BRNO UNIVERSITY OF TECHNOLOGY

FACULTY OF CIVIL ENGINEERING

Resumen español

Análisis de la tensión de una grieta en un disco brasileño

Maria Hevia Villanueva

Supervisor: assoc. prof. Stanislav Seitl, Ph.D.
Supervisor especialista: Ing. Petr Miarka

1. Introducción y objetivo

La durabilidad y la sostenibilidad de los materiales frágiles es un tema que está creciendo en interés para los ingenieros. Por lo tanto, es necesario encontrar un ensayo que pueda utilizarse en probetas cilíndricas y proporcionar información sobre los parámetros de fractura.

La resistencia a la tracción de los materiales frágiles, tales como el hormigón y la roca es, con mucho, su menor resistencia. En muchos ensayos la medida de la tensión en el punto de iniciación de la fractura es errónea por culpa de concentraciones de tensión debidas a condiciones superficiales irregulares. Un ensayo de tracción que no parece incluir estas deficiencias es el ensayo brasileño. Se trata de un ensayo de tracción indirecta en la que se induce un esfuerzo de tracción en un cilindro o disco sometido a compresión. La simplicidad del ensayo y preparación de las probetas es una ventaja adicional de este método.

El ensayo brasileño tiene varias ventajas prácticas y este trabajo se propone investigar la validez del ensayo como un medio para determinar los parámetros de fractura de un material frágil.

Una compresión simple no es adecuada para el estudio de la fractura, debido a la carga mixta (modo I + modo II). El ensayo brasileño, puede realizarse aplicando fuerzas de compresión en dos lados opuestos de un cilindro: esto provoca un esfuerzo de tracción uniforme en el plano que contiene el eje del cilindro y la generatriz, llevando al modo I de fractura. La ventaja de esta prueba es evitar costes excesivos de fabricación de las probetas.

En esta investigación, se han llevado a cabo algunas pruebas experimentales para estudiar el mecanismo de iniciación y propagación de grietas en especímenes que contienen grietas con diferentes ángulos de inclinación.

Los parámetros que se estudiarán son los factores de intensidad de tensión en la punta de la grieta para predecir el estado de estrés. Estos parámetros se calcularán creando un modelo numérico en el software de elementos finitos (FE) ANSYS. Para estudiar diferentes tamaños de grieta, el ratio a / R aumentará en un rango de 0,1 a 0,9 y el ángulo entre el plano horizontal y la grieta variará de 0º a 90º en intervalos de 5º.

Esta contribución también tiene como objetivo preparar una curva de calibración para la evaluación de los resultados experimentales obtenidos de los discos brasileños.

2. Fundamentación teórica

2.1 Ensayo Brasileño

El ensayo brasileño es un método indirecto para obtener la resistencia a la tracción de materiales frágiles, como el hormigón, la roca y materiales similares a rocas. En este ensayo, un disco es comprimido hasta que falla. En la figura 1 se muestran cuatro configuraciones de carga típicas.

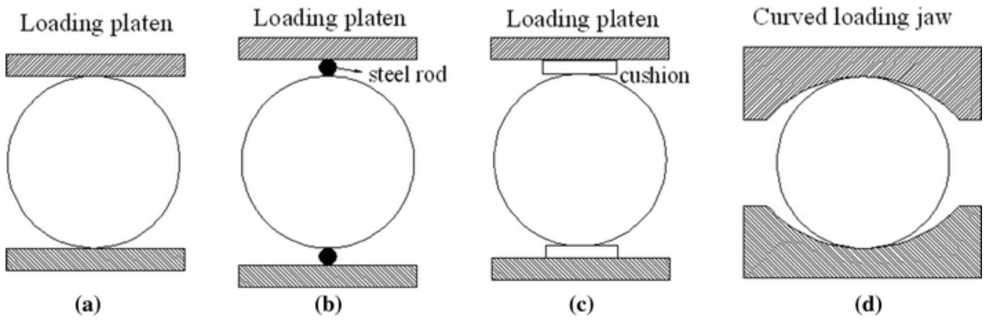


Figure 1. Configuraciones de carga típicas para el ensayo brasileño.

El ensayo tiene como objetivo determinar la resistencia a la tracción de probetas cilíndricas sometiéndolas a una fuerza de compresión aplicada en una franja estrecha sobre toda su longitud, por lo que la fuerza de tracción resultante provoca la rotura de la probeta.

Aunque el ensayo está destinado a realizarse en probetas cilíndricas, también es posible llevar a cabo el ensayo utilizando probetas prismáticas o cúbicas. En este caso, es necesario tener en cuenta los coeficientes de corrección para los resultados que proporciona la norma.

La fórmula sugerida para calcular la fuerza de tracción σ_t [MPa] basada en el ensayo brasileño es (ASTM 2008; ISRM 1978):

$$\sigma_t = \frac{2P}{\pi D t} = 0.636 \frac{P}{Dt} \qquad (1)$$

donde P es la carga a la rotura [N], D es el diámetro de la probeta [mm], y t es el espesor de la probeta medida en el centro [mm].

2.2 Mecánica de la fractura

La mecánica de la fractura es el campo de la mecánica que se ocupa del estudio de la propagación de las grietas en los materiales. Utiliza métodos analíticos de mecánica de sólidos para calcular la tensión que predomina en una grieta y métodos experimentales para caracterizar la resistencia a la fractura del material.

En 1920 Griffith observó que la baja resistencia de los materiales frágiles se debía a la existencia de pequeñas fisuras o grietas superficiales. La presencia de una grieta conduce a la concentración de tensiones en la punta de la misma, de modo que la tensión que actúa sobre ellos puede ser mucho mayor que la aplicada. La fractura ocurrirá cuando esa tensión exceda la resistencia del material.

Existen tres métodos de aplicación de la carga que provocan la propagación de una grieta.

- **Modo I** – Modo de apertura (se produce un esfuerzo tensional perpendicular a la grieta).
- **Modo II** – Modo de cizallamiento (Esfuerzos tangenciales actúan paralelos a las caras en la grieta pero en direcciones opuestas).
- **Modo III** – Modo de rasgado (Esfuerzos tangenciales que actúan paralelos pero perpendiculares a la cara de la placa y opuestos entre sí).

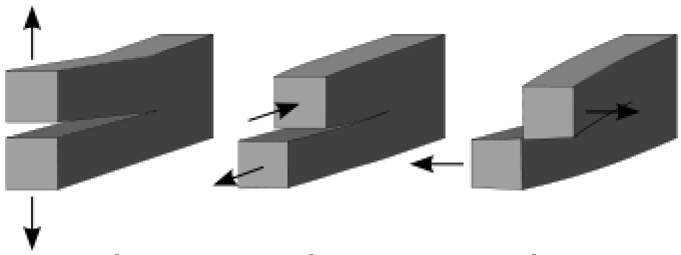


Figura 2. Tres modos de fractura

En teoría, la tensión en la punta de la grieta, donde el radio es casi cero, tendería al infinito. Esto sería considerado una singularidad de la tensión, que no es posible en aplicaciones del mundo real. En la práctica, se ha encontrado que la concentración de tensiones en la punta de una grieta dentro de materiales reales tiene un valor finito pero es mayor que la tensión nominal aplicada a la muestra.

2.3 Factor de Intensidad de Tensiones

El factor de intensidad de tensión, K, se utiliza en la mecánica de fractura para predecir el estado de tensión ("intensidad de tensión") cerca de la punta de la grieta causada por una carga remota o tensiones residuales. Es un parámetro teórico usualmente aplicado a un material elástico lineal homogéneo y es útil para proporcionar un criterio de rotura para materiales frágiles.

La magnitud de K depende de la geometría de la muestra, el tamaño y ubicación de la grieta, y la magnitud y la distribución de las cargas sobre el material.

La teoría elástica lineal predice que la distribución de tensiones (σ_{ij}) cerca de la punta de la grieta, en coordenadas polares (r, θ) con origen en la punta de la grieta, tiene la forma:

$$\sigma_{i,j}(r,\theta) = \frac{K_I}{\sqrt{2\pi r}} f_{ij}^{I}(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} f_{ij}^{II}(\theta) + T\delta_{1i}\delta_{1j} \qquad (2)$$

donde K es el factor de intensidad de tensión $[MPa\sqrt{m}]$ y f_{ij} es una cantidad adimensional que varía con la carga y la geometría. Esta relación se descompone muy cerca de la punta (r pequeño) porque como r tiende a 0, la tensión σ_{ij} tiende al infinito. La distorsión plástica ocurre típicamente con tensiones elevadas y la solución elástica lineal ya no es aplicable cerca de la punta de la grieta. Sin embargo, si la zona plástica de la punta de la grieta es pequeña, se puede suponer que la distribución de la tensión próxima a la grieta sigue siendo determinada por la relación anterior.

El factor de intensidad de tensión para el modo I se denomina K_I y se aplica al modo de apertura de grieta. El factor de intensidad de tensión de modo II, K_{II}, se aplica al modo de deslizamiento de grietas y el factor de intensidad de tensión de modo III, K_{III}, se aplica al modo de desgarramiento. Estos factores se definen formalmente como:

$$K_I = \lim_{r\to\infty} \sqrt{2\pi r}\, \sigma_{yy}(r,0) \qquad (3)$$

$$K_{II} = \lim_{r\to\infty} \sqrt{2\pi r}\, \sigma_{yx}(r,0) \qquad (4)$$

$$K_{III} = \lim_{r\to\infty} \sqrt{2\pi r}\, \sigma_{yz}(r,0) \qquad (5)$$

La tenacidad a la fractura (el factor de intensidad de tensión crítico), K_c, es una propiedad intrínseca del material, y es una medida de la energía requerida para crear una nueva area superficial en un material. Cuando el factor de intensidad de tensiones en la región de la punta de la grieta $K > K_c$, se supone que la fractura se inicia y se propaga hasta que K es menor que K_c. Los parámetros del factor de intensidad de tensiones y el indicador de tenacidad de material, K_c, y el límite de rendimiento, σ_Y, son importantes porque ilustran muchas cosas sobre el material y sus propiedades. Por ejemplo, si K_c es alto, entonces se puede deducir que el material es duro, mientras que si σ_Y es alto, se sabe que el material es más dúctil.

3. Modelo numérico

Para la creación del modelo numérico se utiliza el software de elementos finitos (FE) ANSYS. Ha sido modelado como un modelo 2D con condiciones de deformación plana.

3.1 Geometría

El diámetro del disco modelado es D = 100 mm. Para estudiar diferentes tamaños de grieta, la longitud relativa de la grieta a / R varía en un rango de 0,1 a 0,9.

3.2 Características del material

El modelo del material se creó como un elástico lineal con las siguientes propiedades que suelen ser para materiales de hormigón:

- Módulo de Young: E = 40 GPa
- Coeficiente de Poisson: ν = 0.2

3.3 Condiciones de contorno y carga

La probeta se carga con una fuerza constante P = 100 N en todos los modelos investigados. En la práctica, esta carga se aplicaría en dos puntos opuestos del diámetro del disco y el disco se giraría para variar el ángulo de la grieta. Para simplificar el modelado en ANSYS la carga se aplicará sólo en un punto del disco y el punto opuesto se fijará. Por lo tanto, las condiciones de contorno son:

• En el punto donde se aplica la carga, el desplazamiento en las direcciones x e y es 0.

• En el punto opuesto, la rotación y la distancia al centro del disco son fijas.

3.4 Malla

Aunque el tipo de elemento recomendado para un modelo de fractura bidimensional es PLANE2, el modelo numérico ha sido mallado con el tipo de elemento PLANE183 para tener en cuenta la singularidad de la punta de la grieta.

PLANE183 es 2-D de orden superior, puede ser un elemento de 8 o 6 nodos. PLANE183 tiene un comportamiento de desplazamiento cuadrático y es muy adecuado para modelar mallas irregulares.

3.5 Modelling the crack

La región más importante en un modelo de fractura es la región alrededor del borde de la grieta. Esta se llama punta de grieta en un modelo 2-D y frente de grieta en un modelo tridimensional. Esto se ilustra en la Figura 4.

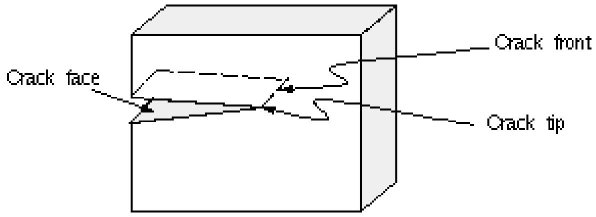


Figura 4. Punta de grieta (crack tip) y frente de grieta (crack front).

En problemas elásticos lineales, se ha demostrado que los desplazamientos cerca de la punta de la grieta varían en función $\sqrt{r}$, donde r es la distancia desde la punta de la grieta. Las tensiones y deformaciones son singulares en la punta de la grieta, variando en función de $1/\sqrt{r}$. Para simular la singularidad en la deformación, las caras de la grieta deben ser coincidentes, y los elementos alrededor de la punta de la grieta deben ser cuadráticos. Tales elementos se llaman elementos singulares. La figura 5 muestra ejemplos estos elementos para un modelo 2-D.

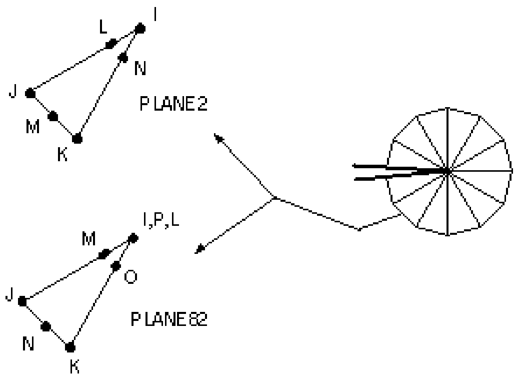


Figura 5. Ejemplo de elementos singulares para el modelo 2-D.

La primera fila de elementos alrededor de la punta de la grieta debería ser singular. El comando PREP7 **KSCON** es particularmente útil en un modelo de fractura. Genera automáticamente elementos singulares alrededor del punto especificado. Otros campos del comando permiten controlar el radio de la primera fila de elementos, el número de elementos en la dirección circunferencial, etc. La Figura 6 muestra un modelo de fractura generado con la ayuda de **KSCON**.

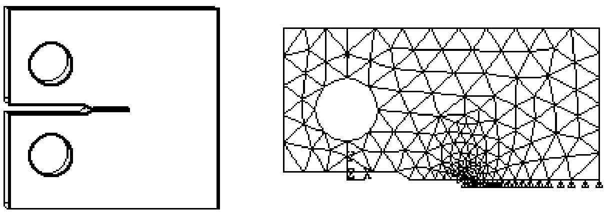


Figura 6. Una muestra de fractura y su modelo 2-D FE.

3.6 Stress Intensity Factors

El comando POST1 **KCALC** calcula los factores de intensidad de tensión en modo mixto K_I, K_{II} y K_{III}. Este comando se limita a problemas elásticos lineales con un material homogéneo e isotrópico cerca de la región de la grieta.

Para usar **KCALC** de manera adecuada han de seguirse los siguientes pasos.

1. Definir un sistema local de coordenadas en la punta de grieta, con X paralelo a la cara de la grieta y Y perpendicular a la grieta, como se muestra en la Figura 7.

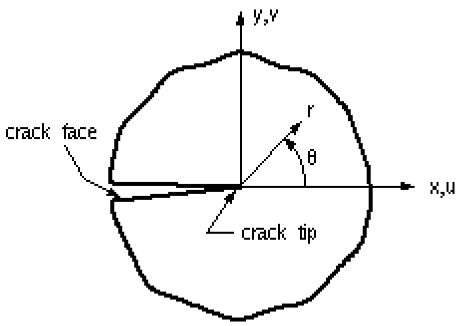


Figure 7. Sistema de coordenadas de la grieta para un modelo 2-D.

2. Definir una trayectoria a lo largo de la cara de la grieta. El primer nodo en la trayectoria debe ser el nodo de la punta. Para un modelo de grieta completo, donde se incluyen ambas caras de la grieta, se requieren cuatro nodos adicionales: dos a lo largo de una cara de la grieta y dos a lo largo de la otra. La Figura 8 ilustra los dos casos para un modelo 2-D.

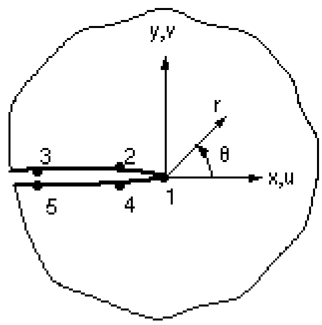


Figura 8. Definiciones de trayectoria típicas en un modelo de grieta completo.

3. Calcular K_I, K_{II} y K_{III}. (KCALC, KPLAN, MAT, KCSYM, KLOCPR). El campo KPLAN del comando KCALC específica si el modelo es deformación plana o de tensión plana. El campo KCSYM específica si el modelo es un modelo de media grieta con condiciones de simetría, un modelo de media grieta con condiciones de anti-simetría o un modelo de grieta completa.

4. Resultados

4.1 Factores de intensidad de tensión

Los siguientes gráficos muestran los resultados obtenidos mediante el uso del software de elementos finitos ANSYS. En las Figuras 9 – 10, se muestra la dependencia de los valores del factor de intensidad de tensión y el ángulo de la grieta bajo la misma carga P = 100 N.

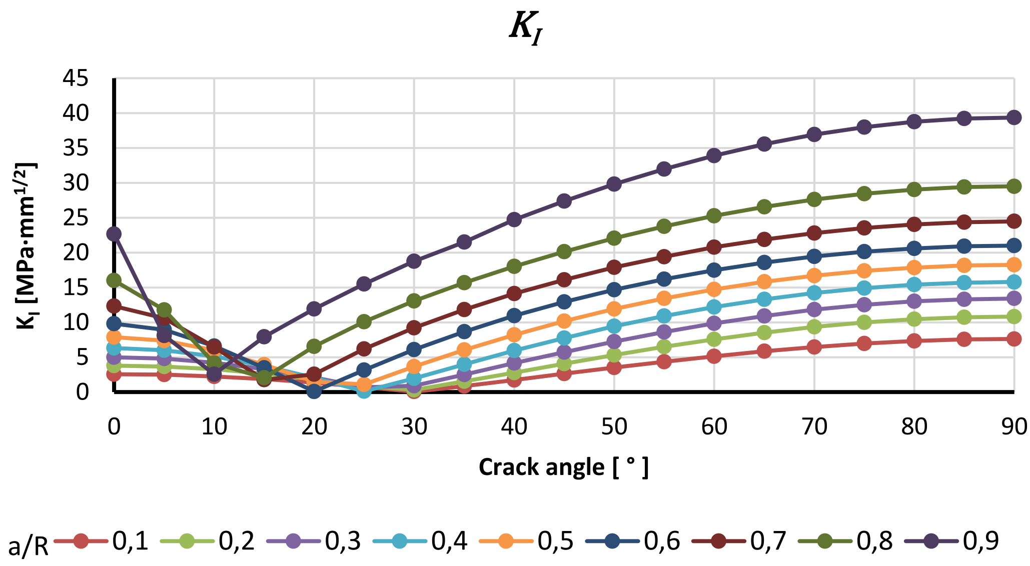


Figura 9. Factor de intensidad de tensión para el modo I frente al ángulo.

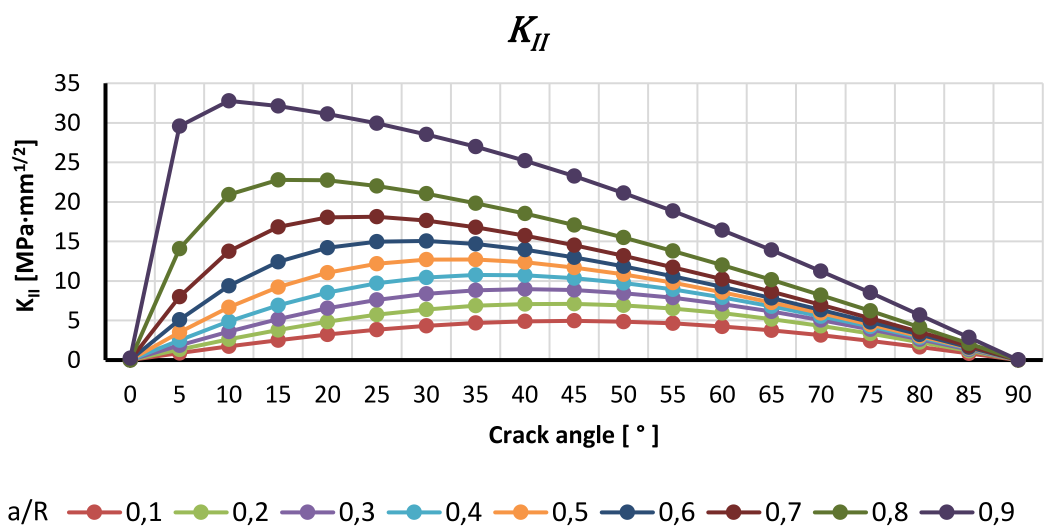


Figura 10. Factor de intensidad de tensión para el modo II frente al ángulo.

4.2 Curvas de calibración

A partir de la geometría mencionada anteriormente, las curvas de calibración se pueden calcular usando las siguientes ecuaciones:

$$f_I(\alpha) = \frac{K_I\sqrt{\pi}RB}{P\sqrt{a}}\sqrt{1-\frac{a}{R}} \qquad (6)$$

$$f_{\mathrm{II}}(\alpha) = \frac{K_{II}\sqrt{\pi}RB}{P\sqrt{a}}\sqrt{1-\frac{a}{R}} \qquad (7)$$

Al sustituir los valores de los factores de intensidad de tensión obtenidos de ANSYS y la geometría de la probeta se pueden obtener las curvas de calibración para diferentes ángulos y diferente tamaño de grieta como se muestra en las figuras 11 y 12.

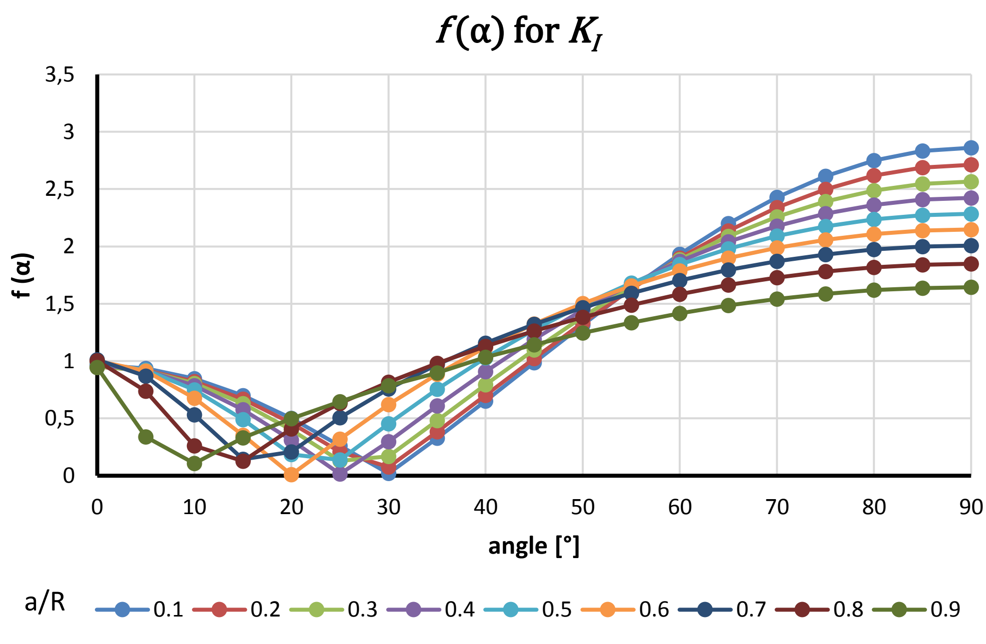


Figura 11. Curva de calibración $f_I(a/R)$ (modo I) para varios ángulos α y ratio a/R.

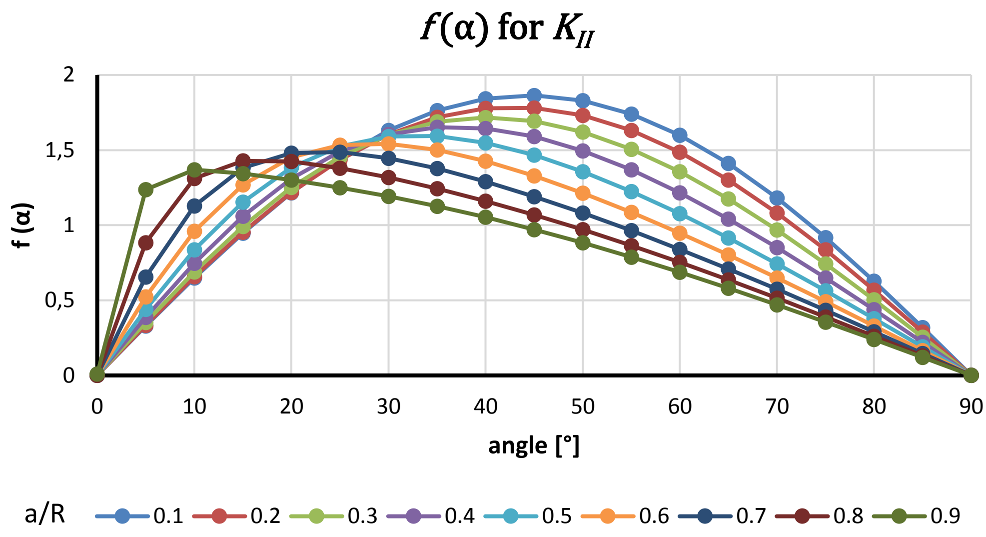


Figura 12. Curva de calibración $f_{II}(a/R)$ (mode II) para varios ángulos α y ratio a/R.

4.3 Intensidad de tensión

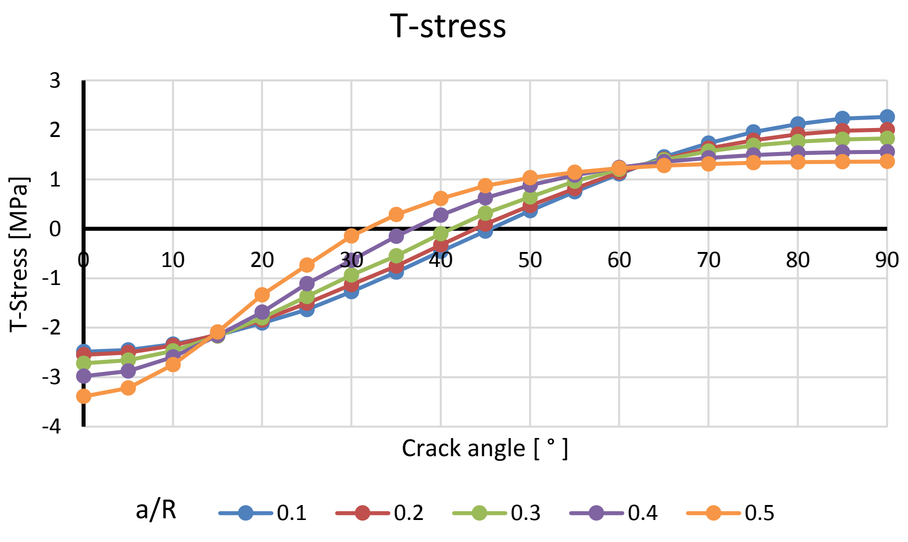


Figura 13. Curva intensidad de tensión.

A partir de la curva de intensidad de tensión se puede observar que la tensión cambia los valores de negativo a positivo y los que valen cero están próximos al modo de cizallamiento puro.

5. Discussion

Los gráficos muestran que para cada tamaño de grieta el factor de intensidad de tensión bajo el modo I tiende a 0 para un ángulo particular. Esto significa que no hay tensión de tracción perpendicular al plano de la grieta, por lo que sólo existe esfuerzo de cizalla actuando sobre la grieta en ese ángulo.

En la primera figura, la carga se aplica en el plano horizontal (ángulo 0 °), por lo que sólo existe tensión de tracción normal (modo I) y ambas caras tienen los mismos valores de intensidad de tensión.

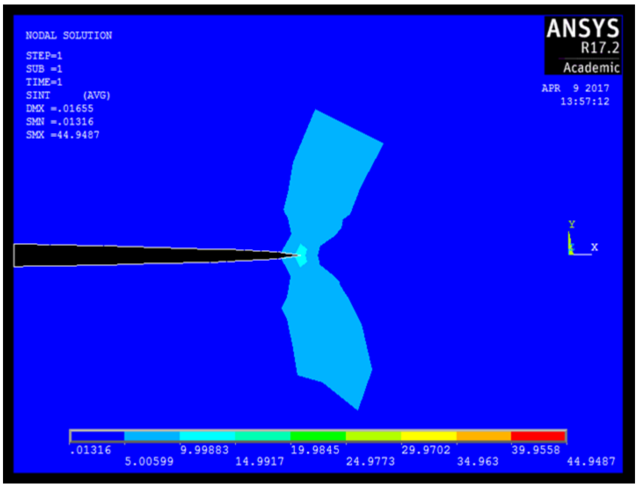


Figure 14. Intensidad de tensión en la punta de la grieta para: $a/R = 0.4$ α=0°

Por el contrario, la Figura 15 muestra el campo de intensidad de tensión donde K_I es casi 0 y sólo hay esfuerzo de cizalla (modo II).

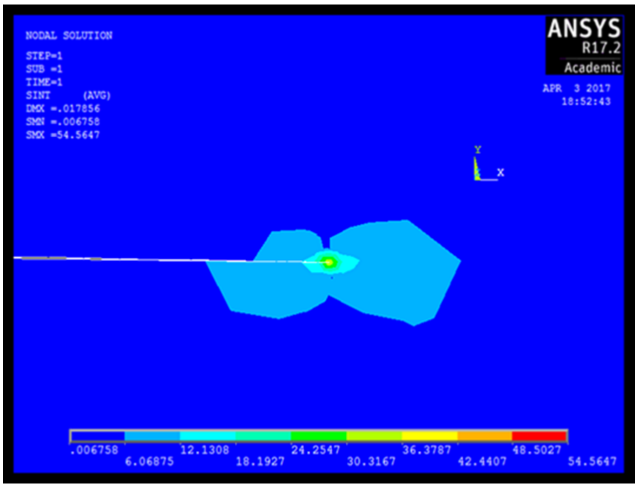


Figure 15. Intensidad de tensión en la punta de la grieta para: $a/R = 0.4$ α=25°

En la figura 16, se puede observar que cada cara de la grieta tiene diferentes valores de tensión, esto significa que la grieta está en modo mixto (modo I + modo II).

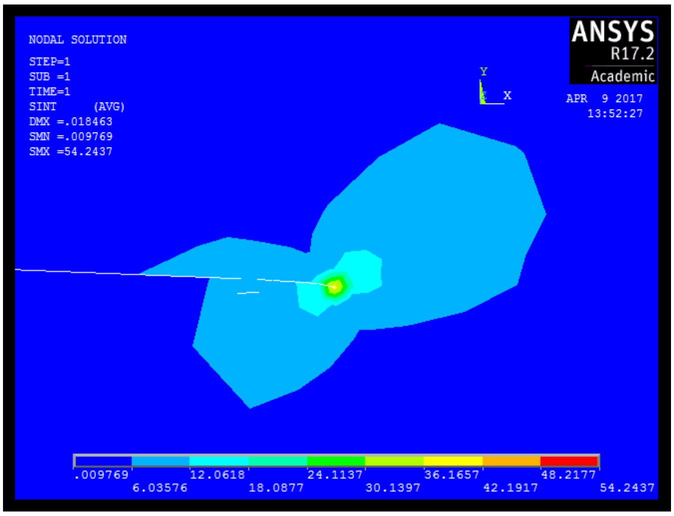


Figure 1. Intensidad de tensión en la punta de la grieta para: $a/R = 0.4$ α=45°

6. Conclusion

Se han realizado análisis de la mecánica de la fractura para analizar el comportamiento de una grieta en un disco sometido a compresión. Basándose en estos análisis, se propone que un simple ensayo brasileño, que se utiliza normalmente para la determinación de la resistencia a la tracción, puede usarse para medir la tenacidad a la fractura de materiales frágiles.

La comparación entre los valores de tenacidad a la fractura obtenidos mediante otros métodos ha demostrado que el ensayo brasileño es un método alternativo prometedor para la medición de la resistencia a la fractura de materiales tipo roca. Se pueden observar varias ventajas del ensayo brasileño propuesto.

- La probeta para un ensayo brasileño es más simple que la de cualquier método disponible de cálculo de la tenacidad a la fractura.
- Es posible realizar la prueba utilizando una máquina convencional de compresión. No se requiere equipo complicado o adicional.
- La interpretación de los resultados es sencilla solo se requiere un cálculo simple.
- La prueba nos permite obtener información sobre ambos modos (modo I + modo II) de fractura.

En las muestras de disco con grieta central, las alas de la grieta se propagan en una trayectoria curvada y continúan su crecimiento en una dirección (aproximadamente) paralela a la dirección de la carga de compresión máxima. Estas alas se inician en las puntas de la grieta para todos los ángulos de inclinación de grieta mayores de 15 °. Debe tenerse en cuenta que las alas de la grieta pueden no iniciarse desde las puntas de la grieta cuando el ángulo de inclinación está cerca de 90 ° (ángulo recto respecto a la línea de compresión aplicada). Por otro lado, el espécimen puede fallar debido al efecto de tracción indirecta exactamente igual que un disco sin grieta sometido a un ensayo brasileño convencional.

Stress Analysis of Crack in Brazilian Disc

Maria Hevia Villanueva

Supervisor: assoc. prof. Stanislav Seitl, Ph.D.
Supervisor specialist: Ing. Petr Miarka

May 2017

Abstract

Durability, sustainability of quasi-brittle materials is getting more into an interest of civil engineers. Therefore it is necessary to find out test method that could be used on core drill specimen and give us information about behavior under the mixed mode. For evaluation of concrete fracture properties, fracture toughness, compliance function, values of double-K model etc., the calibration curves are used. The contribution is focused on numerical model of Brazilian disc specimen with notch. Numerical models are prepared by using finite element s software ANSYS.

Abstracto

La durabilidad y la sostenibilidad de los materiales frágiles es cada vez más un interés de los ingenieros civiles. Por lo tanto, es necesario encontrar un método de ensayo que podría utilizarse para una probeta cilíndrica y darnos información sobre el comportamiento en el modo mixto. Para la evaluación de las propiedades de la fractura del hormigón, la tenacidad a la fractura, valores del modelo de doble K, etc., se utilizan las curvas de calibración. Esta contribución se centra en el modelo numérico de una probeta de disco Brasileño con una grieta en el centro. El modelo numérico se prepara utilizando el software de elementos finitos ANSYS.

Keywords

Brazilian disc, fracture mechanics, numerical study, calibration curve, T-stress, constraint, mixed mode, K_I/K_{II}

Palabras clave

Disco brasileño, mecánica de la fractura, estudio numérico, curva de calibración, T-stress, restricción, modo mixto, K_I / K_{II}

Citation of thesis

Maria Hevia Villanueva, Stress Analysis of Crack in Brazilian Disc. Brno, 2017. 39 p., Bachelor thesis. Brno University of Technology, Faculty of Civil Engineering, Institute of Structural Mechanics. Supervisor assoc. prof. Stanislav Seitl, Ph.D., Supervisor specialist Ing. Petr Miarka

Acknowledgment

The author acknowledges the support of Czech Sciences foundation project No. 16-18702S. This thesis has been carried out under the project No. LO1408 "AdMaS UP – Advanced Materials, Structures and Technologies", supported by Ministry of Education, Youth and Sports under the „National Sustainability Programme I".

Table of contents

7. Introduction

Durability and sustainability of quasi-brittle materials are increasingly becoming an interest for civil engineers. Therefore, it is necessary to find out a test method that could be used on core drill specimens and provide information about fracture parameters.

The tensile strength of brittle materials, such as concrete and rock, is by far their lowest strength, the knowledge of the tensile strength of brittle materials is, therefore, a prerequisite for the design engineer. In many tests e. g. bending beam, direct pull, disc with concentric hole, the stress level at the point of fracture initiation is obscured by the presence of extraneous bending effects and/or stress concentrations due to irregular surface conditions. A form of tensile testing which does not appear to include these shortcomings is the Brazilian test. The Brazilian test is an indirect tensile test in which a tensile stress is induced across the loaded diametral plane of a cylinder or disc subjected to compression. The simplicity of specimen preparation and testing is a further advantage of this test.

The bending test has some problems. The stress-gradient is linear, however some other problems appear such as the self-weight of the specimen (which is relatively large for the size of the specimens needed), that leads to uncontrolled fracture at the end of a test. Moreover, crushing damage near the supports may occur due to local compressive stress concentrations, and frictional restraint in the roller support affects the free lateral displacement of the supports. As a consequence confinement of the growing crack, or neglect of energy dissipation outside the crack zone may result in erroneous values of the fracture energy of the material from such tests.

The Brazilian test has several practical advantages and this paper sets out to probe the validity of the test as a means of determining the fracture parameters of a brittle material. The modified Griffith's fracture theory is used to predict fracture initiation in a loaded disc with notch and it is shown that failure must originate at the center of the disc if the test is to be a valid tensile test. It is also shown that under certain conditions fracture can initiate at the loading points and thereby invalidate the test. The results of these tests suggest that it is possible to establish the validity of each test by examining the specimen after failure.

8. Aim

A simple compression is not suited for crack studies, because of mixed loading (mode II + compressive mode I cracking). The cylinder splitting test, known as the Brazilian test, can be carried out by applying compressive forces on two opposite sides of a cylinder: this causes a uniform tensile stress on the plane containing the axis of the cylinder and the generatrix, leading to mode I cracking. The advantage of this test is to avoid expensive and random machining of brittle samples. This study shows that the Brazilian test is well adapted for the measurement of fracture parameters of materials whose room temperature behavior is brittle.

In this research, some experimental works have been established to study the mechanism of crack initiation and crack propagation emanating from CSBDC [¡Error! No se encuentra el origen de la referencia.] specimens containing different crack inclination angles. Figure 2Figure 2 shows an example of the geometry of the crack in a Brazilian disc.

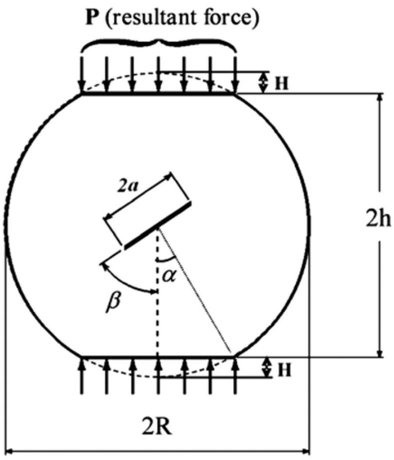


Figure 2. Geometry of Brazilian disc

The parameters that will be studied are the stress intensity factors at the crack tip in order to predict the stress state. These parameters will be calculated creating a numerical model in the finite element (FE) software ANSYS [9]. In order to study different crack sizes the ratio of the relative crack length *a/R* will increase in a range from 0.1 to 0.9, and the angle between the horizontal plane and the crack will vary from 0° to 90° in intervals of 5°.

The contribution also aims to prepare a calibration curve for the evaluation of experimental results obtained from Brazilian disc tests. The calibration curves are prepared for a selected angle between notch and load points.

9. Theoretical background

9.1 Brazilian test

The Brazilian test is a simple indirect testing method to obtain the tensile strength of brittle materials such as concrete, rock, and rock-like materials. In this test, a thin circular disc is diametrically compressed to failure. Four typical loading configurations are shown in Figure 3Figure 3.

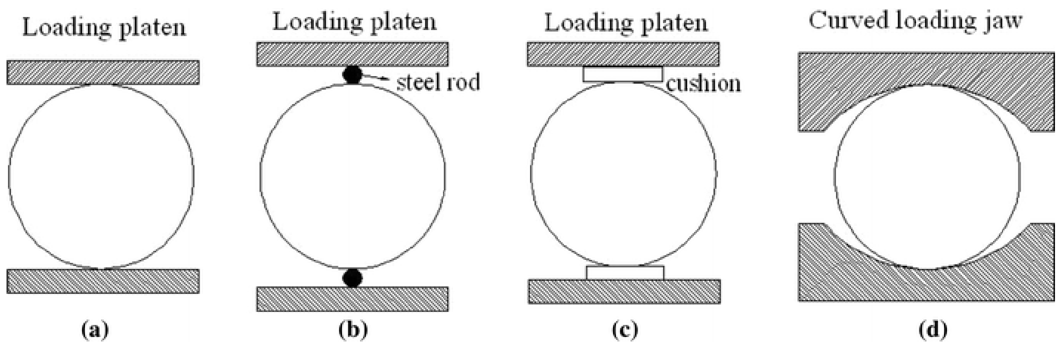


Figure 3. Typical Brazilian tensile test loading configurations: a) flat loading platens, b) flat loading platens with two small-diameter steel rods, c) flat loading platens with cushion, and d) curved loading jaws. Adopted from [2]

The test aims to determine the indirect tensile strength of cylindrical specimens by subjecting them to a compressive force applied in a narrow strip over its entire length, consequently the resulting tensile force causes the break of the specimen. The indirect tensile strength is typically calculated based on the assumption that failure occurs at the point of maximum tensile stress, i.e., at the center of the disc.

Although the test is intended to be carried out on cylindrical specimens, it is also possible to carry out the test using prismatic or cubic specimens. In this case, it is necessary to take into account the correction coefficients for the results that the standard provides.

The suggested formula for calculating the splitting tensile strength σ_t [MPa] based on the Brazilian test is (ASTM 2008; ISRM 1978):

$$\sigma_t = \frac{2P}{\pi D t} = 0.636 \frac{P}{Dt} \qquad (8)$$

where P is the load at failure [N], D is the diameter of the test specimen [mm], and t is the thickness of the test specimen measured at the center [mm].

9.2 Fracture Mechanics

Fracture mechanics is the field of mechanics concerned with the study of the propagation of cracks in materials. It uses methods of analytical solid mechanics to calculate the driving force on a crack and those of experimental solid mechanics to characterize the material's resistance to fracture.

In 1920 Griffith observed that the low strength of brittle materials was due to the existence of small fissures or superficial cracks. The presence of a crack leads to the concentration of stresses at the tip of it, so that the stress acting on them can be much greater than the applied one. The fracture will occur when that stress exceeds the strength of the material.

There are three ways of applying a force to enable a crack to propagate:

- **Mode I fracture** – Opening mode (tensile stress normal to the plane of the crack).
- **Mode II fracture** – Sliding mode (shear stress acting parallel to the plane of the crack and perpendicular to the crack front).
- **Mode III fracture** – Tearing mode (shear stress acting parallel to the plane of the crack and parallel to the crack front).

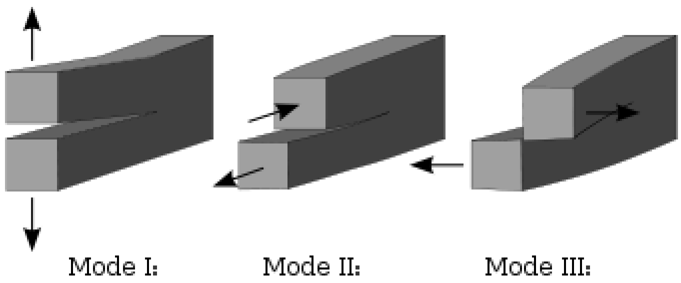


Figure 4. The three fracture modes

In theory the stress at the crack tip where the radius is nearly zero, would tend to infinity. This would be considered a stress singularity, which is not possible in real-world applications. In practice, the stress concentration at the tip of a crack within real materials has been found to have a finite value but larger than the nominal stress applied to the specimen.

9.3 Stress Intensity Factor

The stress intensity factor, K, is used in fracture mechanics to predict the stress state ("stress intensity") near the crack tip caused by a remote load or residual stresses. It is a theoretical parameter usually applied to a homogeneous, linear elastic material and it is useful for providing a failure criterion for brittle materials.

The magnitude of K depends on sample geometry, the size and location of the crack, and the magnitude and the modal distribution of loads on the material.

Linear elastic theory predicts that the stress distribution (σ_{ij}) near the crack tip, in polar coordinates (r, θ) with origin at the crack tip, has the form:

$$\sigma_{i,j}(r,\theta) = \frac{K_I}{\sqrt{2\pi r}} f_{ij}^{I}(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} f_{ij}^{II}(\theta) + T\delta_{1i}\delta_{1j} \qquad (9)$$

where K is the stress intensity factor $[MPa\sqrt{m}]$ and f_{ij} is a dimensionless quantity that varies with the load and geometry. This relation breaks down very close to the tip (small r) because as r goes to 0, the stress σ_{ij} goes to infinity. Plastic distortion typically occurs at high stresses and the linear elastic solution is no longer applicable close to the crack tip. However, if the crack-tip plastic zone is small, it can be assumed that the stress distribution near the crack is still given by the above relation.

The stress intensity factor for mode I is designated K_I and applied to the crack opening mode. The mode II stress intensity factor, K_{II}, applies to the crack sliding mode and the mode III stress intensity factor, K_{III}, applies to the tearing mode. These factors are formally defined as:

$$K_I = \lim_{r\to\infty} \sqrt{2\pi r}\, \sigma_{yy}(r,0) \qquad (10)$$

$$K_{II} = \lim_{r\to\infty} \sqrt{2\pi r}\, \sigma_{yx}(r,0) \qquad (11)$$

$$K_{III} = \lim_{r\to\infty} \sqrt{2\pi r}\, \sigma_{yz}(r,0) \qquad (12)$$

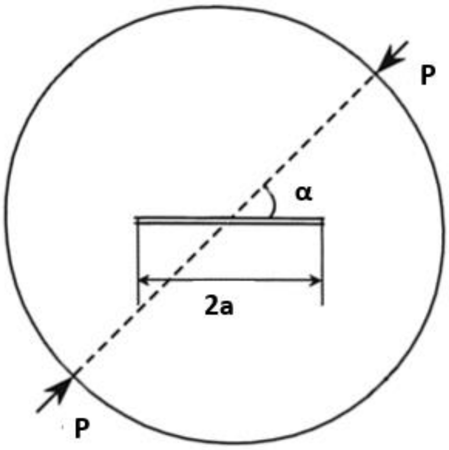
Fracture toughness (the critical stress intensity factor), K_c, is an intrinsic material property, and is a measure of the energy required to create a new surface area in a material. When the stress intensity factor in the region of the fracture tip $K > K_c$, the fracture is presumed to initiate and propagate until K, is less than K_c. The parameters of the stress intensity factor and indicator of material toughness, K_C, and the yield stress, σ_Y, are of importance because they illustrate many things about the material and its properties. For example, if K_C is high, then it can be deduced that the material is tough, whereas if σ_Y is high, one knows that the material is more ductile.

10. Numerical model

For creating a numerical model the finite element (FE) software ANSYS is used. It has been modeled as a 2D model with plane strain conditions.

10.1 Geometry

The modeled disc diameter D = 100 mm. In order to study different crack sizes the ratio of relative crack length a/R varies in a range from 0.1 to 0.9.



10.2 Material characteristics

The material model was created as a linear-elastic with the following properties which are usually for concrete materials:

- Young's modulus: E = 40 GPa
- Possion's ratio: v = 0.2

10.3 Boundary conditions and load

The specimen is loaded with constant force P=100 N in all investigated models. In practice this load would be applied at two opposite points of the disc diameter and the disc would be rotated to vary the angle of the crack. To simplify the modeling in ANSYS the load will be applied only at one point of the disc and the opposite point will be fixed. So the boundary conditions are:

- At the point where the load is applied the displacement in x and y directions is constrained
- At the opposite point the rotation and distance to the center of the disc are fixed.

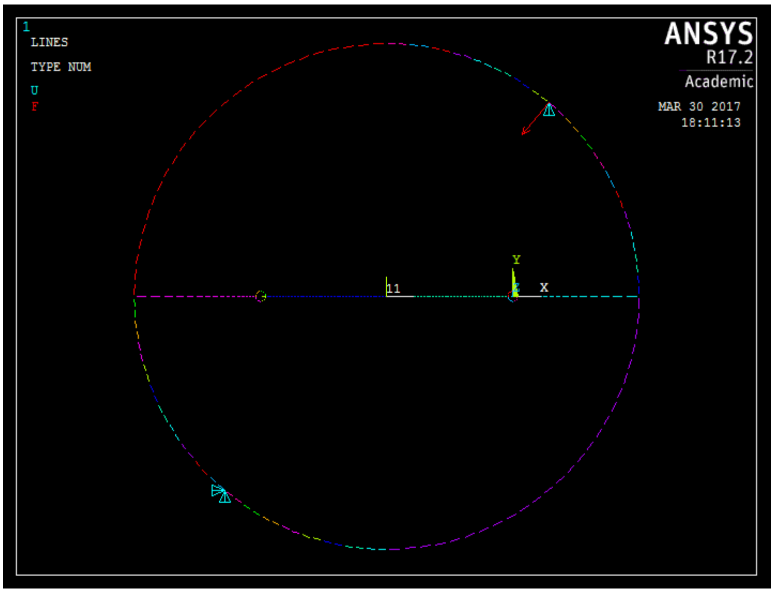


Figure 5. Load and boundary conditions in ANSYS for Brazilian disc

10.4 Mesh

Although the recommended element type for a two-dimensional fracture model is PLANE2, the six-node triangular solid, the numerical model has been meshed with element type PLANE183 in order to take into account the crack tip singularity.

PLANE183 is a higher order 2-D, it can be 8-node or 6-node element. PLANE183 has quadratic displacement behavior and is well suited to modelling irregular meshes.

This element is defined by 8 nodes or 6 nodes having two degrees of freedom at each node: translations in the nodal x and y directions. The element may be used as a plane element (plane stress, plane strain and generalized plane strain) or as an axisymmetric element.

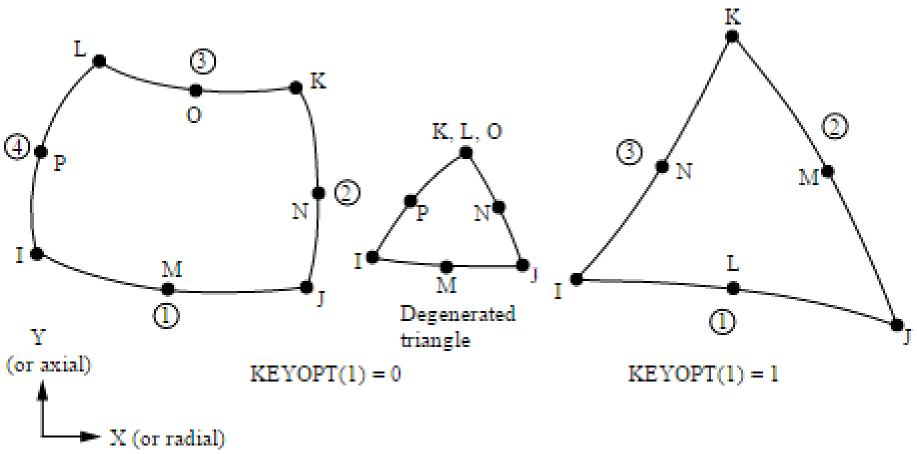


Figure 6. Element type PLANE183 adopted from [9].

10.5 Modelling the crack

The most important region in a fracture model is the region around the edge of the crack. It is called ***crack tip*** in a 2-D model and ***crack front*** in a 3-D model. This is illustrated in Figure 7. **Crack tip and crack front**Figure 7.

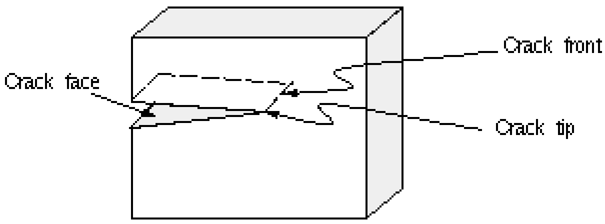


Figure 7. Crack tip and crack front adopted from [9].

In linear elastic problems, it has been shown that the displacements near the crack tip (or crack front) vary as $\sqrt{r}$, where r is the distance from the crack tip. The stresses and strains are singular at the crack tip, varying as $1/\sqrt{r}$. To pick up the singularity in the strain, the crack faces should be coincident, and the elements around the crack tip (or crack front) should be quadratic, with the mid-side nodes placed at the quarter points. Such elements are called ***singular elements***. Figure 8 shows examples of singular elements for 2-D model.

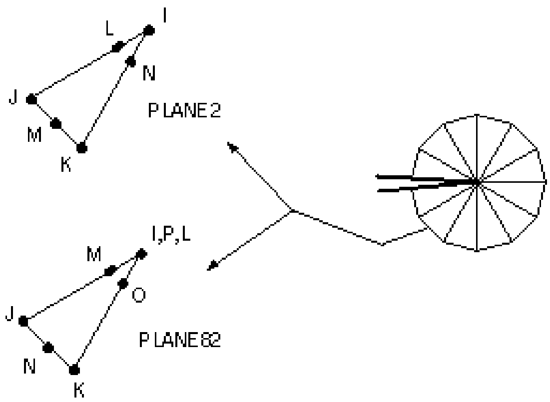


Figure 8. Examples of singular elements for 2-D models adopted from [9].

The first row of elements around the crack tip should be singular, as illustrated in Figure 8Figure 8. The PREP7 **KSCON** command, which assigns element division sizes around a keypoint, is particularly useful in a fracture model. It automatically generates singular elements around the specified keypoint. Other fields on the command allow you to control the radius of the first row of elements, number of elements in the circumferential direction, etc. Figure 9Figure 9 shows a fracture model generated with the help of **KSCON**.

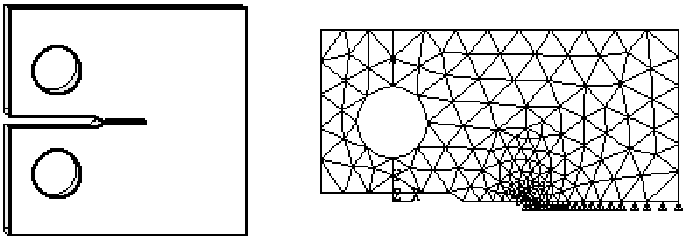


Figure 9. A fracture specimen and its 2-D FE model adopted from [9].

The following images are examples from ANSYS of the numerical model:

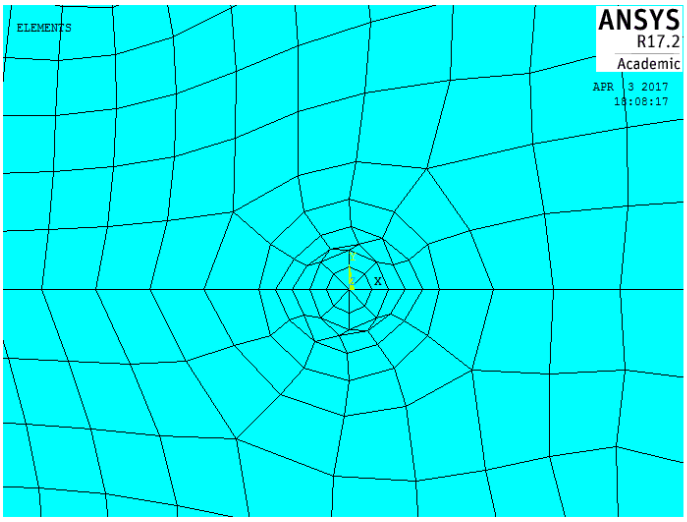


Figure 10. Mesh at the crack tip in ANSYS

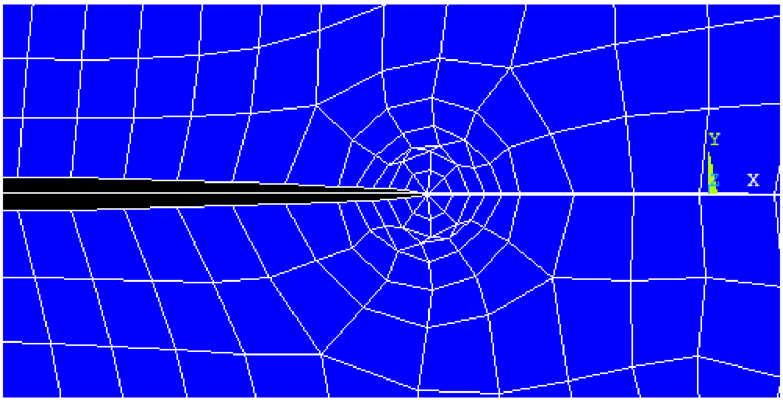


Figure 11. Deformed shape at the crack tip in Brazilian disc

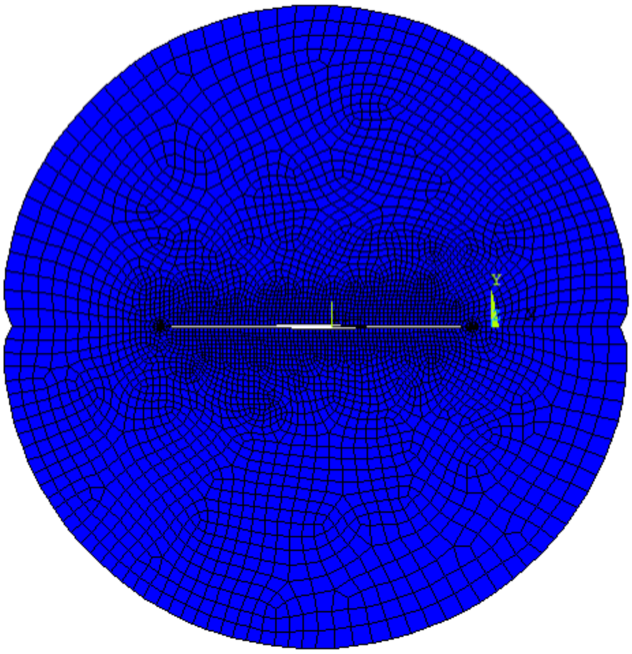


Figure 12. Deformed shape of the Brazilian disc (a/R = 0.5; α=0°)

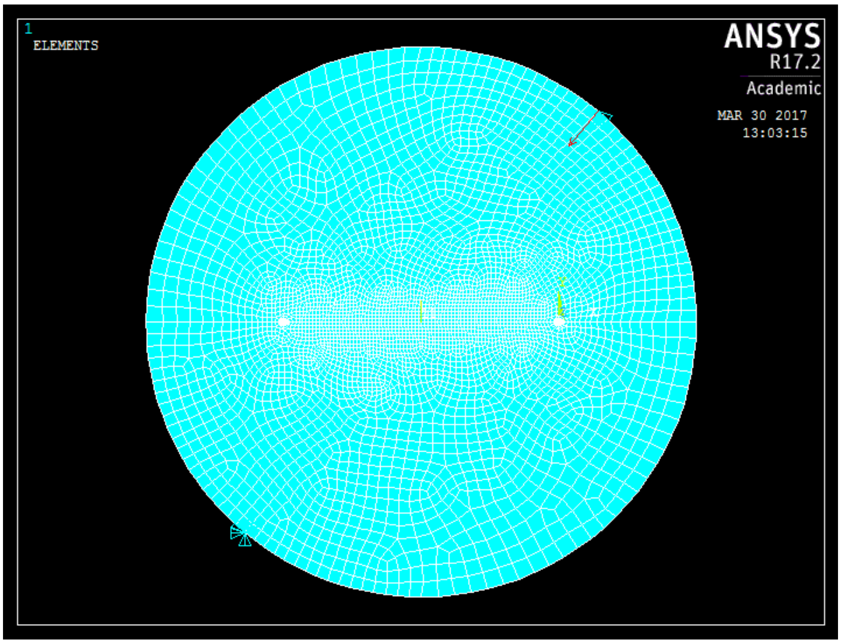


Figure 13. Mesh of Brazilian disk with boundary conditions and load (red arrow).

10.6 Stress Intensity Factors

The POST1 **KCALC** command calculates the mixed-mode stress intensity factors K_I, K_{II}, and K_{III}. This command is limited to linear elastic problems with a homogeneous, isotropic material near the crack region.

To use **KCALC** properly, these steps must be followed:

4. Define a local crack-tip or crack-front coordinate system, with X parallel to the crack face and Y perpendicular to the crack face, as shown in Figure 14 Figure 14.

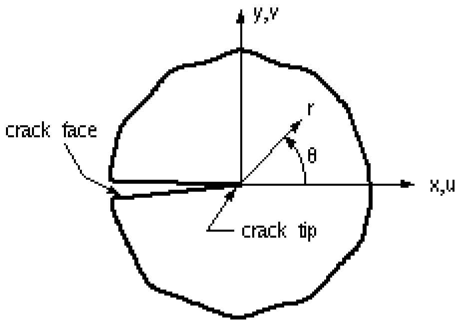


Figure 14. Crack coordinate systems for 2-D models adopted from [9]

5. Define a path along the crack face. The first node on the path should be the crack-tip node. For a full-crack model, where both crack faces are included, four additional nodes are required: two along one crack face and two along the other. Figure 15Figure 15 illustrates the two cases for a 2-D model.

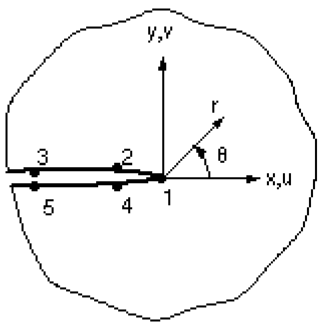


Figure 15. Typical path definitions a full-crack model

6. Calculate K_I, K_{II}, and K_{III}. (**KCALC**, *KPLAN*, *MAT*, *KCSYM*, *KLOCPR).* The *KPLAN* field on the **KCALC** command specifies whether the model is plane-strain or plane stress. Except for the analysis of thin plates, the asymptotic or near-crack-tip behavior of stress is usually thought to be that of plane strain. The *KCSYM* field specifies whether the model is a half-crack model with symmetry boundary conditions, a half-crack model with anti-symmetry boundary conditions, or a full-crack model.

The description of constrain level at the crack tip is done by two parameters. In plane conditions constrains are characterized by *T*-stress. *T*-stress represents the normal stress parallel to crack line, it is the second (constant) term (following the first singular term) in the Williams equation of the stress field:

$$\sigma_{i,j}(r,\theta) = \frac{K_I}{\sqrt{2\pi r}} f_{ij}^{I}(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} f_{ij}^{II}(\theta) + T\delta_{1i}\delta_{1j} \qquad (13)$$

To calculate stress intensity factor (SIF) mode I (K_I) from equation (13) for θ = 0° whole formulation can be simplified to:

$$K_I = \sigma_{xx}\sqrt{2\pi r} \qquad (14)$$

To calculate SIF mode II (K_{II}) from eq. (1) for θ = 180° whole formulation can be simplified to:

$$K_{II} = \sigma_{xx}\sqrt{2\pi r} \qquad (15)$$

For SIFs K_I and K_{II} in finite element software ANSYS, software uses following equations (16)-(17) for θ = ±180°:

$$K_I = \sqrt{2\pi}\frac{2G}{1+\kappa}\frac{|v|}{\sqrt{r}} \qquad (16)$$

$$K_{II} = \sqrt{2\pi}\frac{2G}{1+\kappa}\frac{|u|}{\sqrt{r}} \qquad (17)$$

10.7 Modelling procedure

In order to apply the commands mentioned before the disc must be divided into different areas. The numerical model has been created as follows:

- First step is to insert material and geometric properties.
- Then the keypoints that define the model must be specified. There must be one keypoint for every point where the load will be applied.
- After all keypoints are defined they are connected with lines that will later delimit the areas. At the crack 2 same lines will be created so there can be one area for the upper part of the disc, and another one for the lower part.

- 6 areas will be created, one for each half of the disc and two around each crack tip that will also be divided into two halves, as shown in Figure 18.

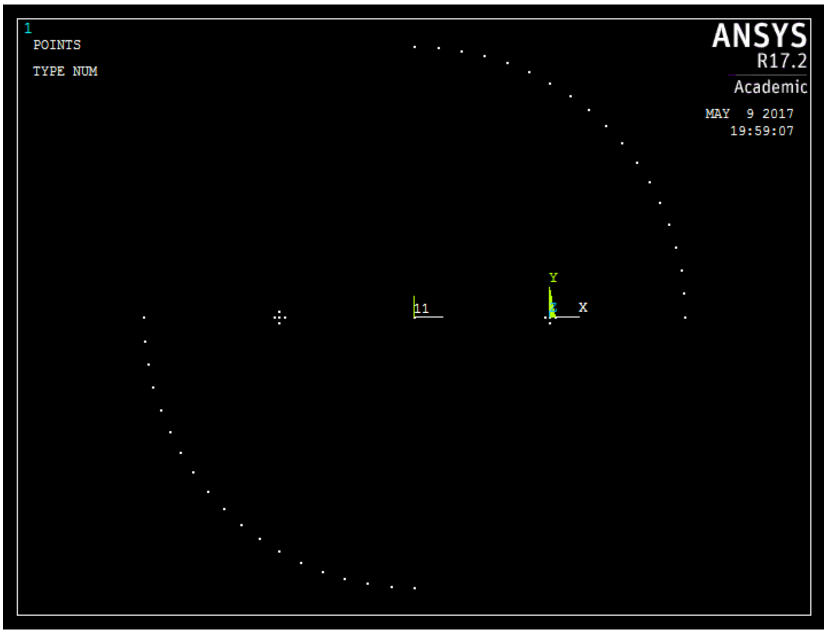


Figure 16. Keypoints used for modelling the Brazilian disc.

In Figure 16 it can be observed that there are only keypoints at two quarters of the disc, this is due to the fact that the load is applied at two opposite points between 0° and 90° with respect to the horizontal.

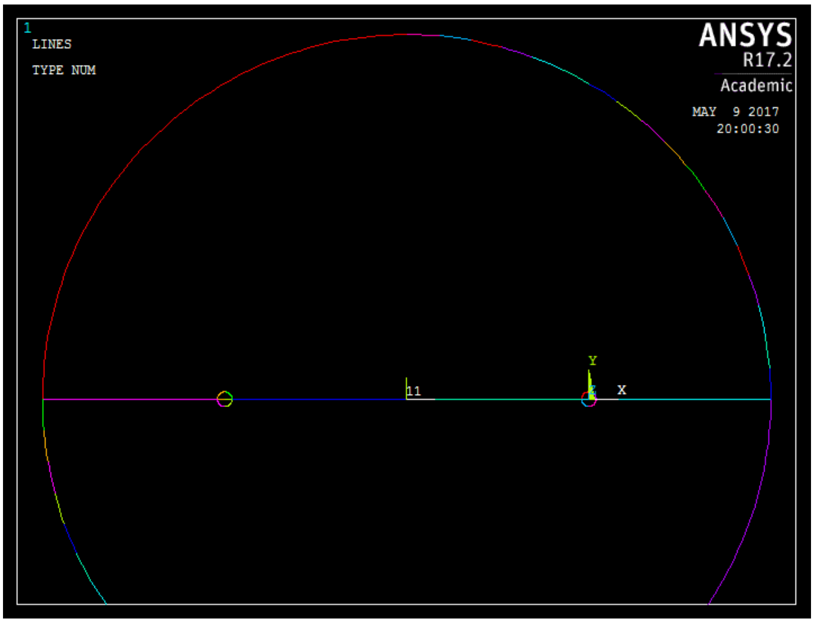


Figure 17. Lines used for modelling Brazilian disc.

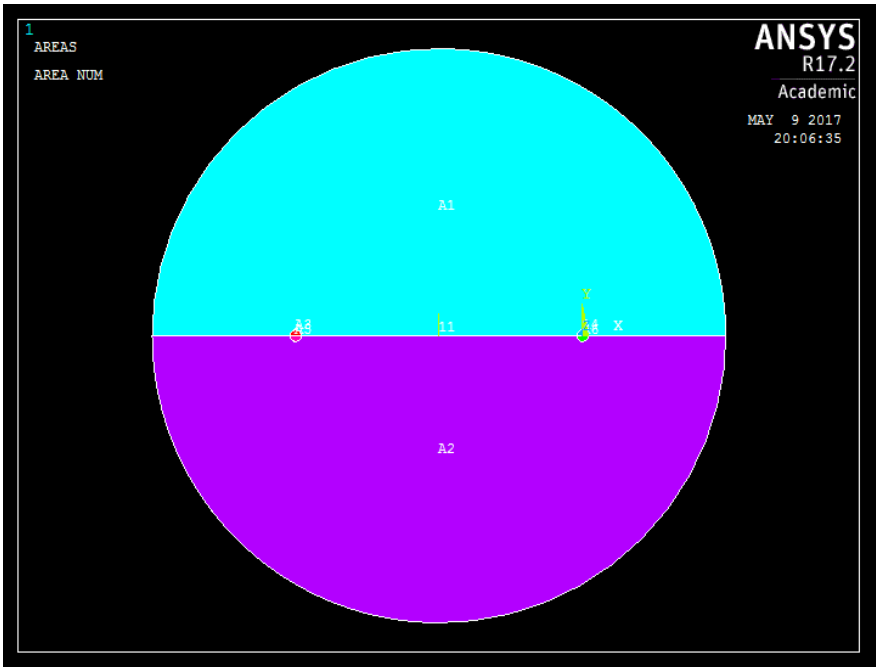


Figure 18. Areas used for modelling Brazilian disc.

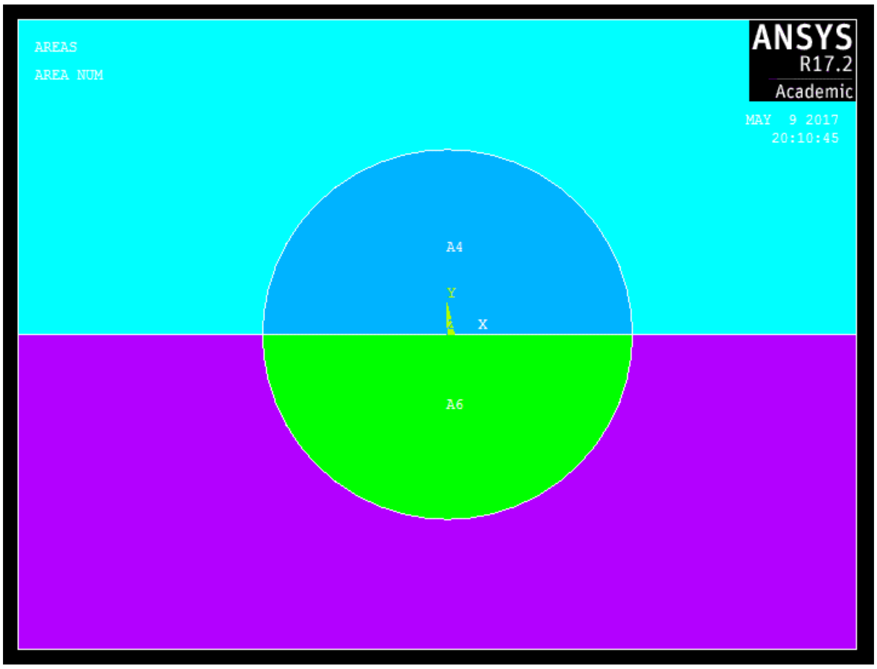


Figure 19. Zoom of the areas near the crack tip**.**

This division of the areas in two halves allows the crack to open when performing the numerical simulation as shown before in Figure 11 andFigure 12.

11. Results

11.1 Stress intensity factors

The following graphs show the results for Brazilian disk obtained by using FE software ANSYS [9]. In Figures 19- 20, there is shown dependency of SIF values on the crack angle under the same load *P* = 100 N.

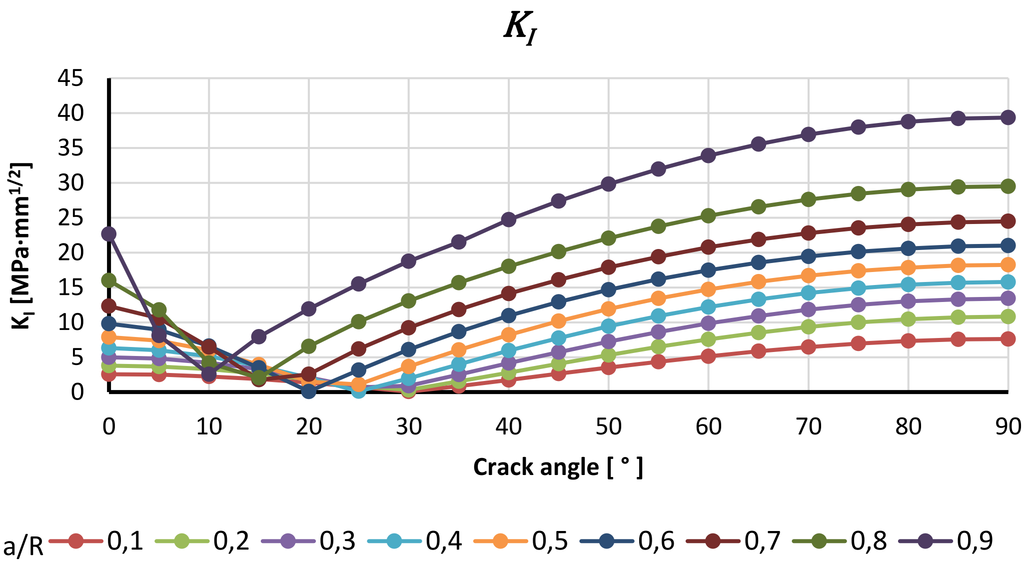


Figure 20. Stress Intensity Factor for mode I versus the crack angle.

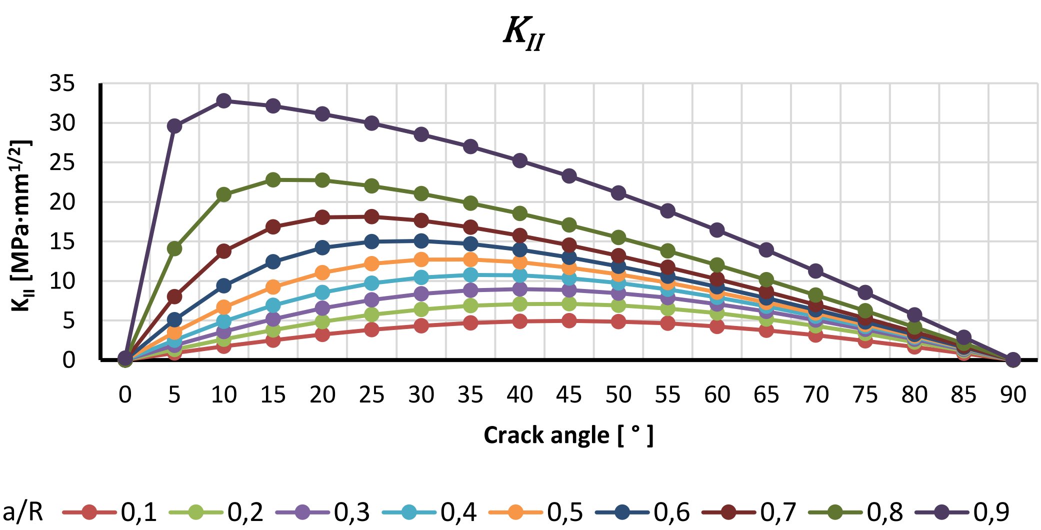


Figure 21. Stress Intensity Factor for mode II versus the crack angle.

11.2 Geometry calibration curves

From the geometry mentioned above the calibrations curves can be calculated using the following equations:

$$f_I(\alpha) = \frac{K_I\sqrt{\pi}RB}{P\sqrt{a}}\sqrt{1-\frac{a}{R}} \qquad (18)$$

$$f_{\mathrm{II}}(\alpha) = \frac{K_{II}\sqrt{\pi}RB}{P\sqrt{a}}\sqrt{1-\frac{a}{R}} \qquad (19)$$

By substituting the values of the SIFs obtained from ANSYS and the geometry of the specimen the calibration curves can be obtained for various angle and different ratio of relative crack length as it is shown in Figure 22 and Figure 23.

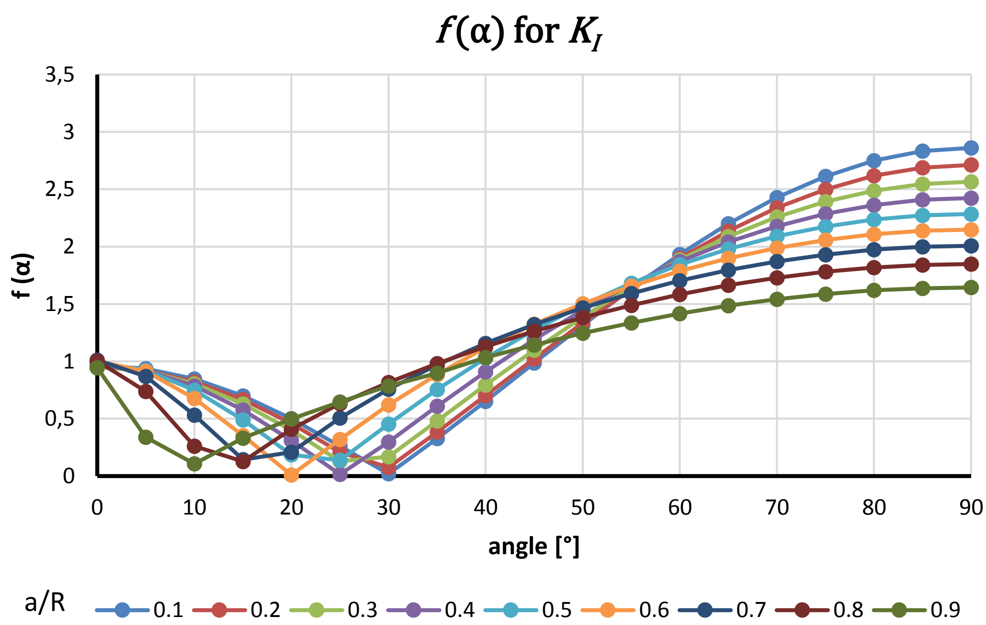


Figure 22. Calibration curve $f_I(a/R)$ (mode I) for various angle α and ratio a/R

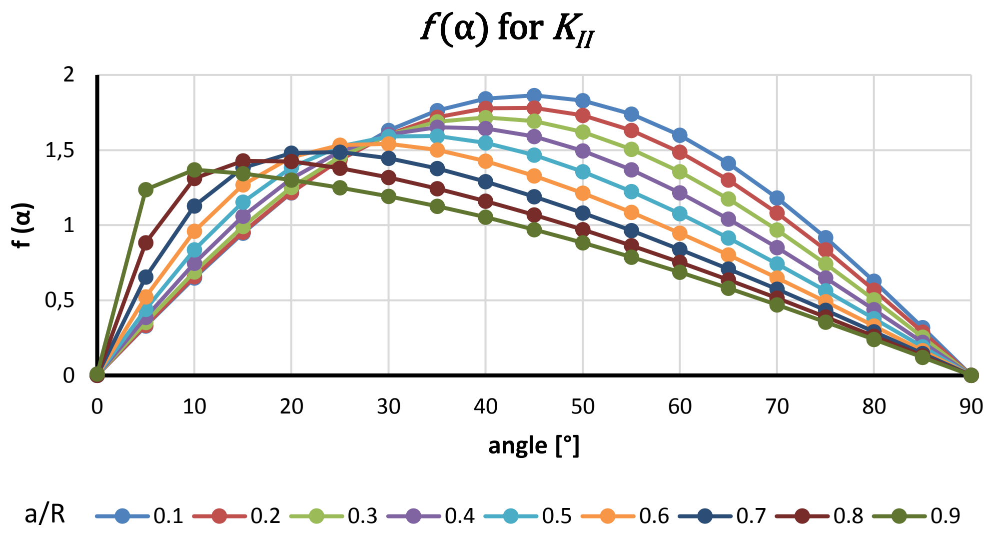


Figure 23. Calibration curve $f_{II}(a/R)$ (mode II) for various angle α and ratio a/R

With this values it is posible to obtain the stress intensity factor for any geometry with the following equations:

$$K_I = \frac{P\sqrt{a}}{\sqrt{\pi}RB}\frac{1}{\sqrt{1-\frac{a}{R}}}f_I(\alpha) \qquad (20)$$

$$K_{II} = \frac{P\sqrt{a}}{\sqrt{\pi}RB}\frac{1}{\sqrt{1-\frac{a}{R}}}f_{II}(\alpha) \qquad (21)$$

12. Example of evaluation data from experiment

In order to calculate the fracture toughness the load must be substituted for the fracture load. I used the load from article [3] to calculate this K_c and compare the results obtained from FE modelling in this thesis..

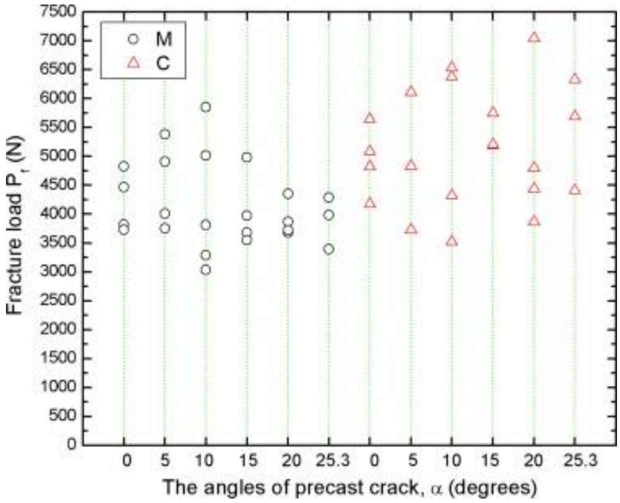


Figure 24. Experimental values of facture load P_f adopted from [3].

In order to obtain the normative K_c equation (18 is used and the geometric values of the specimen are substituted , the properties of the sample that will be studied are:

- Diameter: 70 mm
- Thickness: 30 mm
- Semi-crack length: 14 mm

The following images show the comparison between my results and the ones from the article.

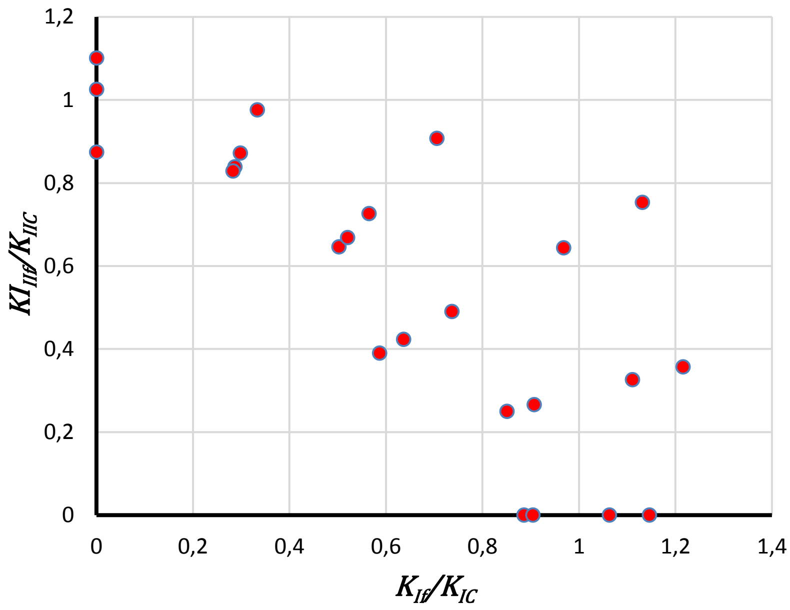


Figure 25. Fracture resistance of mortar specimens.

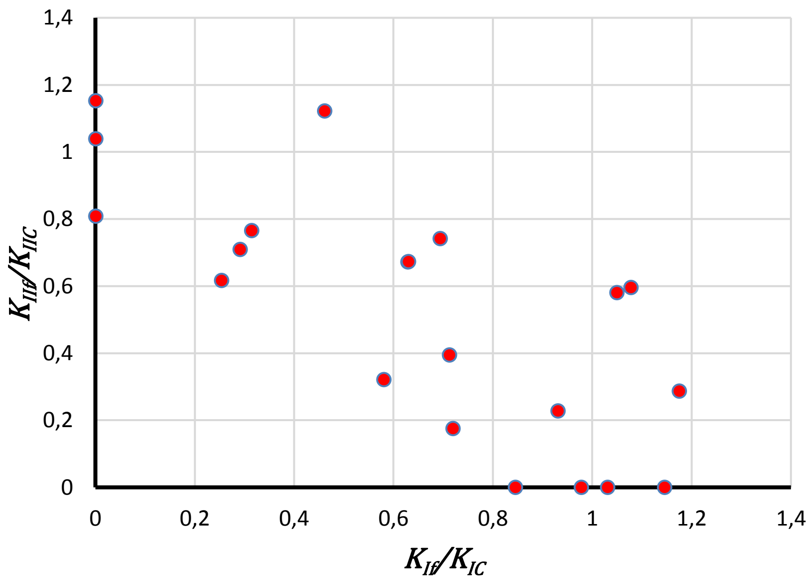


Figure 26. Fracture resistance of concrete specimens.

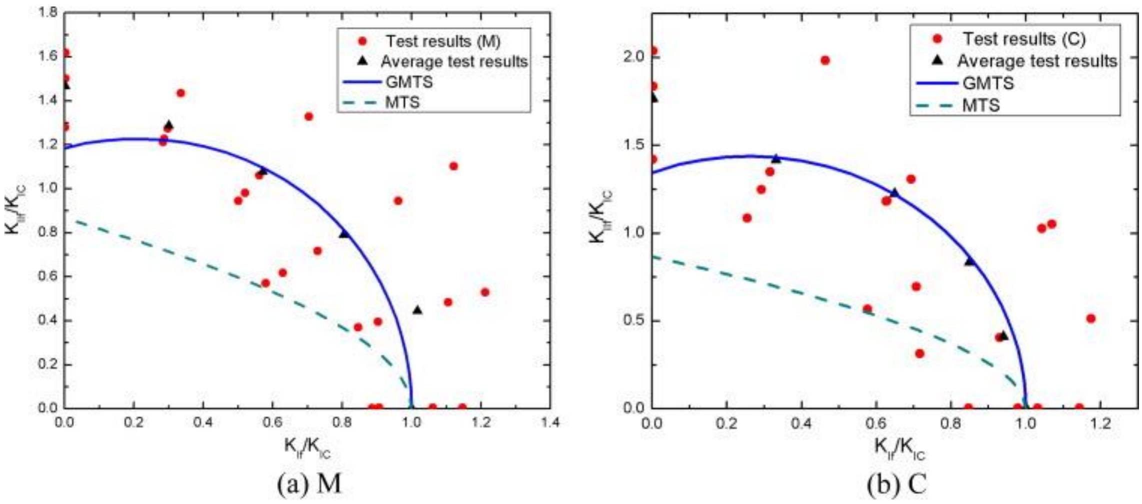


Figure 27. Results of mixed mode (I and II) from article [3]

It can be observed that the results are not exactly the same because the graphs from the article [3] plot K_{IIf}/K_{IC} in the vertical axis instead of K_{IIf}/K_{IIC}.

12.1 T-stress

A calculation of the *T*-stress at the crack tip has been carried out. Using ANSYS software a path along the crack tip has been created. It contains 50 nodes and shows the stress intensity along the X and Y directions. Once those values are obtained for every crack length and angle, they are plotted in order to obtain the trend line which will be the value of the *T*-stress for that particular geometry.

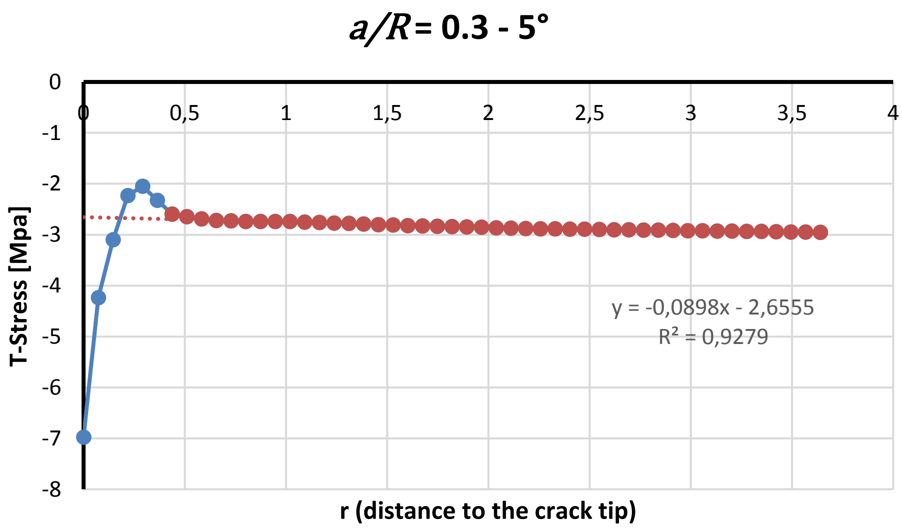


Figure 28. Example of T-stress value for a/R =0.3 and α=5°

Figure 28 is an example of how to determine the *T*-stress from the values from ANSYS. The blue line represents all the stress values and the red line is the trend line. The independent value of the linear regression is the final value of the *T*-stress. By repeating this procedure for every geometry the *T*-stress curved is obtained.

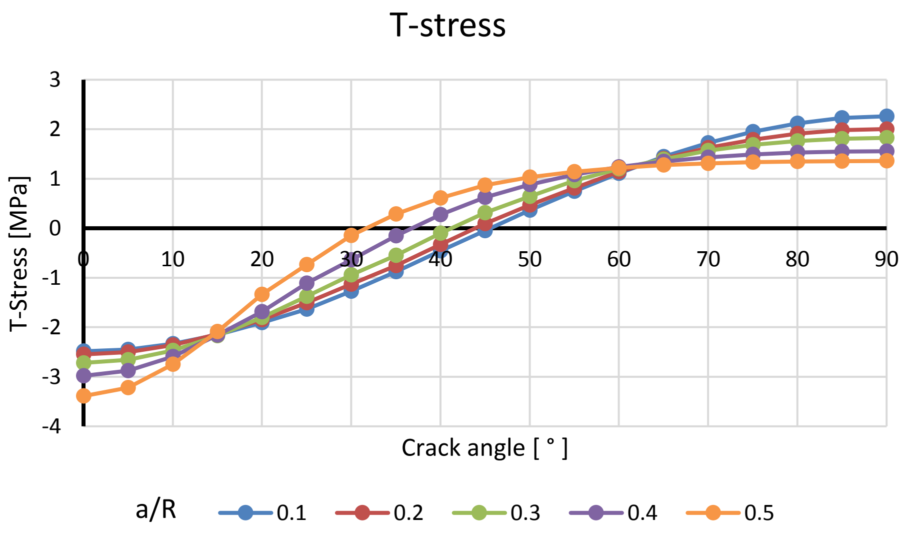


Figure 29. T-stress curve.

From the *T*-stress curve it can be noticed, that the *T*-stress changes values from negative to positive, and zero values are near to the pure shear mode.

12.2 Stress intensity field at the contact points

The following images show the stress intensity field around the point where the load is applied and also at the crack tip for different loading angles and crack length. The maximum tensile stress and the maximum tensile strain are both found to occur about 5 mm away from the two loading points along the compressed diameter of the disc, instead of at the centre of the disc surface. Therefore, the crack initiation point of the Brazilian test for rocks may be located near the loading point when the tensile strain meets the maximum extension strain criterion, but at the surface centre when the tensile stress meets the maximum tensile strength criterion.

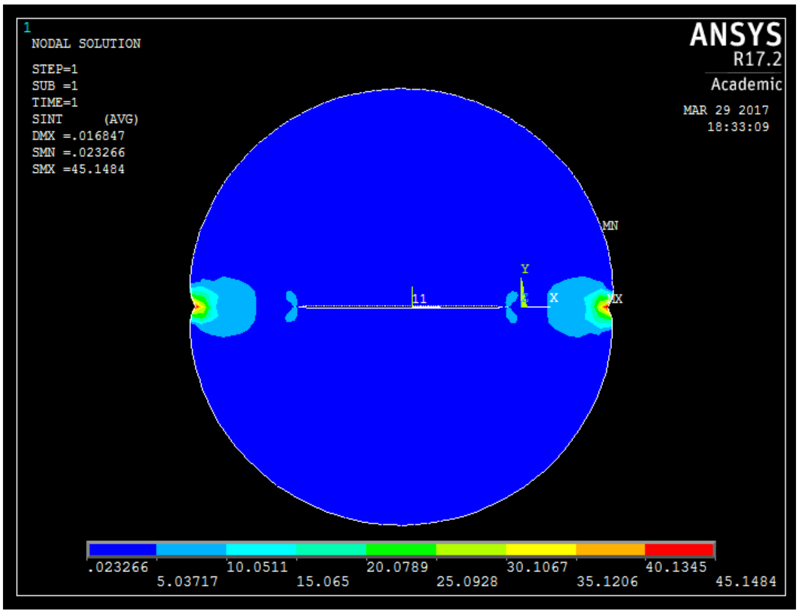


Figure 30. Stress intensity field for geometry: $a/R = 0.5$ α=0°

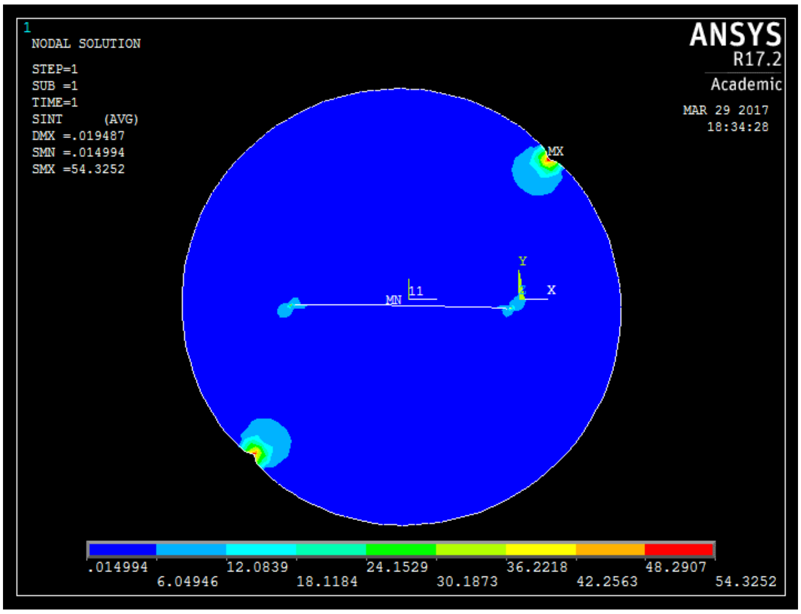


Figure 31. Stress intensity field for geometry $a/R = 0.5$ α=45°

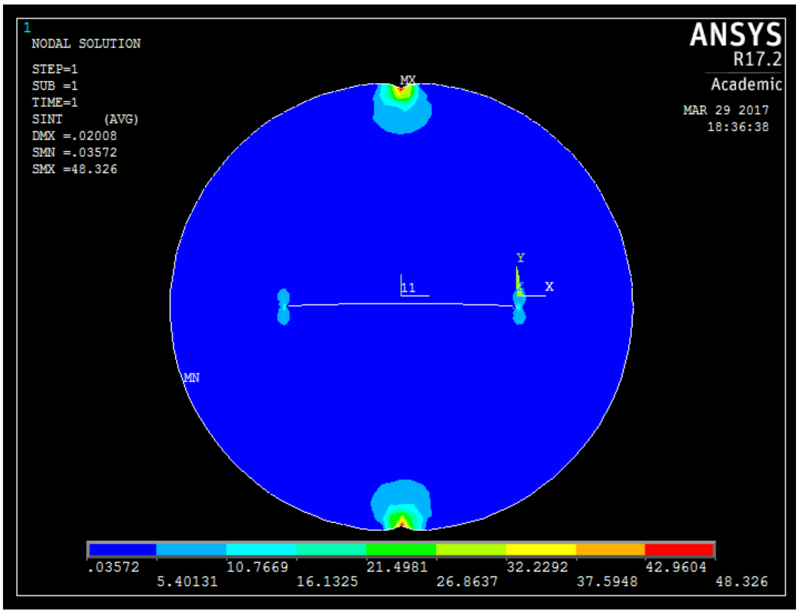


Figure 32. Stress intensity field for geometry $a/R = 0.5$ α=90°

12.3 Pure shear mode

In the following table is represented for every ratio of relative crack length the approximated angle for which there is only mode II and K_I=0 so there is only shear stress.

a/R	α [°]
0.1	30
0.2	29
0.3	27
0.4	25
0.5	24
0.6	20
0.7	16
0.8	14
0.9	8

Table 1. Angles for K_I=0 0 (pure mode II)

It can be observed that the higher the length of the crack, the smaller the angle at which K_I=0.

13. Discussion

The graphs show that for each relative crack length the stress intensity factor under mode I tends to 0 at some particular angle. This means that there is no tensile stress normal to the plane of the crack so there is only shear stress acting upon the crack at that angle.

In the first figure, the load is applied on the horizontal plane (angle 0°), so there is only normal tensile stress (mode I) and both faces have the same stress intensity values.

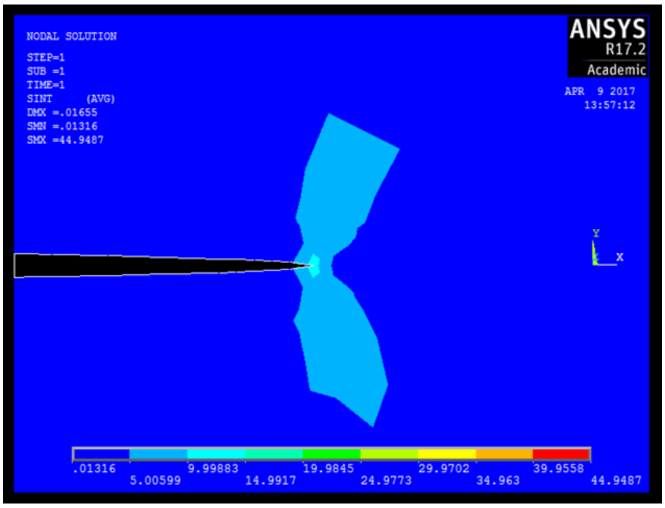


Figure 33. Stress intensity around the crack tip for geometry: $a/R = 0.4$ α=0°

On the contrary, Figure 34 shows the stress intensity field where K_I is almost 0 and there is only shear stress.

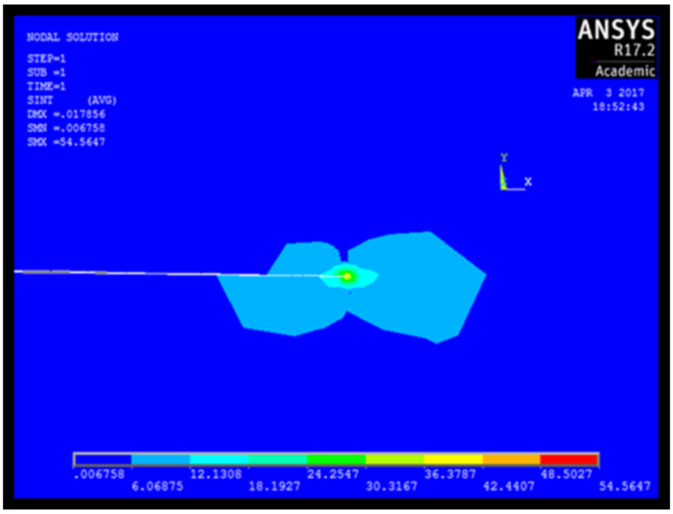


Figure 34. Stress intensity around the crack tip for geometry: $a/R = 0.4$ α=25°

In Figure 35, it can be observed that each face of the crack has different stress values, this means that the crack is under mixed mode (mode I + mode II).

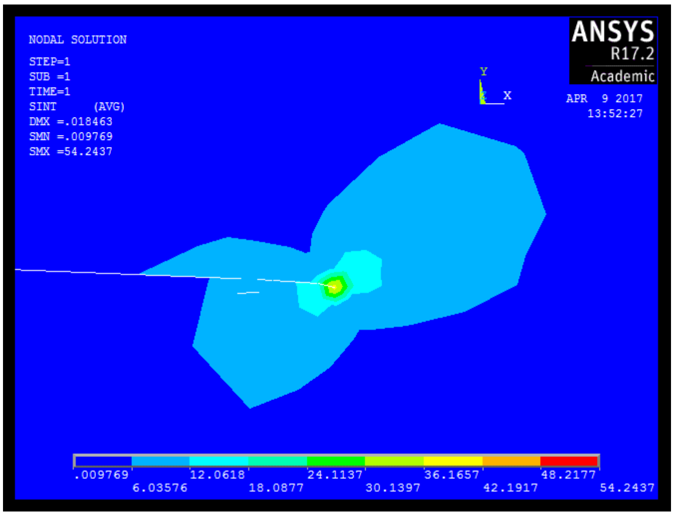


Figure 35. Stress intensity around the crack tip for geometry: $a/R = 0.4$ α=45°

14. Conclusion

Fracture mechanics analyses on the diametral crack behavior in a disc with diametral compression were carried out. Based on these analyses, it is proposed that a simple conventional Brazilian test, which is normally used for the determination of rock tensile strength, can be applied to measure rock fracture toughness.

The comparison between the fracture-toughness values obtained by other methods has shown that the Brazilian test is a promising alternative method for rock fracture-toughness measurement. Several advantages of the proposed Brazilian method can be realized.

- The specimen of the Brazilian test is simpler than that of any available fracture-toughness measurement method.
- The test is convenient to perform using a conventional compressive-test machine. No complicated or additional equipment is required, and only a diametral load-displacement graph is required for recording purposes.
- The interpretation of the results is straightforward; only a simple calculation is required.
- The test allows us to get information about both modes (mode I + mode II) of fracture.

In the single cracked disc specimens, the wing cracks propagated in a curved path and continue their growth in a direction (approximately) parallel to the direction of maximum compressive load. These wing cracks are initiated at the original tips of the cracks for all crack inclination angles greater than 15°. It should be noted that the wing cracks may not start their initiation from the original tips of the single crack when the inclination angle are close to 90° (that is at right angle to the direction of applied compressive line load). On the other hand, the specimen may fail away due to the indirect tensile effect (axial splitting) exactly like that of the un-cracked Brazilian disc specimen in a conventional Brazilian test.

15. Nomenclature

CSBDC (Central Straight Through Crack Brazilian Disk)

FE (finite element)

σ_t(tensile strength)

[1] K_c(fracture toughness)

[2] a (half length of the crack)

[3] R (radius)

[4] P (load)

[5] D (diameter)

[6] t/B (thickness)

[7] K (stress intensity factor)

[8] K_I (stress intensity factor – mode I)

[9] K_{II} (stress intensity factor – mode II)

[10] K_{III} (stress intensity factor – mode II)

16. Author's publications

Miarka, P., Hevia Villanueva, M., Seitl, S., 2017. Stress analysis of crack Brazilian disc: Pilot calibration curves.19th Conference Applied Mechanics 2017 – conference proceedings, 2017, pp 73-77.

17. References

[1] Guo, H., Aziz, N.I. and Schmidt, L.C., 1993. Rock fracture-toughness determination by the Brazilian test. *Engineering Geology*, *33*(3), pp.177-188.

[2] Li, D. & Wong, L.N.Y., 2013. The Brazilian Disc Test for Rock Mechanics Applications: Review and New Insights. *Rock Mechanics and Rock Engineering*, 46 (2), pp. 269-287.

[3] Hou, C., Wang, Z., Liang, W., Li, J., Wang, Z., 2016. Determination of fracture parameters in center cracked circular discs of concrete under diametral loading: A numerical analysis and experimental results. Theoretical and Applied Fracture Mechanics, 85 (B), pp. 355-366.

[4] Amadei, B. Principles and procedure of Brazilian test, University of Colorado, CVEN 5768, lecture Notes 8.

[5] Claessona, J. and Bohlolib, B., 2002. Brazilian test: stress field and tensile strength of anisotropic rocks using an analytical solution. *International Journal of Rock Mechanics and Mining Sciences*, 39 (8), pp. 991–1004.

[6] van Miera, J.G.M., van Vlietb, M.R.A., 2002. Uniaxial tension test for the determination of fracture parameters of concrete: state of the art. *Engineering Fracture Mechanics*, 69 (2), pp. 235-247.

[7] Proveti, J.R.C. & Michot, G., 2006. The Brazilian test: a tool for measuring the toughness of a material and its brittle to ductile transition. *International Journal of Fracture*, 139 (3), pp. 455-460.

[8] Haeri, H., Shahriar, K., Fatehi Marji, M., Moarefvand, P. On the crack propagation analysis of rock like Brazilian disc specimens containing cracks under compressive line loading.

[9] ANSYS, Finite element software www.ansys.com

[10] Anderson, T. L., 2005. Fracture Mechanics: Fundamentals and applications. Taylor & Francis group

[11] Williams, M.L., 1957. On the stress distribution at the base of a stationary crack. ASME J Appl. Mech. Nr 24, pp. 109-114

[12] Ayatollahi, M.R., Aliha, M. R. M. Fracture initiation angle in the centrally cracked circular disk specimen. Iran University of Science and Technology.

18. CV

PERSONAL DATA

Name: **María Hevia Villanueva**

Birth: **1993 (23)**

Address: **Camino Quiñones 175 209-2, 33203 Gijón**

Phone number**: 677529113**

Email: **mheviavillanueva@gmail.com**

EDUCATION

- Currently studying a bilingual degree in **Mechanical Engineering** at the University of Oviedo (4th year).
- Bachelor thesis in Brno University of Technology at the Faculty of Civil Engineering (2017)

OTHER COURSES

- 2015 - **Basic Course in Nuclear Science and Technology**. ("Curso Básico de Ciencia y Tecnología Nuclear" Organizado por Jóvenes Nucleares, comisión de la Sociedad Nuclear Española (SNE), en colaboración con la Escuela Politécnica de Ingeniería de la Universidad de Oviedo).

LANGUAGES

ENGLISH: ***Certificate in Advanced English*** (Cambridge University)

SPANISH: Native

COMPUTER SKILLS

- **AUTOCAD** and **AUTODESK INVENTOR PROFESSIONAL** - Advanced user level.
- Microsoft Office **Word** and **Power Point** - Advanced user level.
- Microsoft Office **Excel** – Advanced user level.

19. List of figures