Checking the fatigue limit from thermographic techniques by means of a probabilistic model of the epsilon-N field

Alfonso Fernández-Canteli[a], Enrique Castillo[b], Antonio Argüelles[a], Pelayo Fernández[a], Margarita Canales[c]

[a] *Department of Construction and Fabrication Engineering, University of Oviedo, Spain*
[b] *Department of Applied Mathematics, University of Cantabria, Spain*
[c] *Technological Institute of Aragón, Spain*

Abstract

The determination of the $\varepsilon - N$ field for a certain material, similar to the $S - N$ case implies a relatively high number of costly and time consuming fatigue tests at different strain amplitudes. For specific applications, the mere estimation of the fatigue limit seems to be sufficient for guaranteeing lifetime safety in fatigue design allowing practical methods to be advantageously applied as a result of the notable reduction of the experimental program implied. In this work, the reliability of the fatigue limit provided by the Risitano thermographic method is investigated in order to check its validity for practical applications. With this aim, an experimental program on a C55E steel under strain control is performed, from which the probabilistic $\varepsilon - N$ field is assessed using the Weibull regression model of Castillo et al., which provides a global solution in both low-and long-cycle fatigue regions. The fatigue limit is derived, primarily as stress range and then compared with that resulting from the Risitano methodology. With the purpose of improving the estimation of the fatigue limit, a novel assessment procedure of the thermographic results is presented that allows an improved correspondence between the fatigue limits arising from both models, thus confirming the suitability of the thermographic technique in practical applications.

Keywords: Fatigue limit, probabilistic fatigue models, thermographic technique

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1. Introduction

Contrary to other models used to reproduce the $S-N$ and the $\varepsilon-N$ fields, the regression models proposed by Castillo et al. (see [1] or [2]) provide a global, probabilistic description of the lifetime problem in both low- and long-cycle fatigue regions. In addition, they supply the value of the *true* fatigue limit in the sense of the maximum stress range, or strain amplitude allowing an unlimited number of cycles to be applied without failure. This fatigue limit happens to be unique for any probability of failure thus being insensitive to the size effect. Even when using such advanced models as those mentioned above, which evade some of the limitations exhibited by other current methods, the determination of the $S-N$ or $\varepsilon-N$ fields, implies a relatively high number of costly and time consuming fatigue tests at different stress or strain ranges or amplitudes, respectively.

For certain applications, the estimation of the fatigue limit appears to be sufficient for guaranteeing lifetime safety in fatigue design so that some alternative methods can be applied. Since the early contribution of [3] a number of researches has been devoted to the study of the rapid estimation of high cycle fatigue properties, particularly using the temperature changes during cyclic loading (see [4] and [5]). In particular, the thermographic technique([6], [7], [8] and [9]) can be advantageously considered for the estimation of the fatigue limit with a notable reduction of the experimental programme. In this work, an experimental fatigue program under strain control on C55E steel is performed aiming at investigating the reliability of the Risitano thermographic method in practical applications.

The $\varepsilon-N$ field is determined from the resulting data using the Weibull regression model of Castillo et al. (see [2]) and subsequently transformed into the fatigue limit, as stress range, using the model proposed in [10]. Thereafter, this value is compared with that resulting from the thermographic technique. The work confirms the utility of the Risitano method pointing out some of its limitations concluding that the thermographic fatigue limit does not supply the *true fatigue limit* but a non-conservative value of it that could be rather understood as an *engineering fatigue limit* corresponding to a reasonably high number of cycles in accordance to the usual fatigue lifetimes most found in practice. Despite its effective utility, an enhancement of the assessment procedure of the thermographic results is also presented that allows an improved correspondence between the fatigue limits arising from both models, thus reinforcing the suitability of the thermographic technique in practical applications.

2. The $\varepsilon - N$ model proposed by Castillo et al.

The relation between the total strain amplitude ε_a and fatigue life N_f, i.e., the strain-life curve, as a necessary fatigue material information, is traditionally based on former proposals of Basquin ([11]), Coffin ([12]) and Manson ([13]) unified by Morrow ([14, 15]) as:

$$\varepsilon_a = \varepsilon_a^e + \varepsilon_a^p = \frac{\sigma'_f}{E}\left(\frac{2N_f}{N_0}\right)^b + \varepsilon'_f\left(\frac{2N_f}{N_0}\right)^c, \tag{1}$$

where the superscripts e and p are used for the *elastic* and *plastic* strains, N_f is the number of cycles, σ'_f the fatigue strength coefficient, b the fatigue strength exponent, ε'_f the fatigue ductility coefficient, c the fatigue ductility exponent, E the Young modulus and N_0 a reference number of cycles used to render (1) dimensionless.

According to this model, both the elastic and plastic components are represented by straight lines on a log-log plot so that the total strain is not linear anymore. The estimation of the four model parameters is done by fitting two regression lines corresponding to the elastic and the plastic components, respectively, then the total strain is obtained by adding both components (see [16]).

Some of the limitations implied in Morrow's approach have been outlined by [2], among them the impossibility of existence of a fatigue limit and the cumbersome application of the probabilistic analysis of data. Contrary to this, the model proposed by [2] allows a probabilistic definition of the strain-life field based on a Weibull regression model supported by sound statistical and physical assumptions. It joins the elastic and the plastic local strain as a total strain, permits dealing with run-outs, defines the whole strain-life field as quantile curves using the fatigue limit as one of the model parameters, and allows a damage accumulation model for lifetime prediction to be proposed. The fatigue limit happens to be unique for any probability of failure being insensitive to the size effect.

According to this model, the fatigue life for fixed strain amplitude ε_a is a random variable, the cumulative distribution function (cdf) of which is denoted $F(N_f; \varepsilon_a)$. The probability of fatigue failure p of a piece when subjected to N_f cycles at a stress range amplitude ε_a is given by the Weibull

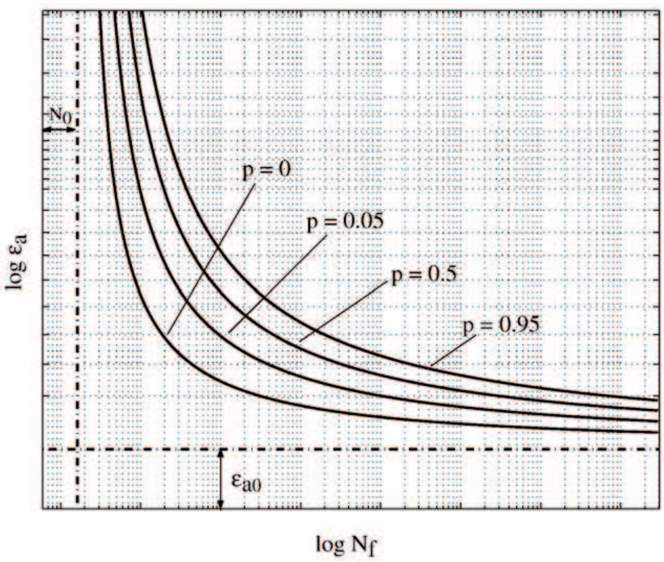

Figure 1: Percentile curves representing the relationship between lifetime, N_f, and strain amplitude, ε_a, in the $\varepsilon - N$ field for the fatigue model ([2]).

model (see [17]):

$$p \;=\; F(N_f;\varepsilon_a) = 1 - \exp\left[-\left(\frac{\log(N_f/N_0)\log(\varepsilon_a/\varepsilon_{a0}) - \lambda}{\delta}\right)^{\beta}\right]; \qquad \log(N_f/N_0)\log(\varepsilon_a/\varepsilon_{a0}) \geq \lambda, \tag{2}$$

where (see Figure 1) N_0 is the threshold value of lifetime, ε_{a0} the fatigue limit of ε_a, λ the parameter defining the position of the corresponding zero-percentile hyperbola, δ the scale parameter, and β the shape parameter.

It is interesting to note that (2) has a dimensionless form, and reveals that the probability of failure p depends only on the product

$$N_f^*\varepsilon_a^* = (log(N_f/N_0))(log(\varepsilon_a/\varepsilon_0)),$$

where

$$N_f^*\varepsilon_a^* \sim W(\lambda,\delta,\beta) \quad \Leftrightarrow \quad N_f^* \sim W\left(\frac{\lambda}{\varepsilon_a^*},\frac{\delta}{\varepsilon_a^*},\beta\right), \tag{3}$$

i.e., $N_f^*\varepsilon_a^*$ and N_f^* have a Weibull distribution allowing to represent the whole $\varepsilon - N$ field by a unique Weibull distribution function. The parameters $N_0, \varepsilon_{a0}, \lambda, \delta$ and β of the model can be estimated using several well

Table 1: Chemical composition of the steel C55E used in this work.

C	Si	Mn	S	P
0.52-0.60	< 0.40	$0.60 - 0.90$	$0.02 - 0.04$	< 0.035
	Cr	Ni	Mo	Additional
	< 0.40	< 0.40	< 0.10	$Cr - Ni - Mo < 0.63$

established methods proposed in the fatigue literature (see, for example, [18, 1, 17]). Accordingly, the proposed model is an adequate candidate to represent the $\varepsilon - N$ field. As an alternative to the Weibull model, the Gumbel model

$$p = F(N_f; \varepsilon_a) = 1 - \exp\left[-\exp\left(\frac{\log(N_f/N_0)\log(\varepsilon_a/\varepsilon_{a0}) - \lambda}{\delta}\right)\right]; \quad \log(N_f/N_0)\log(\varepsilon_a/\varepsilon_{a0}) \geq \lambda \qquad (4)$$

can be assumed adding some advantages for lifetime assessment in the case of varying loading. This relaxation is reasonably justified for Weibull shape parameters β greater than 6.

Finally, run-outs are not necessarily excluded of the evaluation. This depends on their position with respect to the zero-percentile curve, as discussed in [17].

3. A practical example

In this section, a carbon steel C55E is considered for fatigue testing and subsequent checking of the correspondence between the fatigue limits resulting from the model proposed in Section 2 and the Risitano method, respectively. The chemical composition of this material is shown in Table 1, whereas its stress-strain curve and the specimen geometry used in the tests are depicted in Figure 2.

The experimental program consisted in fatigue tests carried out under strain control for variable strain amplitude and constant $R = -1$ on a MTS 250 kN dynamic machine. The results obtained are shown in Table 2.

The following parameter estimates were obtained when fitting data to the Weibull model: $\log N_0 = -0.3197$; $\log \varepsilon_{a0} = -7.7095$; $\lambda = 14.7482$; $\delta = 7.6531$; $\beta = 10.3684$. Because $\beta > 6$ the model can be approximated

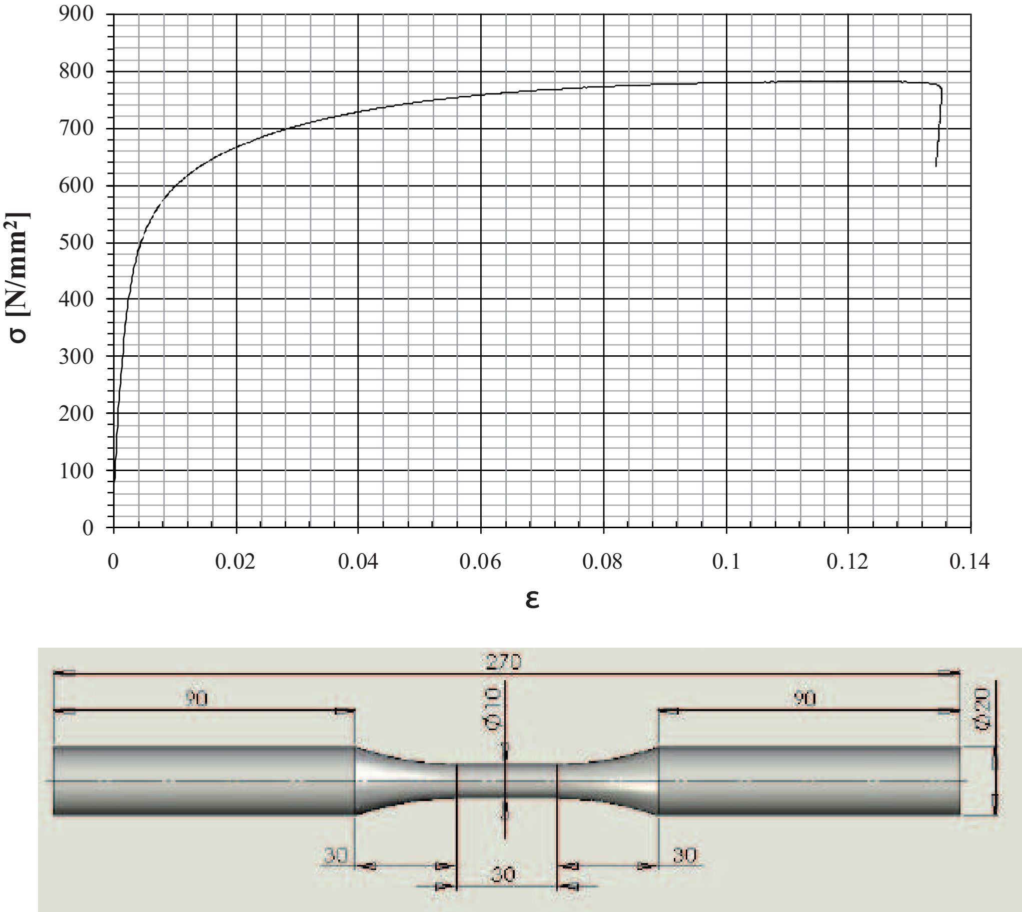

Figure 2: Ideal strain-stress curve for the material C55E and geometry used in the present specimens.

Table 2: Test data resulting from the experimental program carried out (F=failure, NF=run-out).

Specimen number	ε_{max} [‰]	ε_{min} [‰]	ε_a [‰]	Number of cycles	Result
1	4.623	-4.623	4.623	110678	F
2	5.367	-5.367	5.367	8454	F
3	6.673	-6.673	6.673	2770	F
4	3.317	-3.317	3.317	36928	F
5	2.540	-2.540	2.540	101532	F
6	1.928	-1.928	1.928	1609754	F
7	2.143	-2.143	2.143	525136	F
8	2.044	-2.044	2.044	899905	F
9	3.714	-3.714	3.714	18732	F
10	4.954	-4.954	4.954	11628	F
11	6.209	-6.209	6.209	3202	F
12	7.169	-7.169	7.169	1066	F
13	2.771	-2.771	2.771	55692	F
14	2.275	-2.275	2.275	280078	F
15	1.928	-1.928	1.928	3000000	NF
16	2.044	-2.044	2.044	2498674	F
17	7.169	-7.169	7.169	1516	F
18	7.318	-7.318	7.318	2736	F
19	7.517	-7.517	7.517	1803	F
20	4.772	-4.772	4.772	6858	F
21	5.714	-5.714	5.714	5442	F

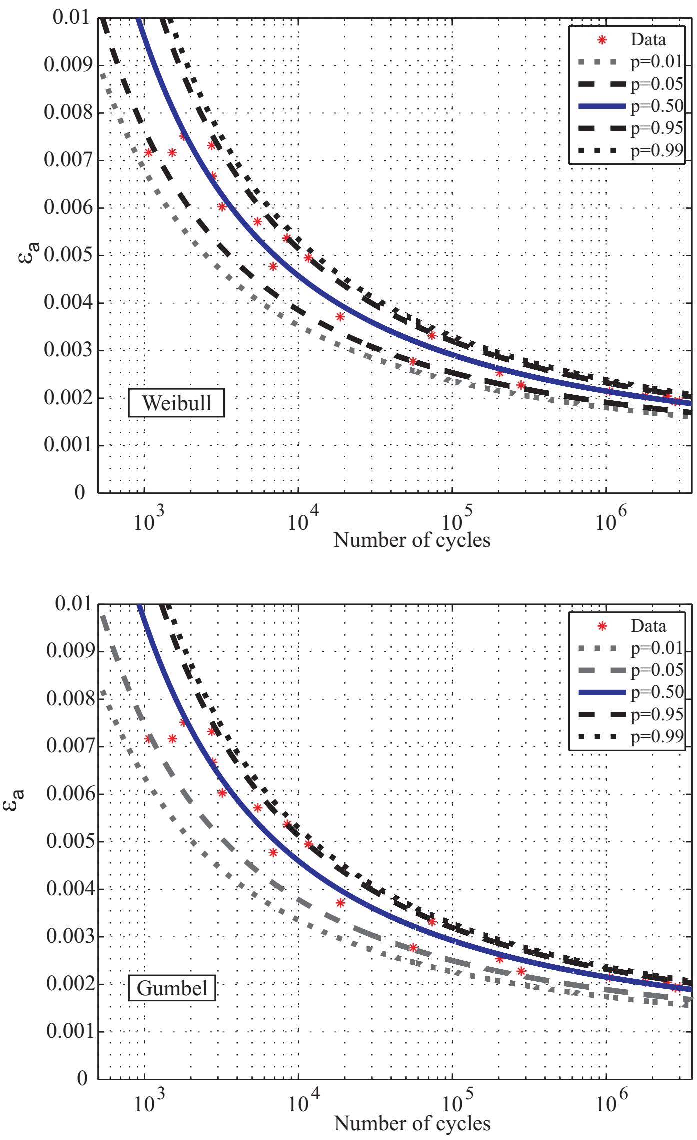

Figure 3: Data and fitted delta strain-lifetime for the Weibull and Gumbel models showing the $0.01, 0.05, 0.50, 0.95$ and 0.99 percentiles.

by the Gumbel option for which the following parameters are estimated: $\log N_0 = -0.3203$; $\log \varepsilon_{a0} = -7.7096$; $\lambda = 22.4413$; $\delta = 0.7124$.

The resulting data points, the $0, 0.05, 0.50, 0.95$ and 0.99 percentiles and the expected run-out value corresponding, respectively, to the Weibull and Gumbel models for the $\varepsilon - N$ curves are represented in Figure 3. The fit to the data shows a small scatter leading to a fatigue limit $\Delta\varepsilon_0 = 2\varepsilon_{a0} = 8.970 \times 10^{-4}$ in the elastic region of the strain field that corresponds to a stress range fatigue limit $\Delta\sigma_0 = E\Delta_{\varepsilon_0} = 188.40$ MPa, which is the value to be compared with the fatigue limit resulting from the thermographic methodology.

4. Application of thermography to estimation of the fatigue limit

The thermographic method known after Risitano (see [8]) is based on the empirical observation of the temperature increase of a specimen subjected to fatigue loading. According to this method, the fatigue limit can be identified as the maximum value of the stress range that does not induce measurable damage, i.e., a rise of temperature. Some of the advantages provided by this method are:

- The possibility of recognition of the critical location where the fatigue crack originates or, alternatively, of providing a representative measurement of the temperature change in the zone where the crack is formed.
- The non-destructive character of the tests needed to determine the fatigue limit.
- The reduced number of specimens implied in the procedure.
- The simplicity of the methodology applied. According to these authors, it suffices to state that the temperature is stationary during the number of cycles applied what, in general, happens at a relative low amount of cycles, say, between 10.000 and 50.000 cycles.

In a first phase, during which the stress range $\Delta\sigma$ applied does not surpass the fatigue limit $\Delta\sigma_0$ and, consequently, no crack propagation takes place, it is expected that the results show a straight line trend with flat slope. As soon as the stress range applied exceeds the fatigue limit, the temperature evolves exhibiting a noticeable change and the plot becomes steeper as a function of the stress range. According to those authors, the simple construction of

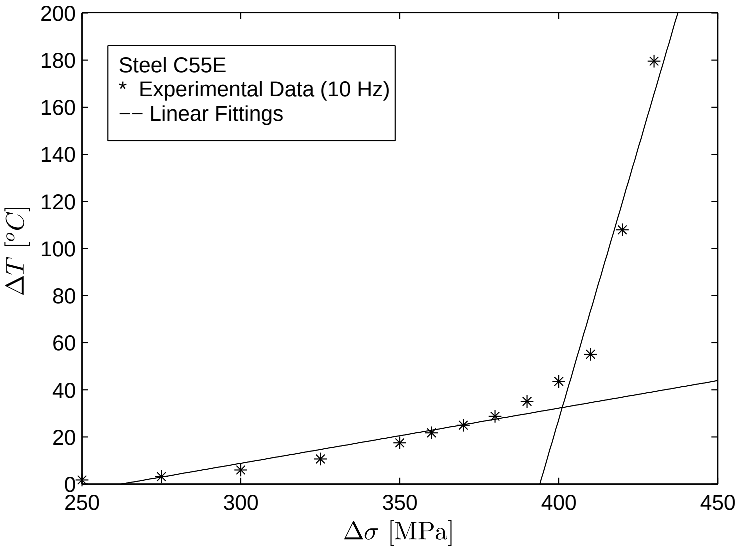

Figure 4: Linear approximation of the infrared thermal detection for determining the fatigue limit $\Delta\sigma_0 \approx 400$ MPa) of a carbon steel C55E according to [8].

a diagram, plotting the temperature increase ΔT, respectively, temperature gradient $\Delta T/\Delta N$ referred to the number of cycles considered during the stability of the temperature change with respect to the stress range $\Delta\sigma$ allows us to recognize the fatigue limit as the intersection of both characteristic straight regression lines as illustrated in Figure 4 or, simply, as the cut of the steep regression straight line with the abscissa axis.

In the present case, an infrared camera A320G of Flir Systems (see Figure 5) has been used for obtaining the thermographic results using two different frequencies: 5, respectively, 10 Hz. The measurements performed are summarized in Table 3. Applying the procedure suggested by [8] to the higher frequency, a fatigue limit of $\Delta\sigma_0 \cong 400$ MPa is estimated (see Figure 5), which differs notably from the value found in the analytical method for the true fatigue limit, namely, $\Delta\sigma_0 = 188.40$ MPa. The estimation for the lower frequency yields even higher values, and will be handled in Section 5. Nevertheless, applying the analytical model proposed in (3), respectively in (4), a very high lifetime median value of 4.24×10^{12} cycles is predicted for the original strain amplitude $\varepsilon_{a0} = \Delta\sigma_0/2E = 4.485 \times 10^{-4}$ corresponding to the above stress range.

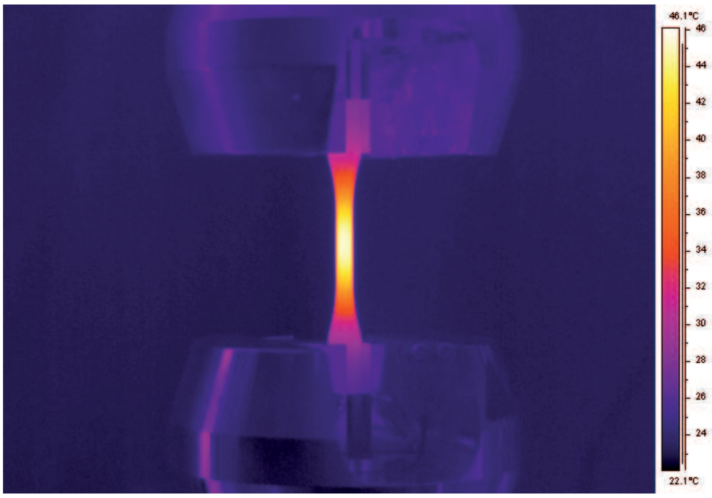

Figure 5: Infrared thermal detection of a carbon steel C55E during the fatigue test using a A320G Flir camera.

Table 3: Experimental results of the thermographic measurements obtained for two frequencies, 5 and 10 Hz.

Data set for 10 Hz			Data set for 5 Hz		
n	$\Delta\sigma$	ΔT	n	$\Delta\sigma$	ΔT
1	250	1.66	1	250	1.92
2	275	3.06	2	260	2.76
3	300	5.96	3	275	3.68
4	325	10.6	4	300	5.9
5	350	17.4	5	325	9.34
6	360	21.64	6	350	13.02
7	370	24.96	7	365	16.34
8	380	28.72	8	370	17.38
9	390	35.06	9	420	36.64
10	400	43.5	10	450	106.86
11	410	55.06			
12	420	107.92			
13	430	179.5			

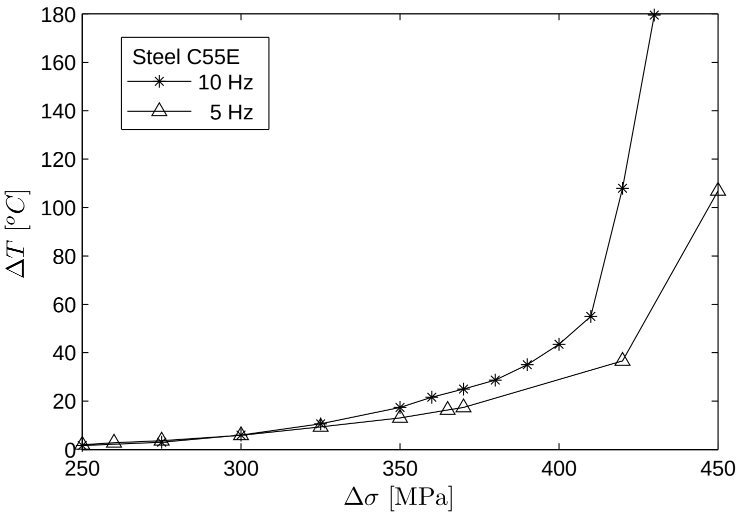

Figure 6: Plot of thermographic data obtained in the present experimental tests for two different frequencies (10 and 5 Hz).

5. Discussion

Using the 5-parameter Weibull regression model of [2], a probabilistic definition of the $\varepsilon - N$ field is achieved allowing us to define analytically the fatigue limit for the carbon steel C55E, which can be converted to the fatigue limit in terms of stress, and compared subsequently with the value found using the thermographic method proposed by [8].

The value obtained using the thermography technique does not agree with the *true* fatigue limit provided by the regression model, i.e., that one understood as the maximum stress range allowing an unlimited number of cycles to be applied without failure, but it corresponds to a value of the fatigue limit that could be denoted *engineering fatigue limit* associated with a sufficiently high number of cycles in accordance to realistic fatigue lifetimes referred to in practice. It should be noticed that for other materials the model parameters found would lead to a significantly less favorable ratio between the *true* and *engineering* fatigue limits proving that the Risitano method does not provide homogeneous safety coefficients for design. In fact, the results provided by that method have been recurrently validated using results obtained from the staircase method, which is indefectibly related to a fixed limit number of cycles, usually $2 \cdot 10^6$ or 10^7 (see [19]).

Moreover, the trend of data for this and other infrared thermal measurements points out that a steady increase of the thermal change happens from the very beginning of the thermal measurements associated to the lengthy and progressive crack growth process experienced in the crack rather than a sudden change in the temperature or temperature gradient, as implied in the assessment of the Risitano method. It follows that an estimation of the fatigue limit, alternative to the procedure currently applied according to the Risitano method, seems advisable.

Note that the result for the fatigue limit is, undoubtedly, influenced by:

- The uncertainty inherent to the thermal measurements using thermography at the level of the accurateness attainable by the thermographic equipment used and the external disturbances caused by the environmental influences. This affects the fatigue limit determination that should be ascertained, theoretically, for vanishing variations of the temperature.

- The number of cycles considered to achieve the state of the thermal stability of the critical zone in the specimen.

All these factors point out that the value of the fatigue limit furnished by the Risitano method entails certain restrictions, which are seemingly related to a) the number of cycles associated with the presumably stability of the thermal change, b) the procedure for determining the fatigue limit that implies fitting the data through a simple straight line, c) the practical limits of the thermographic technique applied to the vanishing thermal change measurement, and d) the sensitivity of the fitting procedure variables considered and the scale used for the determination of the fatigue limit. While the practical utility of the method is not under question, its limitations concerning uncertainty in the evaluation of the fatigue limit suggest the need of an enhancement in the methodology applied.

A final issue that illustrates the limitations of the current assessment method and deserves further investigation is the influence of the frequency of fatigue loading on the thermographic results. According to a number of experimental programs referred to in the literature (see for instance [20]), frequency is confirmed as exerting no noteworthy influence on the fatigue limit even for a much broader range of variability than that considered here. Nonetheless, the experimental thermographic curves obtained for 10 and 5

Hz (see Table 3 and Fig. 6) prove to be frequency dependent and would provide two sufficiently differentiated fatigue limits (about 400 MPa and 420 MPa, respectively).

Since both left tails of the thermal curve seems to join at a lower value of the stress range, it suggests that the lower flat left tail of the curve rather than that of the steep right tail gives the clue for determining the real fatigue limit. This fact adds new arguments for a critical interpretation of the evaluation method being currently used. Furthermore, the scatter of classical fatigue tests and the temperature measurement under cyclic loading, as suggested by [21] and [22] seems to be amenable for investigation using the proposed model.

6. The proposed novel procedure

The model to be proposed in this section considers that the evolution of the temperature range may be better represented by a continuous function $\Delta T = \Delta T(\Delta\sigma)$ that can be deduced in the following way.

First, a normalized stress range is achieved making

$$\Delta\sigma^* = \frac{\Delta\sigma - \Delta\sigma_0}{\Delta\sigma_{up} - \Delta\sigma_0}, \tag{5}$$

where $\Delta\sigma_0$ is the fatigue limit for $N \to \infty$ and $\Delta\sigma_{up}$ represents an upper limit of the stress range for which theoretically the thermal emission grows without limit. Because the normalized variable $\Delta\sigma^*$ is a monotonically increasing function of ΔT and its range is the interval $[0, 1]$, can be represented by a cumulative distribution function. Assuming a Gumbel function for minima with zero location parameter leads to the proposed model:

$$\log \Delta T^* = -\delta \log[-\log \Delta\sigma^*] = -\delta \log\left[-\log\left(\frac{\Delta\sigma - \Delta\sigma_0}{\Delta\sigma_{up} - \Delta\sigma_0}\right)\right], \tag{6}$$

where $\Delta T^* = \Delta T/\Delta T_0$, δ is the scale parameter of the Gumbel distribution, and ΔT_0 is a reference temperature that can be associated with a location parameter representing a constant temperature increment resulting from the elastic loading and the possible increase of temperature originated in the testing machine both not due to the crack growth.

Table 4: Parameter estimates resulting for the three cases.

Case	Q	$\hat{\Delta T_0}$	$\hat{\delta}$	$\hat{\Delta\sigma_0}$	$\hat{\Delta\sigma^1_{up}}$	$\hat{\Delta\sigma^2_{up}}$
1	0.156	8.962	1.413	242.73	455.07	–
2	0.037	9.670	1.250	244.01	–	483.61
3	0.255	9.422	1.275	244.74	450.06	483.22

6.1. Parameter estimation

To estimate the parameters of model (6), we use a least squares method. More precisely, we minimize with respect to ΔT_0, δ, $\Delta\sigma_0$ and $\Delta\sigma_{up}$

$$Q = \sum_{i=1}^{n} \left(\log \Delta T_i - \log \Delta T_0 + \delta \log \left[-\log \left(\frac{\Delta\sigma_i - \Delta\sigma_0}{\Delta\sigma_{up} - \Delta\sigma_0} \right) \right] \right)^2, \quad (7)$$

where $\{(\Delta\sigma_i, \Delta T_i); i = 1, 2, \ldots, n\}$ are the data points, subject to the following constraints.

$$\begin{aligned} \delta &> 0 & (8)\\ \Delta\sigma_0 &< \min_i(\Delta\sigma_i) & (9)\\ \Delta\sigma_{up} &> \max_i(\Delta\sigma_i) & (10) \end{aligned}$$

6.2. Application to the termographic data

In this section the proposed model is applied to the experimental data results of the thermographic measurement according to Risitano in Table 3.

Three different estimations were done, as follows:

Case 1. Using data set #1 alone.

Case 2. Using data set #2 alone.

Case 3. Using both data sets #1 and #2 and considering two models with common parameters ΔT_0, δ and $\Delta\sigma_0$ and two different values $\Delta\sigma^1_{up}$ and $\Delta\sigma^2_{up}$ of $\Delta\sigma_{up}$.

The parameter estimates are given in Table 4.

Figure 7 represents the fitted curves for the above three cases, showing a good agreement between the theoretical model and the experimental data.

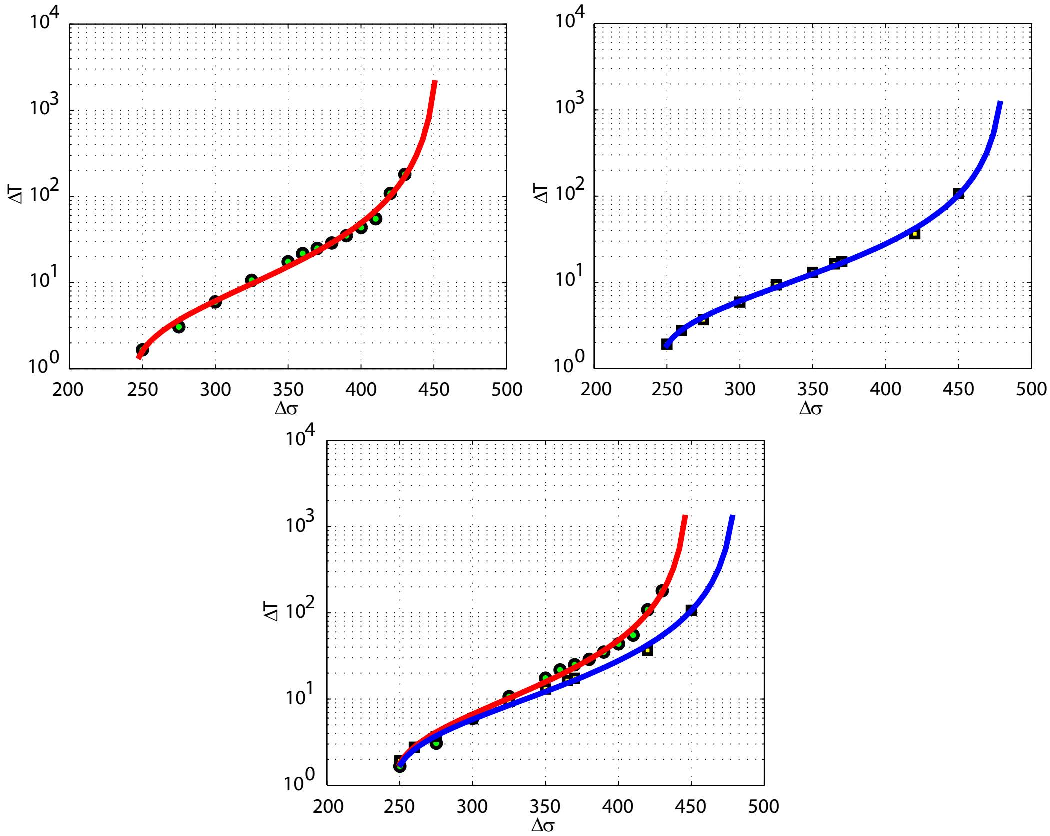

Figure 7: Representation of the new relation proposed between ΔT and $\Delta\sigma$. The upper curves correspond to the separate models, and the lower curve to the joint model.

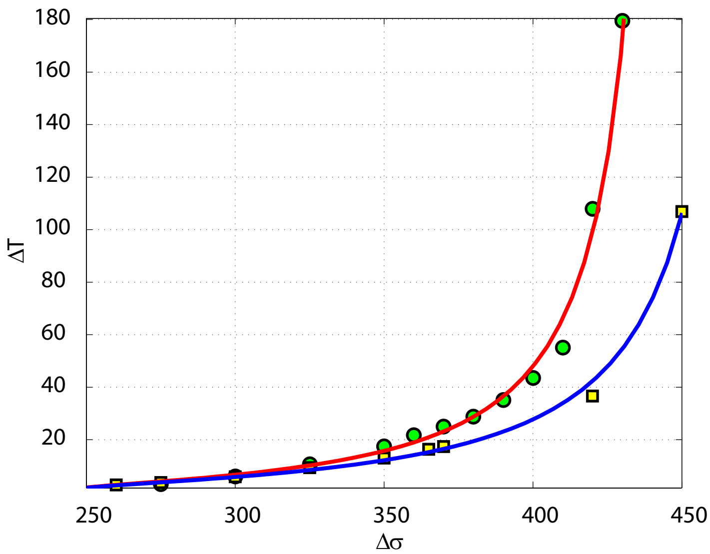

Figure 8: Thermographic data obtained in the present experimental tests for two different frequencies fitted by the proposed model.

In figure 8 the initial information is recovered, that is, the one in Figure 6.

We note that the estimated values of $\Delta\sigma_0$ are very similar for all the three cases. This justifies a common fatigue limit for all frequencies, contrary to the different fatigue limits resulting from the application of the conventional Risitano's procedure.

When considering the two different data sets obtained for the two frequencies 5 and 10 Hz considered in the testing program, a practical coincidence in the estimation of the true fatigue limit is obtained. No contradiction is then observed and the usefulness of the proposed method is thus reinforced even for different testing frequency.

Thus, the revision proposed here underlines the interest of considering different frequencies in the thermographic model to further improvement of the current assessment procedure.

7. Conclusions

The main conclusions of this paper are the following:

1. A Weibull regression model, based on physical and compatibility conditions, was applied for the definition of the probabilistic $\varepsilon - N$ field of a carbon steel C55E. A direct conversion of the strain-life curve to a stress-life curve can be performed allowing an analytical definition of the fatigue limit in the $S - N$ field to be made.
2. The model provides either the *true* value of the fatigue limit of the material for an unlimited lifetime, being insensitive to the probability of failure, or the fatigue limit corresponding to a determined number of cycles, in which case different values are rendered according to the probability of failure chosen.
3. The thermographic method proposed by La Rosa and Risitano was applied to estimate the fatigue limit of the carbon steel C55E. The results were compared with those provided by the probabilistic model.
4. The Risitano method provides a non-conservative estimate of the *true* fatigue limit, understood as the maximum stress range allowing an unlimited number of cycles to be applied without failure. Instead, it seems to provide a conservative value of an*engineering* fatigue limit corresponding to a sufficiently high number of cycles (even for low probability of failure) in accordance to current fatigue lifetimes referred to in the practical design.
5. Thus, the applicability of the thermographic method in practice is confirmed once more though the necessity of differentiating between the *true* and the *engineering* fatigue limits is suggested.
6. An alternative to the current assessment method is proposed. It provides an improved fatigue limit approximation to that resulting from the probabilistic regression model of [1] or [2], and proves to be independent of the test frequency used. Thus, another perspective for the interpretation of the Risitano methodology, and its utility is improved.

8. Acknowledgements

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References

[1] E. Castillo, A. Fernández Canteli, A general regression model for lifetime evaluation and prediction, International Journal of Fracture (107) (2001) 117–137.

[2] E. Castillo, A. Fernández-Canteli, H. Pinto, M. López-Aenlle, A general regression model for statistical analysis of strain-life data, Materials Letters D 62 (2008) 3639–3642.

[3] E. Lehr, Die dauerfestigkeit, ihre bedeutung fr die praxis und ihre kurzfristige ermittlung mittels neuartiger prfmaschinen, Glasers Ann. Gew. (1184) (1926) 109–114.

[4] J. Bérard, S. Rathery, A. S. Béranger, Dtermination de la limite d'endurance des matriaux par thermographie infrarouge, Materials and Technology (1-2) (1998) 55–57.

[5] M. Poncelet, C. Doudard, S. Calloch, B. Weber, F. Hild, Probabilistic multiscale models and measurements of self-heating under multiaxial high cycle fatigue, Journal of the Mechanics and Physics of Solids 27 (58) (2010) 578–593.

[6] M. P. Luong, Infrared thermographic scanning of fatigue in metals, Nuclear Engineering and Design 2-3 (158) (1995) 363–376.

[7] J. C. Krapez, D. Paco, C. Bertin, Application of lock-in thermography to a rapid evaluation of the fatigue limit in metals, in: Proceedings of the 5th AITA International Workshop on Advanced Infrared Techn. and Appl., Grinzato, E., et al. (Eds.), Venezia, Italy, 1999, pp. 379–385.

[8] G. La Rosa, A. Risitano, Thermographic methodology of the fatigue limit of materials and mechanical components, International Journal of Fatigue (22) (2000) 65–73.

[9] F. Cura, G. Curti, R. Sesan, A new iteration method for the thermographic determination of fatigue limit in steels, International Journal of Fatigue 27 (4) (2005) 453–459.

[10] A. Fernández-Canteli, E. Castillo, H. Pinto, M. López-Aenlle, Estimating the S-N field from strain-lifetime curves, Strain in press.

[11] O. Basquin, The exponential law of endurance tests, Tech. rep., ASTM, Philadelphia, PA (1910).

[12] L. Coffin, A study of the effect of cyclic thermal stresses on a ductile materials, Transactions of ASME 76 (1954) 931–950.

[13] S. Manson, Fatigue: a complex subject-some simple approximation 5 (1965) 193–226.

[14] J. Morrow, Fatigue properties of metals, section 3.2 of *fatigue design handbook*, Tech. rep., Warrendale, Pa. (1964).

[15] J. Morrow, Cyclic plastic strain energy and fatigue of metals. internal friction, damping and cyclic plasticity, Tech. rep., Warrendale, Pa. (1965).

[16] C. Williams, Y. L. Lee, J. Rilly, A practical method for statistical analysis of strain-life fatigue data, International Journal of Fatigue 25 (2003) 427–436.

[17] E. Castillo, M. López Aenlle, A. Ramos, A. Fernández Canteli, V. Esslinger, R. Kieselbach, Specimen length effect on parameter estimation in modelling fatigue strength by weibull distribution, International Journal of Fracture (28) (2006) 1047–1105.

[18] E. Castillo, A. S. Hadi, Modeling lifetime data with application to fatigue models, Journal of the American Statistical Association 90 (1995) 1041–1054.

[19] E. Castillo, A. Ramos, R. Koller, M. López-Aenlle, A. Fernández-Canteli, A critical comparison of two models for assessment of fatigue data, International Journal of Fatigue 30 (1) (2008) 45–57.

[20] R. Munier, C. Doudard, S. Calloch, B. Weber, Towards a faster determination of high cycle fatigue properties taking into account the influence of a plastic pre-strain from selfheating measurements, Procedia Engineering (2) (2010) 1741–1750.

[21] C. Calloch, F. Hild, P. Cugy, A. Galtier, Identification of the scatter in high cycle fatigue from temperature measurements, Comptes Rendues de Mcanique 332 (10) (2004) 795–801.

[22] C. Doudard, S. Calloch, Influence of hardening type on self-heating of metallic materials under cyclic loadings at low amplitude, European Journal of Mechanics and Solids 28 (2009) 233–240.