PROBABILISTIC FAILURE ANALYSIS FOR REAL GLASS COMPONENTS UNDER GENERAL LOADING CONDITIONS

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ABSTRACT

In this paper, the generalized local model (GLM) is applied to derive the primary failure cumulative distribution function (PFCDF) of annealed glass in order to achieve the failure prediction of structural glass. The uniqueness of the glass characterization is demonstrated irrespective of the test, specimen size and geometry used. Consequently, the strength of glass is unequivocally derived in a probabilistic way as a material property, so that the definition of normalized testing specimens in international standards might be put under question. Furthermore, the application of the GLM to the results assessment allows to ensure a correct transferability of the laboratory data from simple specimens to the practical design of real glass components and vice-versa. The feasibility of the GLM to characterize the strength of annealed glass from different test types is illustrated by means of an extensive experimental program.

Keywords: Structural Glass; Experimental Program; Probabilistic Design.

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Nomenclature

β Weibull shape parameter

δ Weibull scale parameter

λ Weibull location parameter

σ_I, σ_{II}, σ_{III} Principal stresses

4PS Four-Point Bending test (small)

4PS Four-Point Bending test (large)

CL Coaxial double ring test (large)

CS Coaxial double ring test (small)

GLM generalized local model

GP Generalized parameter (which the failure criterion is referred to)

GP_{ij} Generalized parameter at the element i for load condition j

GP_{max} Maximum value of the generalized parameter reached at failure conditions

PFCDF Primary failure cumulative distribution function

PIA Principle of independent action

$P_{fail_{ij}}$ Probability of failure for the element i for the load condition j

S_i Size of the element i

S_{ref} Reference size of the PFCDF

1. Introduction

The use of structural glass elements has increased and diversified into the construction industry, combining the concepts of sustainability, functionality and aesthetics with performance on roofs, facades and interiors of high or unique buildings.

Despite various studies done[1-3], some critical points that prevent a real understanding of the limit state of glass under different types of loading persist. Therefore, it is necessary to develop new rules, considering glass specific characteristic as a brittle material with the inevitable presence of micro-cracks that allow implementation safety and reliability conditions in the same conditions to other conventional structural materials such as steel, concrete or wood.

Nowadays, there are some standards to regulate the characterization of glass materials[2, 4]. Nevertheless, there is a certain lack in the regulation of glass design for structural proposes. Both the American code[5] and the Australian code[6] refer to glass as a panel not intended mainly for structural use, while in Europe there is no consensus for an Eurocode for glass.

The difficulty of stablishing a proper failure criterion for glass resides in the adequate description of the material behavior and in the precise choice of the reference parameter. In the literature there are several failure criteria based on the Elastic-Linear Fracture Mechanics depending on the comparison magnitude used and subsequently on its application to brittle materials, i.e. the one proposed by Erdogan and Sih[7, 8] based on local stress and strain concentrations instead of energy release rates[8-10]. Moreover, Lo et al.[11] propose a unified model considering a generalization of Irwin´s approach and Kalthoff and Podleschny[8, 9, 12] confirmed experimentally that the crack direction under mixed mode I-II conditions agrees with the one resulting of Erdogan and Sih approach.

Moreover, although shear stress is generally not considered in current failure models for glass, Reid[13] indicated that it might affect the glass strength in ring-on-ring tests. In addition, recently Yankelevsky[14] illustrated the problem of determining the strength distribution of glass studying a square plate subjected to bending but neglecting bulk flaws and Kinsella el al.[15] presented a numerical method for predicting the glass failure of small specimens under double ring bending test based on fracture mechanics and the weakest–link principle, assuming a preexisting population of surface cracks.

Furthermore, the strength of glass components presents a large dispersion, which is usually motivated by random introduction of surface defects during the manufacturing process, so for mechanical characterization is essential to use probabilistic methods, that includes the scale effect[1, 16]. In previous authors works[4, 17], the so-called generalized local model (GLM) was introduced to derive the primary failure cumulative distribution function (PFCDF) for a generalized parameter based on experimental data even from specimens with different geometry and size. Accordingly, the PFCDF of the generalized parameter may be interpreted as a material property allowing failure of specimens or components to be predicted independently of the specimen shape and size and test type selected for the experimental program.

The formulation of the GLM as an alternative to obtain the strength of materials opens a new perspective on the design and evaluation of experimental programs. Taking into account that the strength is a material property, it could be derived in a probabilistic way from any kind of test, and for this reason the definition of *standard specimens* on *international standards* does not make sense anymore.

On the other hand, it is important to notice that the geometry, size and load conditions of a structural component are commonly very different to that ones suggested by the standards to characterize the material. For this reason, the construction industries usually require to validate the strength of a component by testing it directly on the laboratory, in spite of characterizing the material by standard tests with small specimens. This requirement is mainly motivated by the ignorance of the correct way to overcome the scale effect, and causes an increment of the experimental procedure cost and adds considerable difficulty to the interpretation of results. Furthermore, the results obtained from such expensive tests are usually misspend because the lack of a proper standard methodology to derive the primary and fundamental information of the material behavior.

In this paper, a methodology based on the GLM to overcome all these limitations is presented, demonstrating that experimental results obtained with real components can be used to obtain the material failure characterization by means of the PFCDF and proving the validity of the GLM to predict the failure of real components of large dimensions subjected to complex load states from standardized experimental results.

The paper is organized as follows: Firstly, a brew introduction of the real case under study is introduced in order to illustrate the applicability of the model. After that, the material characterization according to standard specimens is summarized, and the experimental procedure related to specimens out of the scope of the standards is detailed. Then, the methodology to predict the probability of failure of any component is presented and applied to the case under study. Finally, a critical comparison of the results obtained in the laboratory and the models introduced in this paper is presented, and the main conclusions of the paper are highlighted.

2. Prediction of failure and determination of the PFCDF

2.1. Description of the case under study

With the aim of remaining as close as possible to the real conditions of a real glass component, a new series of tests are proposed. It is noticeable that the dimensions and load conditions used in this work are out of the scope of the standards recommendations[2], although they are closer to the real conditions than the smaller and simpler specimens proposed by them. In this case, a large rectangular plate (1200 x 1200 x 5 mm) under coaxial double rings loading conditions (see Figure 1 and Table 1) is studied.

Table 1. Dimensions of the Coaxial Large (CL) tests

Test	Dimensions [mm]			
CL	r_1=187.5	r_2=525	L = 1200	h = 5

Furthermore, due to the reduced thickness of the plate in comparison with its dimensions, a partial membrane (in-plane stress) behavior is expected in combination with the bending stress plate conditions. Moreover, the displacement ratio remains in the range of large deformation analysis. Thus, there is not analytical proposal to obtain the distribution of stresses analytically on the standards[2].

Figure 1 shows the difference of stress distributions expected under both small and large deformations for the coaxial double ring test[2]. The transferability of results between both cases has been considered until now an open issue, so standards suggest imposing a constant pressure over the surface involved by the small ring[3, 18-20] in order to get a constant distribution of stresses, which leads to an unnecessary increment of the difficulty and cost of the tests development.

For this reason, this paper shows a demonstration of the application of the GLM as a tool to ensure the transferability of results from standard specimens to real components and to prove the possibility to perform the material characterization using specimens and loading conditions not considered in the design standards.

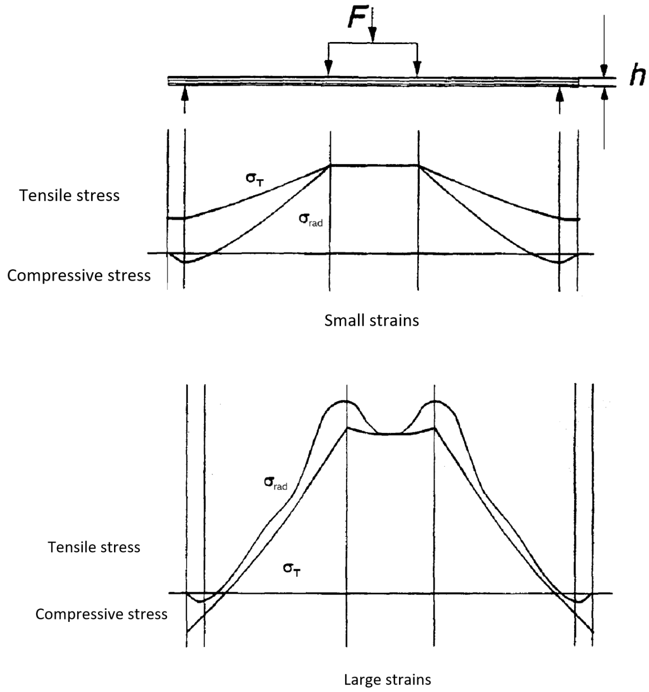

Figure 1. Stress profiles for small and large deformation cases in coaxial rings tests (UNE-EN 1288-1[21]).

2.1.1. Material Characterization

In previous works[1, 4, 17, 22], two experimental campaigns of four-point bending tests with different sizes (4PS-Small and 4PL-Large) and a coaxial double ring tests with small surface areas (CS), see Figure 2 and Table 2, have been carried out to characterize the stress resistance of the glass under study.

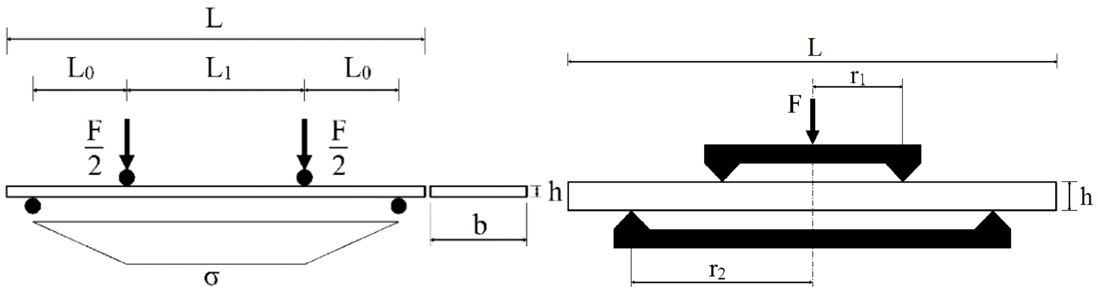

Figure 2. Four-point bending (left) and coaxial double ring tests (right).

Table 2. Experimental campaign used on the characterization of the annealed glass.

Type of test	Dimensions [mm]				Nº of specimens
Coaxial double ring (CS)	r_1=30	r_2=80	L = 250	h = 5	30
Four-point bending (4PS)	L_0 = 50	L_1 = 150	L = 300	b = 100	50
Four-point bending (4PL)	L_0 = 400	L_1 = 200	L = 1100	b = 360	30

Although all tests have been performed following the recommendations of UNE-EN 1288-3 (2000)[18] and UNE-EN 1288-5 (2000)[2], the Generalized Local Model (GLM)[22] was used to determine the material strength. That model is based on an iterative procedure[22] that allows obtaining, from any experimental program, the failure probability of a reference size subjected uniformly to a certain value of the generalized parameter

(GP), as defined by the fracture criterion. In this case, the GP is defined as a combination of the values of the principal stresses σ_I, σ_{II}, and σ_{III}:

$$GP = (\sigma_I^{2.5} + \sigma_{II}^{2.5} + \sigma_{II}^{2.5})^{1/2.5} \quad (1)$$

which is in concordance with other models, such as the Principle of Independent Action (PIA)[23, 24], and has been demonstrated to be a suitable failure criterion for this material.

Once the GP is defined, its distribution all over the specimens tested is obtained by finite element method. Following an iterative process[22], the unequivocal relation between the value of the GP and the probability of failure is established based on the Weibull model[25]:

$$P_{fail} = 1 - \exp\left[-\left(\frac{GP-\lambda}{\delta}\right)^{\beta}\right]; GP > \lambda \quad (2)$$

where λ, β and δ are the location, shape and scale parameters respectively. That relation is defined as the primary failure cumulative distribution function (PFCDF), which is considered a material property as being independent of the geometry, load conditions and size of the tests selected to perform the tests related to its derivation. Finally, it is worth to mention that Eq.(2) allows the probability of failure of any finite element on a mesh of a FEM study to be calculated, so that the prediction of failure for any type of component can be done by applying Eq.(2) in combination with a finite element model.

Finally, Figure 3 (left) shows the maximum values of the GP (GP_{max}) reached for each test from the different experimental programs[4, 8]. Although GP_{max} has been selected as the representative value to illustrate the scatter associated to each experimental program, the PFDCFs shown in Figure 3 (right) are obtained by the GLM, so that the whole distribution of the GP over all specimens is considered. As can be seen, the relation between the GP and the probability of failure obtained from the three different experimental programs is in close agreement.

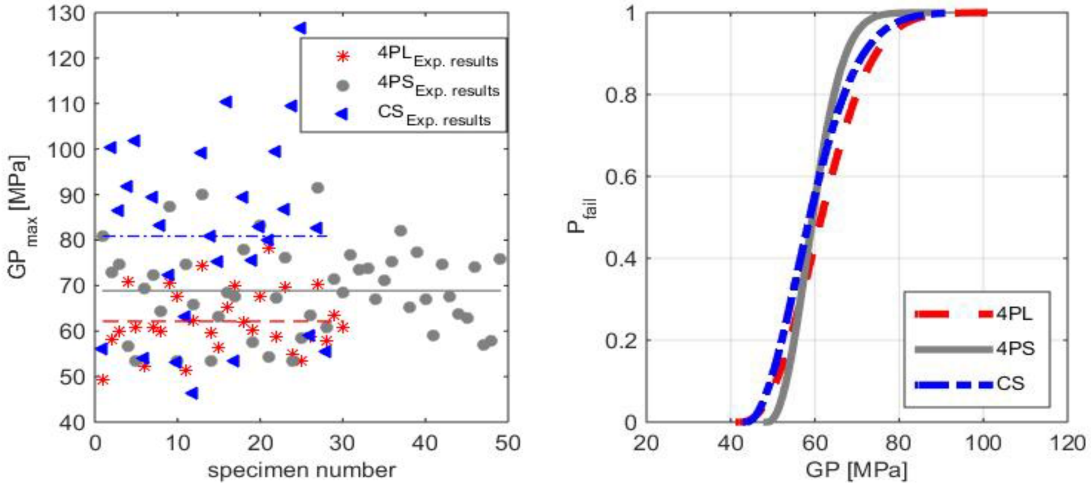

Figure 3. Experimental results and representation of the PFCDF for the annealed glass under study.

2.1.1. Experimental procedure

In order to validate the proposed methodology, a total of 30 specimens were tested in a Walter+bai (100 kN) general testing system in the experimental program (see Figure 4). During testing, load and displacement have been registered. Furthermore, six strain gages were placed on a radius from the mid-point of the plate to the external ring in order to obtain the stress distribution along this direction.

Figure 4. Test set-up for the Coaxial Large (CL) tests.

2.2. Prediction of failure

The hazard maps and the global probability of failure[14] for any load condition j could be obtained according to the weakest link principle:

$$P_{fail_j} = 1 - \prod(1 - P_{fail_{ij}}) = 1 - \prod \exp\left[-\frac{S_i}{S_{ref}}\left(\frac{GP_{ij}-\lambda}{\delta}\right)^{\beta}\right]; GP_{ij} > \lambda \qquad (3)$$

where λ, β, δ are the Weibull values associated to the PFCDF (Figure 3 (right)), whilst S_i and GP_{ij} are the size and Generalized Parameter at the element i for the load condition j.

As can be seen, in order to obtain a prediction of failure for the new geometry under study, a numerical model of that must be implemented. This has been done by using the commercial finite element software ABAQUS/Explicit v6.12. It was used a mesh with reduced integration continuum shell elements (SC8R) for the glass specimen[26], shell elements for the rubber bands and rigid solids for both loading and support rings. A detail of the mesh for the CL specimen is presented in Figure 5.

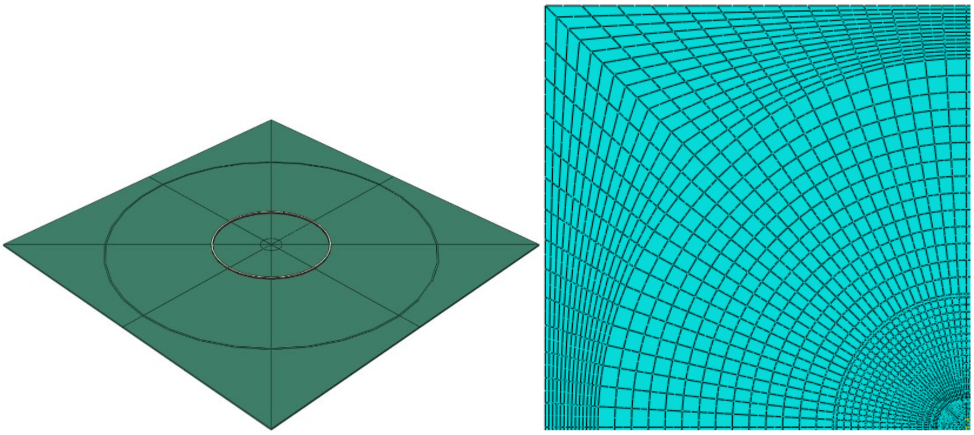

Figure 5. Model and detail of the mesh for the CL specimen.

According to the experimental procedure, the load is incrementally applied by imposing a vertical displacement to the internal ring, and the local principal stresses and element sizes are extracted by Abaqus2Matlab tool[27] for each load step in order to obtain the GP_{ij}, which allows us to compute the evolution of the global probability of failure according to Eq. (3) (see Figure **6**).

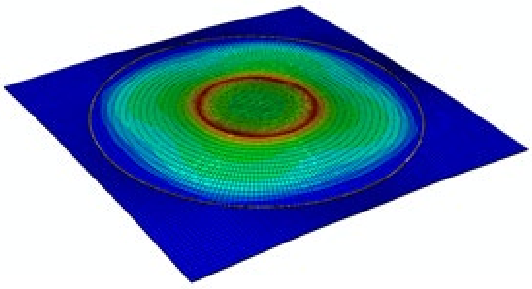

Figure 6. Model and detail of the mesh for the CL specimen

3. Experimental results

3.1. Validation of the prediction of failure

The FE model has been validated by comparing the load-displacement curve obtained experimentally and numerically. Moreover, Figure 7 shows a comparison between the stresses obtained from the six strain gauges placed in a radial direction (dashed lines) and those estimated with the finite element model (points). As can be seen, a good correlation can be observed, which confirms the validation of the model.

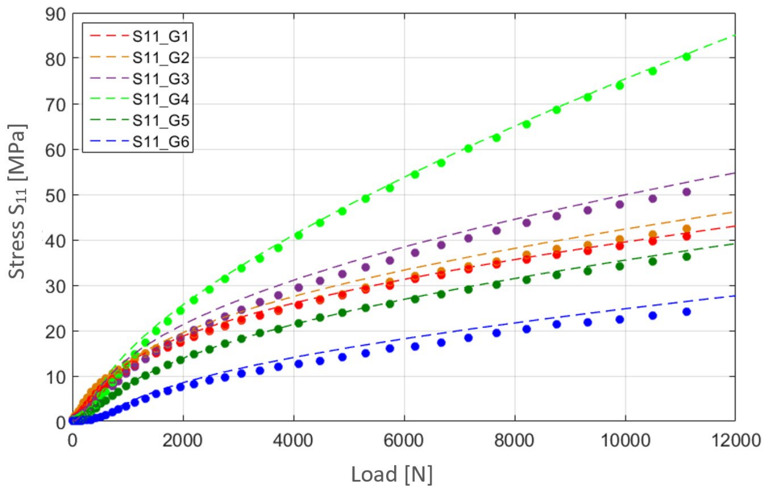

Figure 7. Experimental (dashed lines) and estimated (points) stresses for different radial points and different applied loads on the CL test.

In order to see the stress/strain profiles in the plate in relation to the loading, the experimental values for different time instants were plotted along the radial direction. Figure 8 shows the stress profiles for the plate being the lowest curve for the beginning of the test (load close to zero) and the highest one corresponding to approximately 16 kPa. From these stress profiles (Figure 8), it can be observed that the stress under the loading ring represents the maximum value, what is approximately the double of the value registered at the middle point of the plate. These differences are located entirely out of the standard[2] calculations where the assumption is that the stresses inside the loading ring should be practically constant. Furthermore, it is important to remark the high non-linearity showed by the stress profiles on the new problem under study, which is very different than the linear behavior present on the standard specimens used for the material characterization (section 2.1.1.)

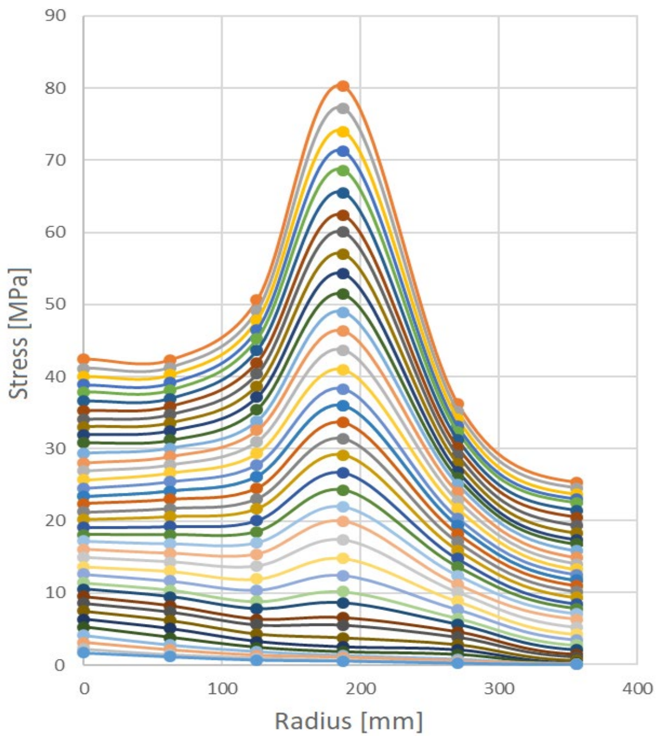

Figure 8. Experimental stress profiles for the plate along the radial direction.

Once the FE model can be considered valid, the prediction of failure made on the previous section must be compared with the experimental failures registered on the laboratory. Table 3 shows the main results obtained at the laboratory related to this experimental program, the actuator displacement and the critical load for each specimen at the failure moment. As can be seen at Figure 9, there is a good agreement between the predictions of failure, obtained by using Eq. (3) in combination with any of the previous PFCDF derived from standard specimens, and the experimental results obtained in the laboratory for the new case under study, which implies that a good transferability can be achieved by the Generalized Local Model, also in case of non-linearity.

Table 3. Experimental results from CL type testings

Specimen	*Displacement* [mm]	*Load* [N]	Specimen	*Displacement* [mm]	*Load* [N]
CL-01	28.53	38877	CL-16	19.76	18375
CL-02	21.85	23193	CL-17	21.05	21303
CL-03	30.02	44296	CL-18	25.93	33089
CL-04	21.13	21480	CL-19	17.47	13659
CL-05	17.45	13635	CL-20	30.14	44718
CL-06	28.14	37668	CL-21	24.75	30258
CL-07	15.53	10256	CL-22	20.56	20134
CL-08	20.20	19337	CL-23	20.54	20113
CL-09	23.87	28123	CL-24	23.93	28279
CL-10	16.82	12469	CL-25	29.21	41396
CL-11	15.62	10404	CL-26	24.72	30189
CL-12	23.43	27043	CL-27	25.02	30895
CL-13	14.20	8234	CL-28	28.82	39951
CL-14	16.61	12092	CL-29	23.16	26378
CL-15	11.05	4461	CL-30	13.19	6869

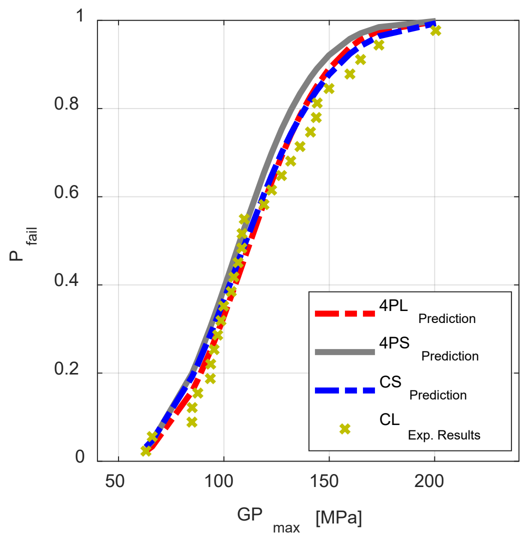

Figure 9. Prediction of failure of CL from the PFCDF obtained by tests developed under standard conditions.

3.2. Validation of the material characterization

According to the GLM, the PFCDF can be obtained from any type of experimental tests as a material property. Thus, the tests performed in this work, to validate the transferability of results from standard test to real components, should allow us to obtain the strength of the material as well as the standard ones.

Figure 10 and Table 4 show the PFCDFs obtained from each type of standard test and from the new geometry under study. As can be observed, there is a good correlation between the values obtained at this work and the previous reported by the authors, so it can be concluded that the PFCDF can be obtained from testing of real components, and it is not necessary to define a standard geometry to do that.

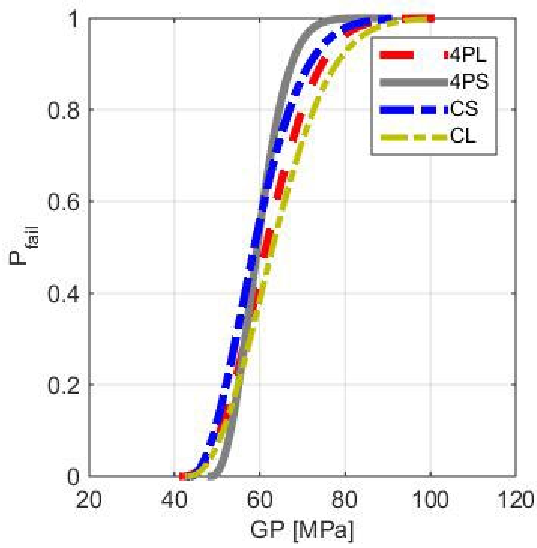

Figure 10. PFCDFs obtained from different type of tests.

Table 4. Weibull parameters for all the test cases.

Test	β	λ [MPa]	δ [MPa]
4PL	2.34	42.08	22.72
4PS	2.17	48.66	18.76
CS	2.01	43.76	18.12
CL	2.15	43.16	23.58

As a final validation of the PFCDF obtained in this work from the real component, the GLM can be used in the opposite direction, that is, to use this PFCDF derived from the real component (CL) for the prediction of the tests under standard conditions (4PL, 4PS, CS). The prediction of failure of the four bending and the double ring cases is performed again by means of the GP_{ij} obtained from finite element methods at each position of each test, and using Eq. (3) in combination with the PFCDF derived for the real component. The results are shown in Figure 11. This fact is relevant since it shows the possibility of the GLM method for using the real component results in order to obtain the PFCDF of the material whose failure characterization is being studied.

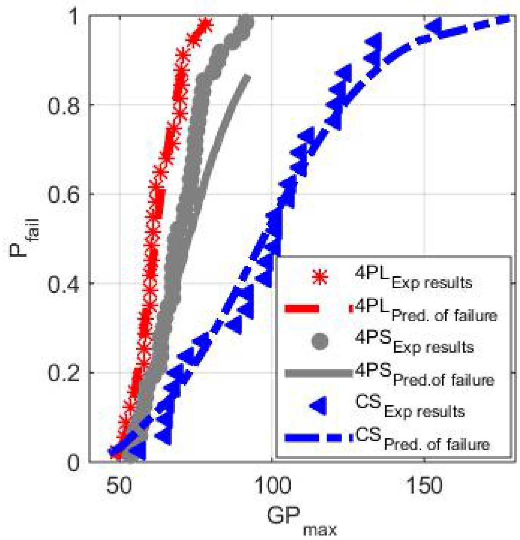

Figure 11. Prediction of failure of different test from the CL case.

4. Conclusions

The main conclusions to be drawn are the following:

1. A working methodology that allows the probabilistic characterization of structural glass has been developed considering glass brittle behavior and the inherent presence of surface defects.
2. Numerical models using the finite element method based on Shell Continuum elements have been developed, allowing a satisfactory simulation of the static tests to be performed and both uniaxial and biaxial states of stresses to be reproduced.

3. Failure criteria have been adopted according to the type of loading applied, proving suitable maximum principal stress criterion for the four-point bending tests and the modified Principle of Independent Actions (PIA) for coaxial double ring tests.
4. The so-called GLM has been applied to results evaluation of the experimental program. This probabilistic model, using Weibull three-parameter functions and adopting a suitable generalized parameter (GP), helps determining the PFCDF from the EFCDF, independently of the eventual complexity of the failure criterion selected, specimen dimensions or test type used.
5. It has been proved that the primary failure distribution function (PFCDF) determined by applying GLM to the results of different tests unequivocally represents a property of the material under study.
6. It has been proved that GLM application guarantees the transferability of laboratory tests results, with simple specimens, to the design of real structural glass elements.

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