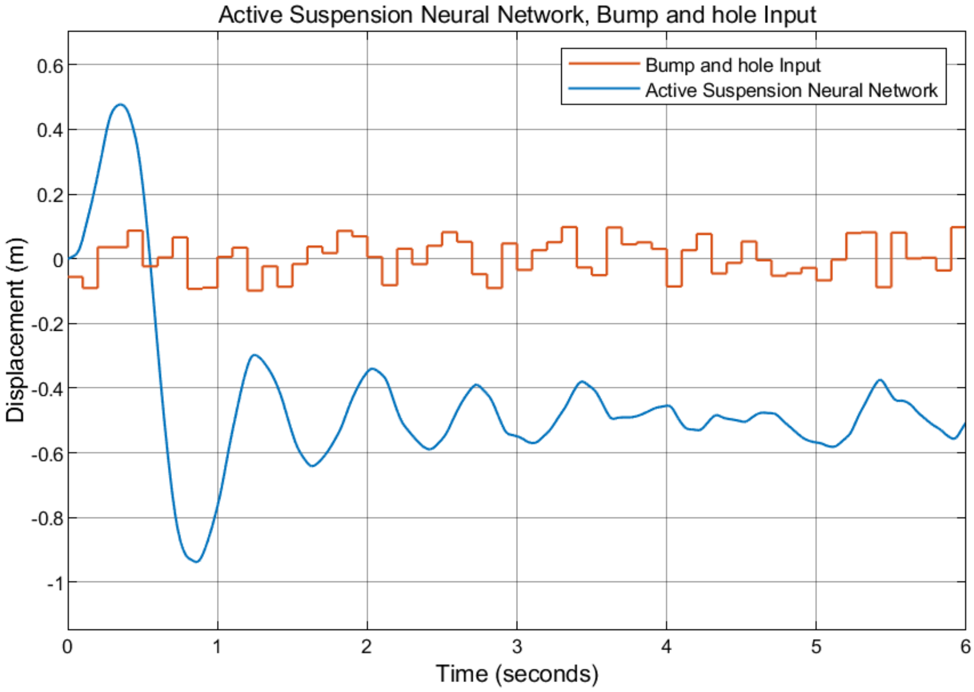


Figure 55. Vertical displacement of chasis using Predictive Controller

7.3. Conclusions

In this section, two different controls have been implemented for the model of a quarter car model. On the one hand, a conventional control methodology, the PID controller, and on the other hand an intelligent control methodology, the predictive control through neural networks, have been implemented.

It has been demonstrated that the PID control has been successfully implemented with ease, obtaining significant improvements in the vertical displacement of the centre of gravity of the chassis for all types of road.

However, the author has not been able to implement a predictive control through neural networks successfully, showing in part that the implementation of such systems is much more complex than that of conventional control systems.

8. CONCLUSIONS AND FUTURE WORK

In the present work three linear models of active suspension have been successfully implemented using a quarter car, half a car and a full car. It has been possible to check the operation of each of them by introducing different disturbances. Using the linear model of a quarter car, a sensitivity analysis has been carried out to estimate which suspension parameters are most important. Active session using quarter car, half car and full ca models. It has been possible to check the operation of each of them by introducing different disturbances. Using the linear model of a quarter car, a sensitivity analysis has been carried out to estimate which suspension parameters become more important.

In the following section a quarter car model of a McPherson suspension has been developed, to be able to represent the nonlinearities due to the kinematics that the suspensions have in real world. This model has been tested and compared with the quarter car model developed in the previous section. It has been found that both suspensions have similar, but not exact, behaviours. It is highlighted as a conclusion that the McPherson suspension multibody model corresponds more accurately to reality, but its computation time is unaffordable for real-time applications.

In the last section, a conventional PID control has been successfully implemented, showing that there is a notable improvement in the behaviour of the vertical dynamics of the vehicle with this controller. Later, an attempt was made to implement a neural network controller. A training set has been successfully generated and the neural network has been trained. But the result has been unsatisfactory. The results show that the system is stable, but it became stable around a wrong reference value, which shows that the training of the neural network has given erroneous results.

As a future work, it is proposed, first, to study further the available literature regarding the implementation of control through neural networks to make a correct application of this type of control.

Another goal that has been decided not to continue in this work, but which is without any doubt of great interest, is the implementation of control strategies, both conventional and intelligent, in a complete multibody model. A successful implementation of intelligent control methodologies, in a multibody model of a car, would allow simulations of great accuracy and usefulness to improve the active control of suspensions.

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