

Coordination of inbound and outbound deliveries in a distribution center

Daniel Velasco Lastra

Supervisor: Prof. dr. Birger Raa

Master's Dissertation submitted in partial fulfillment of the requirements for the student exchange program

Department of Industrial Systems Engineering and Product Design

Chair: Prof. dr. El-Houssaine Aghezzaf

Faculty of Engineering and Architecture

Academic year 2016-2017



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Preface

Six months ago I started this dissertation as an operations research project. The objective was to develop a mathematical model that significantly improved the existing algorithms for the optimization of cyclic inventory routing problems. This paper is the report of this long process. However, it cannot express the long days spent in front of the desktop, developing and testing different linear models with large numbers of different instances.

This project was carried out together with prof. Birger Raa as supervisor within the department of “*Industrial Systems Engineering and Product Design*” at Ghent University. Fortunately, he was always available and willing to answer my queries. I would like to thank him for his excellent guidance and support during this process.

I also wish to thank the different professors that have taught me during this academic year as an exchange student. I have learned many useful concepts that I am sure that will help me to develop a successful career in the future.

My home university also deserves a word of thanks for allowing me to enjoy this experience.

Finally, I would like to thank my family because your supporting and kind words have always helped me to overcome the difficulties and challenges that I have had to face.

Thank you all for your support

Daniel Velasco Lastra

Ghent, June, 2017

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Daniel Velasco Lastra, June 2017

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by

Daniel VELASCO LASTRA

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Academic Year 2016–2017

Promoter: Prof. Dr. Ir. Birger RAA
Faculty of Engineering and Architecture
University of Ghent

Department of Industrial Systems Engineering and Product Design
Chair: Prof. dr. Ir. El-Houssaine AGHEZZAF

Abstract

This paper explains the development and analysis of three mathematical models for a two-echelon supply chain. The main objective of these models is the coordination of inbound and outbound deliveries in a distribution center and the optimal balance between transportation and holding costs at either depot and retailers that minimizes the overall costs of the mentioned supply chain.

Keywords

Supply Chain, Inventory Routing Problem, Two-echelon, cross-dock, replenish, scheduling, retailers.

Coordinating inbound and outbound deliveries in a distribution center

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Supervisor(s): prof. ir. Birger Raa

Abstract— This paper aims to develop and test three different mathematical models for a two-echelon supply chain. Their main objective is the coordination of inbound and outbound deliveries in a distribution center and the optimal balance between transportation and holding costs at either depot and retailers that minimizes the overall costs.

The first model presented in this thesis considers periodic replenishment of the retailers with the same quantity of product delivered each time by making use of the same routes. Furthermore, it schedules the outgoing shipments from the depot in order to minimize the overall costs of the supply chain. The second version of the model, also considers the same periodicity at the retailers but does not necessarily have to make use of the same routes at each delivery and neither delivers the same quantity of product each time. Finally, the third version does not consider periodic replenishment of the different retailers.

Keywords— Supply Chain, Inventory Routing Problem, Two-echelon, cross-dock, replenish, scheduling, retailers.

I. INTRODUCTION

THIS problem presents a two-echelon inventory system, in which a distribution center receives incoming products and then delivers it to retailers that are clustered into several pre-defined route options.

On the one hand, there are several possible options to visit the different retailers from which the model has to choose the optimal ones. On the other hand, demand rates and inventory holding costs are defined constant. The solution of this problem is planned cyclically in an infinite planning horizon, therefore, this thesis could be classified as an example of Cyclic Inventory Routing Problem (CIRP).

This CIRP selects the optimal option to deliver products each day. It also quantifies the product units to be delivered and stored at each retailer in any period and sizes the fleet according to the best trade-off to minimize the overall costs.

II. LITERATURE REVIEW

This master dissertation is an improvement of the paper: “*Fleet optimization for cyclic inventory routing problems*”, published in 2015 by B. Raa [1] and the paper: “*Route and fleet design for cyclic inventory routing*”, published in 2017 by B. Raa and W. Dullaert [13]. The idea of considering fixed vehicle costs, route-specific costs and holding costs at the retailers, which are periodically replenished is extracted from B. Raa [1], as well as the idea of considering fleet size in the cost formula. This paper considers that the central depot has enough inventory available to load all vehicles. Hence, no coordination between inbound and outbound deliveries is carried out.

It neither studies the selection of the optimal routes on each day between a set of possible options. Instead, the model is provided with a set of pre-defined routes that have to be scheduled during the cycle.

The paper from Raa and Dullaert [13], considers the same problem, but there, a metaheuristic is performed.

There is also a master’s dissertation on the same topic published in 2016 by M. Alsina, [3]. This master thesis considers the same optimal cycle time for all the customers clustered on the same route. The main problem of this assumption is that setting the periodicity of a specific retailer different to its optimal value can increase the overall cost of the system. The best solution to solve this problem is to adapt the Travel Salesman Problem (TSP) to this scenario and solve a linear problem that considers the route design phase. However, if the study is extended to real instances with a large number of customers, the computation times of the algorithms would increase exponentially. For that reason, a group of possible route options have been established for each different instance from which the model has to choose the optimal one on each day of the week.

The paper of G. Iassinovskaia et. al., [4], considers an inventory routing problem of returnable transport items in a closed-loop supply chain with time windows at customers. The idea of considering limited storage capacity at each retailer has been extracted from this paper. The possibility of setting time window constraints at the retailers has also been studied. However, this assumption would not add valuable information about how to spread the transportation costs among the time horizon. Thus, this possibility was discarded.

In the paper of K. Shang et. al., [5] and in the one of M. Seifbarghy and M. R. A. Jokar, [6] a two-echelon system is also considered. Each facility faces Poisson independent demands. Therefore, a stochastic is performed in order to obtain the optimal base-stock levels and reorder intervals for all the retailers. On the other hand, there are no papers in the current literature that consider the problem of choosing between a set of possible route options to deliver products to retailers over a period of time.

This idea has raised since not all retailers in a supply chain have normally the same holding costs and demand rates. They also can have different inventory levels and therefore, the optimal routes from one delivery to another one may change. Therefore, considering several route options and choosing the optimal ones on each day of the horizon depending on the necessities of the retailers seems to be a good idea.

Finally, due to similarities with the model, the same framework as in the paper of Raa and Aghezzaf [2] has been used for the design of the different instances during the analysis phase.

III. MATHEMATICAL MODEL

As mentioned above, this paper presents a two-echelon system with a central depot and different retailers with deterministic and continuous demand. The model is aimed to adjust the incoming and outgoing products in order to store them or cross-dock the items with the aim of minimizing the overall costs.

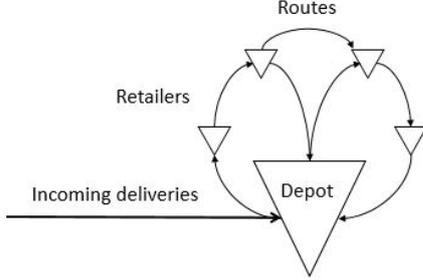


Fig. 1. Two-echelon inventory system

The considered costs of this system are the fixed vehicle cost, the fixed cost of incoming shipments, the cost of making the routes and the inventory holding costs (at the depot and the retailers). Some other constraints are added such as product flows, capacity constraints and limits on the daily total driving time. The entire model is described by the following constraints:

$$O.F. = \text{Min} \sum_{v \in V} F.nT.Z_v + \sum_{r \in R} \sum_{v \in V} \sum_{d \in nT} X_{vrd}C_r + H. \sum_{d \in nT} ID_d + H. \sum_{c \in C} \sum_{d \in nT} IR_{cd} + S. \sum_{d \in nT} Y_d \quad (1)$$

$$Z_v \leq Z_{v-1}; \forall v \in V \quad (2)$$

$$ID_0 = ID_{nT} \quad (3)$$

$$IR_{c0} = IR_{cnT}; \forall c \in C \quad (4)$$

$$IR_{10} = 0 \quad (5)$$

$$IR_{11} \geq IR_{1d}; \forall d \in nT \quad (6)$$

$$\sum_{v \in V} Qdel_{r cd} \leq MM \sum_{v \in V} X_{vrd} \begin{cases} \forall d \in nT \\ \forall r \in R \end{cases} \quad (7)$$

$$Qdel_{r cd} \leq A_{rc} MM \sum_{v \in V} X_{vrd} \begin{cases} \forall d \in nT \\ \forall r \in R \\ \forall c \in C \end{cases} \quad (8)$$

$$QI_d \leq MM Y_d; \forall d \in nT \quad (9)$$

$$ID_d = ID_{d-1} + QI_d - \sum_{r \in R} \sum_{c \in C} Qdel_{r cd}; \forall d \in nT \quad (10)$$

$$IR_{cd} = IR_{cd-1} - Dem_c + \sum_{r \in R} Qdel_{r cd} \begin{cases} \forall d \in nT \\ \forall c \in C \end{cases} \quad (11)$$

$$IR_{cd} \leq IK_c; \forall c \in C \quad (12)$$

$$\sum_{d=1}^{T_c} \sum_{v \in V} \sum_{r \in R} X_{vrd} A_{rc} = 1; \forall c \in C \quad (13)$$

$$Qdel_{r cd} = Qdel_{r cd+T_c} - \begin{cases} \forall d \in nT - T_c \\ \forall r \in R \\ \forall c \in C \end{cases} \quad (14)$$

$$\sum_{r \in R} D_r X_{vrd} \leq MZ_v \begin{cases} \forall d \in nT \\ \forall v \in V \end{cases} \quad (15)$$

$$Sdel_{cd} = \sum_{r \in R} Qdel_{r cd} \begin{cases} \forall c \in C \\ \forall d \in nT \end{cases} \quad (16)$$

$$Z_v, X_{vrd} \in \{0, 1\} \quad (17)$$

$$ID_d, IR_{cd}, Y_d, QO_{rd}, QI_d, Qdel_{r cd}; Sdel_{cd} \geq 0 \quad (18)$$

As mentioned before, the first version of this model, considers that a specific retailer must be delivered the same quantities periodically by making use of the same specific routes each time. However, the second version places the outgoing shipments periodically but not with the same quantity. This means that each retailer $c \in C$ will be replenished every T_c days respectively but can be shipped from one route on a specific delivery and from a different route on the next one. Furthermore, the quantity delivered to this retailer can vary each time. In order to implement this change, constraints (14) are replaced for the (19) ones.

$$\sum_{v \in V} \sum_{r \in R} X_{vrd} A_{rc} = \sum_{v \in V} \sum_{r \in R} X_{vrd+T_c} A_{rc} \begin{cases} \forall c \in C \\ \forall d \in nT - T_c \end{cases} \quad (19)$$

Finally, the third version of the model does not consider periodicity in the replenishment of retailers. Moreover, it neither restricts the route that has to replenish to a specific customer and the quantity delivered can also change from one shipment to another.

In order to implement the mentioned modifications, constraints (13) and (14) of the model must also be replaced by the (20) ones.

$$\sum_{r \in R} A_{rc} \sum_{v \in V} X_{vrd} \leq 1 \begin{cases} \forall d \in nT \\ \forall c \in C \end{cases} \quad (20)$$

IV. DESIGN OF INSTANCES

In the literature of the cyclic inventory routing problem, some datasets are available, e.g. the paper of Sindhuchao et al., [7], the one of Aghezzaf et al., [8] or the paper of Birger Raa [1]. However, these datasets cannot be used in this thesis for several reasons. In Sindhuchao et al., [7], no fixed vehicle cost is considered; in Aghezzaf et al., [8], a single vehicle is assumed so fleet sizing is not an issue; in Birger Raa [1], the route design phase is not considered. Moreover, it does not consider inventory capacity constraints at the retailers.

However, due to the similarity with the model, the instances that have been chosen to test the different models are based on the instances proposed in Raa and Aghezzaf [2] with some modifications. The mentioned instances are generated according to a 5×2^4 Factorial Design in which the different factors that have been analyzed are illustrated in table I.

Factor	Shorthand	Level '-1'	Level '1'
Customer capacity restriction	CCAP	No	Yes
Holding cost rate	H	0.08 / (u. day)	0.8 / (u. day)
Fixed Vehicle cost	F	100 / (u. day)	400 / (u. day)
Number of customers	NR	10 Cust.	15 Cust.

TABLE I
KEY FACTORS CONSIDERED IN THE FACTORIAL DESIGN

V. NUMERICAL RESULTS

The first factor studied was the customer storage capacity restrictions with the levels "Yes" or "No". After analyzing the results, it has been proved that this factor has no influence on the solution for models 1 and 2. However, it has a significant effect on model 3. If customers impose a storage capacity restriction on the third model, the total cost of the solution is increased by 6,79 %.

Secondly, the holding cost rate was studied. In this case, the holding cost has a notorious influence on the total cost of the solutions for the three different models. Furthermore, when holding costs are low, larger deliveries are made causing a decrease on the average stock level. These larger delivery quantities also imply that less customers are visited per tour.

The last considered factor is the number of customers. It has been noticed that there is a small economy of scale when servicing customers. Moreover, in larger instances, more customers are served per tour at the same time that larger quantities are delivered. This last statement is possible because of the increased utilization of the vehicles.

Finally, the two-way interactions of this factors were analyzed. However, the statistical analysis showed that any of this interactions was significant for any of the models. The study also proved that the fixed holding cost had a notorious influence on the total cost, but did not have any effect in the actual solutions for any of the three different models. Therefore, this factor was

excluded.

VI. COMPARISON TO ANOTHER HEURISTIC

In table II, the results of the three different models developed in this paper for a group of instances are compared to the Raa and Dullaert [13] metaheuristic results for the same set of experiments.

For all problem instances, solutions are found that are cheaper than those proposed by Raa and Dullaert [13]. It is worth mentioning that the cost decrease for the first two versions of the model is not significant (On average less than 1 %).

However, for the third version of the model, this cost decrease is on average 4,93 % for small instances and 10,34 % for large instances. Therefore, larger instances strengthen the improvement effect of this solution.

CCAP	F	H	NR	Raa and Dullaert	Ver. 1	Δ	Ver. 2	Δ	Ver. 3	Δ
Yes	400	8c	10	615,2626	612,72	-0,41%	610,92	-0,71%	584,93	-4,93%
Yes	400	8c	15	716,6962	698,71	-2,51%	698,71	-2,51%	642,59	-10,34%

TABLE II
COMPARING THE SOLUTION CHARACTERISTICS

VII. SUGGESTIONS FOR FURTHER RESEARCH

In this problem, delivery options are already created and fixed during the time schedule. They have been created by randomly making subtours of the optimal solutions provided by the Raa and Dullaert algorithm [13]. One interesting improvement would be the design of a heuristic algorithm to find 50 good delivery options which contain the optimal solutions that could serve as input of the models of this dissertation at each period of the horizon.

Another interesting improvement since the version 3 of this dissertation provides really good results compared to another existing algorithm but has excessive running times for real instances would be the development of a heuristic that finds near-optimal solutions for this version of the model in smaller running times. Therefore, this solution could be applied to real instances more easily.

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Chapter 1

Introduction

In this introductory part of the thesis, the purpose, the scope and some motivational factors that have driven the execution of this master's dissertation are provided in order to understand the general framework in which this paper is developed.

1.1 Purpose

Firstly, the main objective of this project involves the development of several mathematical models in CPLEX in order to minimize the overall costs of a two-echelon supply system. The aim of this models is to schedule income and outgoing products in a distribution center at the cheapest overall cost in order to replenish a set of customers. It considers several route options from which the models must choose the optimal ones to make those shipments on each day of the considered cycle. Moreover, the presented models should size the required vehicle fleet and determine the best quantity to be delivered at the retailers.

Finally, the behavior of each version has to be analyzed by changing some key factors in order to obtain a better understanding of these models.

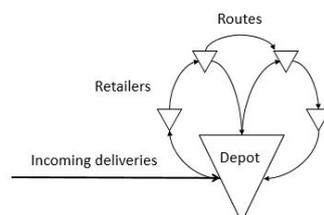


Figure 1.1: Scheme of the IRP of this dissertation

1.2 Scope

Now, the different tasks that are encompassed by the scope of this project are listed in detail.

- First of all, a linear mathematical model that contemplates the issue of choosing the best route options to periodically deliver a group of retailers is developed. This model has been improved into two different versions in order to obtain better results.
- Secondly, a group of datasets have been prepared in excel sheets and the three versions of the model have been run making use of the software “IBM ILOG CPLEX Optimization Studio”.
- Thirdly, the obtained results have been analyzed looking for behavior patterns and some key factors have been modified in order to assess their influence on the overall behavior of the chain.
- Finally, the results and some conclusions have been written as guidelines for further investigation.

1.3 Motivation

This section gives a brief overview of the technical and personal factors that have motivated the execution of this dissertation.

1.3.1 Technical reasons

Firstly, supply chain management plays a very important role in a large variety of societal issues. For example, in 2005 Hurricane Katrina flooded New Orleans, causing a tremendous societal problem and leaving its residents without food or water. As a result, a massive rescue of inhabitants was carried out. During the first weekend, 1.9 million meals and 6.7 million liters of water were delivered. The coordination of all people and the stages in the supply chain posed an important challenge for supply chain management. Another application of this topic is in human healthcare. During a medical emergency, supply chain performance can be the difference between life and death.

In economy, supply chain management encompasses all enterprises and associations in the transformation process from raw materials to the end product as well as the associated information flows.

Moreover, in a context of global recession and a hard-economic crisis, companies try to reduce costs in order to provide goods and services with high quality at low costs and be able to compete in global markets. Between 20 and 40% of the total cost of most products consists of controllable logistics costs: “*inventory, transportation and handling costs*”. Hence, having a deep knowledge on how to manage logistic flows, integrate and coordinate the different stages in a supply chain and control the storage of products is particularly important for an engineer.

Finally, there are a lot of jobs in this field which is still growing at an exponential rate. For example, a new MHI report, states that the logistics business will be looking to fill about 1.4 million jobs, or roughly 270,000 per year, by 2018 in U.S..

1.3.2 Personal reasons

As an industrial engineer, this master's dissertation has given me the opportunity to deepen my knowledge in quantitative methods and analytical skills. Moreover, it has allowed me to acquire a solid dominance of mathematical tools such as "IBM ILOG CPLEX Optimization Studio" or statistical programs such as "R".

On the other hand, supply chain management and logistics are areas that strike me and in which I am encouraged to work and develop a successful career in the future. This master thesis has allowed me to have a first experience in this field and apply the concepts that I have been learning during the year as well as getting an insight into how a future in operations research would be like.

Chapter 2

General overview

This chapter provides context information about the definition and historical development of inventory routing problem solutions from its beginning until the current situation. It also gives a brief description of the main characteristics of the different typologies of IRP and the particular case of the problem tackled in this thesis. Finally, a literature review of the papers that have influenced the development of this thesis is carried out.

This master thesis can be described as a specific case of Inventory Routing Problem where a finite time horizon is considered and supposed to be repeated into the infinity. Moreover, coordination between inbound and outbound deliveries in this distribution center is performed.

2.1 Evolution of the IRP

The Inventory Routing Problems, date back 30 years and are often described as a combination of vehicle routing and inventory management problems in which a supplier has to deliver products to a number of different customers located in several locations subject to side constraints. This kind of problems provide integrated logistics solutions by simultaneously optimizing inventory management, vehicle routing and delivery scheduling.

However, the IRP arises in the context of Vendor-Managed Inventory (VMI). VMI problems are a family of business models in which a customer provides certain information to the supplier, who takes full responsibility for maintaining an agreed inventory of product, usually at the buyer's consumption location. This policy is usually taken in order to

reduce logistics costs and add business value to the supply chain. It is often described as a win-win situation where vendors save on transportation and production costs, as they can coordinate the manufacturing scheduling and shipments made to different customers. Buyers also benefit by not allocating efforts to inventory management and reducing the risk of unintended stock outs at the same time.

With this kind of policies, the supplier normally has to make three simultaneous decisions:

- When to serve a specific customer.
- How much to deliver.
- How to combine customers into the different vehicle routes.

2.2 Typologies of the problem

The existing IRP can be classified attending to two different schemes. The first of them refers to the structural variants present in IRPs whereas the second one is related to the availability of information about demand.

Through the past 30 years, many variants of this problem have raised and there is not a standardized version. Therefore, in this section only the “basic versions” of the IRP are explained, on which most of the research effort has been concentrated through the last years.

This basic versions are presented in Table 2.1.

Table 2.1: Structural variants of the IRP

Criteria	Possible options		
Time horizon	Finite	Infinite	
Structure	One-to-one	One-to-many	Many-to-many
Routing	Direct	Multiple	Continuous
Inventory policy	Maximum level(ML)	Order-up-to level (OU)	Fixed-Time period
Inventory decisions	Lost sales	Back-order	Non-negative
Fleet composition	Homogeneous	Heterogeneous	
Fleet size	Single	Multiple	Unconstrained

Source: Adapted from Coelho et. al. [11]

As it can be seen in the mentioned table, the models can be classified attending to seven different criteria called: time horizon, structure, routing, inventory policy, inventory decisions, fleet composition and fleet size.

In table 2.1, time refers to the horizon taken into account by the model which can be either finite or infinite. The number of suppliers and customers can vary, and therefore the structure of the IRP can be one-to-one when there is only one supplier and one customer, one-to-many in the most common case having one supplier or depot that serves several customers, or many-to-many with more than one suppliers and more than one customers.

Routing on the other side, can be direct when there is only one customer per route, multiple when there are more than one customers clustered on the same route and continuous when there is no central depot.

Inventory policies define pre-established rules to replenish customers. The two most common ones are the maximum level (ML) policy, the order-up-to-level (OU) and the Fixed-Time Period policy. The order-up to level (OU) policy, determines the quantity shipped to each retailer in such a way that the level of its inventory reaches a specific level smaller than the retailer's capacity. However, in the Maximum Level (ML) policy, the quantity shipped to each retailer is such that the inventory cannot exceed its maximum level. Finally, in the Fixed-Time period policy, each customer is visited with fixed frequency and is delivered different quantities of products each time.

Inventory decisions determine how inventory management is tackled. If this inventory is allowed to be negative, then back-ordering occurs and the corresponding demand will be served at a later stage. If there are no back-orders, then the extra demand is lost and is considered as lost sales. In both cases there exist a penalty cost for the stock-out.

Finally, the last two criteria refer to fleet composition and size. The fleet can either be homogeneous or heterogeneous and the maximum number of vehicles used can be fixed at one single vehicle, limited at several vehicles or unconstrained.

The second classification criteria refers to the time at which information on demand becomes known. If it is constant and the information is fully available at the beginning of the planning horizon, the problem is deterministic. If what is known is the probability distribution of demand, the problem is then stochastic and it yields the Stochastic Inventory Routing Problem (SIRP)

2.3 Classification on the basic IRP

In this master's dissertation, a unique distribution center is considered. It receives incoming shipments when needed and then, it delivers to a set of customers already clustered in a set of possible route options depending on their proximity and demand patterns. Demand rates are considered constant and back ordering is not allowed. Therefore, this IRP is an example of deterministic problem.

Attending to the first criteria scheme mentioned in the section 2.2, this Cyclic Inventory Routing Problem can be described as follows:

- Firstly, the considered time horizon of this model is “Finite”, as the cycle of time considered in the model is fixed despite it will be repeated continuously over time.
- Secondly, the structure of this model can be described as “One-to-many”, as only one depot is considered from where products are delivered to a set of possible customers.
- In this model, several customers are clustered in a set of possible route options. Therefore it can be classified as a “Multiple Routing” IRP.
- The inventory is managed with a “Fixed -Time period” policy for the first two versions of this model as each customer is delivered with fixed periodic intervals and the order size can fluctuate. However, the third version makes use of “Fixed-Order Quantity Shipment” as it places the different orders when the inventory levels arrive to 0.
- Back-order and stock-out are not permitted. Therefore, the inventory levels are “Non-negative”.
- Finally, the fleet is “Homogeneous” and its size is “Multiple” as a fixed number of trucks are available.

Table 2.2: Classification on the basic versions of IRP

Reference	Time horizon	Structure	Routing	Inventory policy	Inventory decisions	Fleet com.	Fleet size
Own project	Finite Infinite	One-to-one One-to-many Many-to-many	Direct Multiple Continuous	Maximum Level Order-up to level Periodic Review	Lost sales Backlogging Non-negative	Homogeneous Heterogeneous	Single Multiple Unconstrained
	✓	✓	✓	✓	✓	✓	✓

Source: Adapted from Coelho et. al. [11]

2.4 Literature Review

Referring to the literature review, there are many papers that have had a notorious impact on the development of this master dissertation.

First of all, the master thesis entitled “Coordinating inbound and outbound deliveries in a distribution center” M. Alsina, (2016) [4], has been taking into account in order to structure some parts of this project. Actually, this last thesis has been born as a further application of the abstract “Fleet optimization for cyclic inventory routing problems” B. Raa,(2014) [3].

In B.Raa’s (2014) [3] paper, an infinite horizon is considered, where the optimization of a vehicle fleet is studied in order to periodically repeat a given set of routes considering the overall cost rate that is composed of fixed vehicle costs, route-specific costs and holding costs at the customers. The main difference with this paper is that B. Raa’s paper [3] considers that the central depot has enough stock available to load all the shipments during the time horizon and therefore, no coordination with inbound products is done.

In M.Alsina’s (2016)[4] thesis, the mentioned consideration is removed and therefore, inbound deliveries are considered. Nevertheless, the customers are already clustered in a set of pre-defined routes and therefore, the routes design phase is not considered. Moreover, no inventory capacity is considered at the retailers. Finally, the different routes are unique and have to be scheduled without the possibility of changing the routes to deliver the different customers along time.

In the G. Iassinovskaia’s (2017) paper [5], it is considered a producer, located at a depot, who has to distribute his products to a set of customers. Moreover, each partner has a storage area composed of both empty and loaded RTI stock, as characterized by initial levels and maximum storage capacity. The idea of considering the inventory capacity of each retailer has been extracted from this paper.

In the paper of Raa and Aghezzaf [12], a heuristic column generation approach is proposed, analyzed and evaluated against a comparable state-of-the-art heuristic in order to solve an inventory routing problem. In this paper, any stock-out is permitted and deterministic customer demand rates are assumed, integrating vehicle fleet sizing, vehicle routing and inventory management. Furthermore, some realistic constraints are introduced as in this dissertation such as limited storage capacities or driving time restrictions .

The paper of B. Raa and W. Dullaert [13], has also been considered. The metaheuristic proposed to solve the cyclic inventory routing problem has been used in order to design the different route options of this thesis.

On the other hand, the paper of K. Shang et. al., [6] and the one of M. Seifbarghy and M. R. A. Jokar, [7] also consider a two-echelon system. Each facility faces Poisson independent demands. Therefore, a stochastic is performed in order to obtain the optimal base-stock levels and reorder intervals for all retailers.

Finally, there are no papers in the current literature that consider the problem of choosing between a set of possible route options to deliver products to retailers over a time horizon.

This idea has raised since not all retailers in a supply chain have normally the same holding costs and demand rates. They can also have different inventory levels and therefore, the optimal routes from one deliver to another one may change. However, if the design of the routes was considered as in the TSP, the computing times would increase exponentially with the number of customers. Therefore, considering only several options and choose the optimal ones on each day of the horizon depending on the necessities of the system seems to be a good idea.

Chapter 3

Mathematical model

This chapter explains the three different versions of the mathematical model developed in this project as well as the notation used for data and variables as well as their main differences.

3.1 Description of the problem

As this project is an enhancement of the “Fleet optimization for cyclic inventory routing problems” B. Raa, 2014 [3], and the master thesis of M. Alsina, 2016 [4], the same framework of this papers has been used.

In this two-echelon supply chain system, the demand rates d_j are set constant and the considered time horizon is infinite. Therefore, the obtained schedule for a specific cycle, must be repeated periodically. This cycle is established as the common multiple minimum of the retailer’s optimal cycle times.

Moreover, in this system, there is a depot in one stage, in which incoming and outgoing deliveries must be coordinated and the transport of outgoing products to the retailers is planned. As both parties are involved in the global cost of the supply chain, an integrated supplier-retailer optimum will be achieved, which is more profitable than two separate optimums.

The considered retailers $c \in C$ in this supply chain can be replenished from a set of possible route options $r \in R$ that have been created before-hand attending to proximity and cost criteria. At each time-period, the model must choose between the different delivery options in order to minimize the overall supply chain costs.

On the other hand, the required cycle time of each retailer is denoted by T_r . Each vehicle has a limited truck capacity MM and each retailer has a limited storage capacity IK_c . Moreover, it takes a certain amount of time to complete a route D_r , including traveling from the depot to the retailers, as well as the loading and unloading times of the vehicles at the different places. A vehicle is also allowed to make several routes on one day, but the total driving time per day is limited to M hours. It is therefore assumed that any route cannot take more than this daily amount of time available (to avoid infeasibilities).

As the objective is to minimize the total costs, the cost structure must be defined. The cost rate of the distribution schedule is composed out of the following four components:

- A fixed cost rate per vehicle F . The total number of vehicles that are available for the distribution schedule is denoted by “ V ”. When used, the cost rate per vehicle is charged regardless its utilization, which means that even if one vehicle is only used to make a small route with many days between consecutive iterations, the daily cost is still charged for every day in the cycle.
- The cost for making the route r is denoted by C_r . This cost includes the cost of loading the vehicles, transporting the items through the route and dispatching cost at each retailer. It is worth mentioning that the cost of making each route is highly dependent on the cost and duration of each route.
- The third component is the holding cost at each retailer and at the depot. In order to compute this cost, a constant holding cost rate H per unit per day is charged at both retailers and depot.
- The last component is the fixed cost for replenishing the depot S that is charged per order each time the depot needs to receive incoming products.

Therefore, the cost of a distribution schedule in which a set of routes R is replenishing a group of customers C is given by the following formula:

$$\begin{aligned}
 O.F. = \text{Min} \sum_{v \in V} F.nT.Z_v + \sum_{r \in R} \sum_{v \in V} \sum_{d \in nT} X_{vrd}C_r + H. \sum_{d \in nT} ID_d \\
 + H. \sum_{c \in C} \sum_{d \in nT} IR_{cd} + S. \sum_{d \in nT} Y_d
 \end{aligned} \tag{3.1}$$

The first term of the previous equation corresponds to the fixed vehicle cost of the vehicles used in the distribution schedule. The second term is the cost of making the different routes, while the third and fourth components of the same equation are the holding costs at the depot and the retailers. Finally, the last component corresponds to the cost of replenishing the depot.

3.2 Notation of parameters

The notation of the different parameters used in this project are listed below. At the same time, a small explanation is given as well as its corresponding units:

R Number of possible route options to deliver the different retailers. Each route can deliver either a unique retailer or more than one. A specific retailer can also be delivered from more than one option.

C Number of retailers in the supply chain.

V Maximum number of outgoing delivery vehicles in a distribution schedule.

M Limit of total driving hours per day for each vehicle (h/day).

nT Total number of periods of the cycle (days).

T_c Periodicity of retailer “ c ” (days).

D_r Duration of route “ r ” (hours).

Dem_r Demand rate of retailer “ c ” (Number of units/day).

MM Truck capacity (Units).

A_{rc} Binary matrix that indicates if the route “ r ” visits the retailer “ c ”.

- IK_c Inventory capacity at the retailer “ c ”.
- C_r Cost of making the route “ r ” (€/route).
- F Fixed cost per vehicles in euros per day (€/day).
- S Fixed cost for replenishing the depot (€/replenishment).
- H Holding cost at the retailers and the depot (€/unit.day).

3.3 Notation of variables

The notation of the different variables are the listed below as well as a little explanation of its notation.

- X_{vrd} Binary variable that is “1” if the vehicle “ v ” makes the route “ r ” on day “ d ”.
- Z_v Binary variable that indicates wether the vehicle “ v ” is used.
- ID_d Inventory level at the depot on day “ d ”.
- IR_{cd} Inventory level at the retailer “ c ” on day “ d ”.
- Y_d Number of vehicles needed for replenishments of the depot on day “ d ”.
- QI_d Number of units of product that enters the depot on day “ d ”.
- QO_{rd} Number of units of product that leave the depot through route “ r ” on day “ d ”.
- $Qdel_{rcd}$ Number of units of product delivered through the route “ r ” to retailer “ c ” on day “ d ”.
- $Sdel_{cd}$ Total number of units delivered to retailer “ c ” on day “ d ” as sum of all the possible options.

3.4 Mathematical model (Version 1)

Once the notation has been established, the first mathematical model to solve this problem can be developed.

As mentioned above, this version takes into account the periodicity requirements of each retailer in order to deliver them every T_r days, making use of the same routes during each delivery. Furthermore, the quantity delivered each time must be the same.

This model, schedules the routes in order to minimize the overall costs across the supply chain within a finite space of time that will be repeated cyclically over the time.

The mathematical equations of the model are presented below. After the equations, a brief explanation about each constraint is given.

$$\begin{aligned}
 O.F. = \text{Min} \sum_{v \in V} F.nT.Z_v + \sum_{r \in R} \sum_{v \in V} \sum_{d \in nT} X_{vrd}C_r + H. \sum_{d \in nT} ID_d \\
 + H. \sum_{c \in C} \sum_{d \in nT} IR_{cd} + S. \sum_{d \in nT} Y_d
 \end{aligned} \tag{3.2}$$

$$Z_v \leq Z_{v-1} ; \forall v \in V \tag{3.3}$$

$$ID_0 = ID_{nT} \tag{3.4}$$

$$IR_{c0} = IR_{cnT} ; \forall c \in C \tag{3.5}$$

$$IR_{10} = 0 \tag{3.6}$$

$$IR_{11} \geq IR_{1d} ; \forall d \in nT \tag{3.7}$$

$$\sum_{v \in V} Qdel_{r cd} \leq MM \sum_{v \in V} X_{vrd} \begin{cases} \forall d \in nT \\ \forall r \in R \end{cases} \tag{3.8}$$

$$Qdel_{r cd} \leq A_r c MM \sum_{v \in V} X_{vrd} \begin{cases} \forall d \in nT \\ \forall r \in R \\ \forall c \in C \end{cases} \tag{3.9}$$

$$QI_d \leq MM Y_d ; \forall d \in nT \quad (3.10)$$

$$ID_d = ID_{d-1} + QI_d - \sum_{r \in R} \sum_{c \in C} Qdel_{rcd}; \forall d \in nT \quad (3.11)$$

$$IR_{cd} = IR_{cd-1} - Dem_c + \sum_{r \in R} Qdel_{rcd} \begin{cases} \forall d \in nT \\ \forall c \in C \end{cases} \quad (3.12)$$

$$IR_{cd} \leq IK_c ; \forall c \in C \quad (3.13)$$

$$\sum_{d=1}^{T_c} \sum_{v \in V} \sum_{r \in R} X_{vrd} A_{rc} = 1 ; \forall c \in C \quad (3.14)$$

$$Qdel_{rcd} = Qdel_{rcd+T_c} - \begin{cases} \forall d \in nT - T_c \\ \forall r \in R \\ \forall c \in C \end{cases} \quad (3.15)$$

$$\sum_{r \in R} D_r X_{vrd} \leq MZ_v \begin{cases} \forall d \in nT \\ \forall v \in V \end{cases} \quad (3.16)$$

$$Sdel_{cd} = \sum_{r \in R} Qdel_{rcd} \begin{cases} \forall c \in C \\ \forall d \in nT \end{cases} \quad (3.17)$$

$$Z_v, X_{vrd} \in \{0, 1\} \quad (3.18)$$

$$ID_d, IR_{cd}, Y_d, QO_{rd}, QI_d, Qdel_{rcd}; Sdel_{cd} \geq 0 \quad (3.19)$$

The objective function (3.2) is aimed to minimize the overall costs across the supply chain. The different terms of that equation according to the order in which they appear are the fixed cost per vehicle, the cost of making the routes, the inventory holding costs at the retailers and the depot and finally the total replenishment cost at the depot as mentioned above in this report.

The constraints (3.3) force the model to choose the vehicles in order from the vehicle list (the vehicles used first). Otherwise, the model would choose them randomly and the number of possible solutions would increase. The constraints (3.4) and (3.5) match the inventory levels at the depot and the retailers of two consecutive cycles. The constraints

(3.4) focus on the depot while the constraints (3.5) implement the same restriction for the retailers.

The constraints (3.6) and (3.7) are set in order to break the symmetry of the solution by fixing the inventory level at retailer 1 on day 0 equal to 0 and forcing that the highest shipment to this retailer is made on the first day of the studied cycle. In this way, the number of symmetric solutions is decreased and the running times are improved. Constraints (3.8) force the total quantity delivered on one day to not exceed the capacity of the vehicles that have been used. Constraints (3.9) force the quantity delivered to a specific retailer using a route to be 0 if that route does not visit the retailer. Constraints (3.10) prevent the quantity that enters the depot on day $d \in nT$ from exceeding the vehicles capacity. Constraints (3.11) and (3.12) define the inventory at the depot and the retailers respectively. Constraints (3.13) limit the inventory level at the retailers to not exceed their inventory capacity. Constraints (3.14) ensure that it is only possible to visit a specific retailer one time per cycle, with only one vehicle and using only one route. Constraints (3.15) ensure that the quantity delivered to a retailer $c \in C$ on a day $d \in nT$ using a route $r \in R$ is the same after the periodic interval of that retailer T_c . Constraints (3.16) ensure that there is no vehicle that exceeds the total driving time limit. Constraints (3.17) calculate the total number of units delivered to a given customer $c \in C$ through all the routes. Finally, constraints (3.18) and (3.19) define the decision variables of the problem.

3.5 Periodic replenishment of retailers (Version 2)

In this section, a new mathematical version of the model is presented. Now, a periodic schedule is assumed. However, the quantities delivered to each retailer and the routes used to perform those shipments can vary on different days. The number of possible solutions in this second version of the model increases respecting to the first model as the quantities delivered to each retailer are not restricted to be regular and the routes can vary. Furthermore, the optimal solution of the first model is included here. Hence, the solution of this model can be the same or even better, but the execution time is increased as the number of possible solutions also raises.

The basis of this model is the same as in the last one. However, as the quantities delivered to each customer does not have to be regular, the constraints (3.15) are replaced by constraints (3.20).

$$\sum_{v \in V} \sum_{r \in R} X_{vrd} A_{rc} = \sum_{v \in V} \sum_{r \in R} X_{vrd+T_c} A_{rc} \begin{cases} \forall c \in C \\ \forall d \in nT - T_c \end{cases} \quad (3.20)$$

Equation 3.20 implies that one specific retailer $c \in C$ has to be replenished with its required periodicity T_c . However, it does not fix the route used to visit each retailer nor the vehicle used. Therefore, either the quantity and the route used to visit a specific retailer can vary from time to time.

3.6 Non-periodic replenishment (Version 3)

Finally, a third version of the model is developed. Now, the periodicity constraint is removed. Hence, the solution is not forced to follow the periodicity requirements of the retailers. Therefore, the number of possible solutions of this model also increases considerably with respect to the first two versions. Among all possible solutions, the optimal one of the first two models are also included. Hence, the solution of this model can be the same or better but the execution time is increased exponentially as the number of possible solutions experiment a huge raise. For that reason, the solution of this model can differ from the optimal one if the maximum execution time is limited.

The periodicity constraint of each retailer is no longer used, therefore, there is no need to introduce it into the model. In order to implement this change properly, constraints (3.14) and (3.15) must be replaced by constraints (3.21).

$$\sum_{r \in R} (A_{rc} \sum_{v \in V} X_{vrd}) \leq 1 \begin{cases} \forall d \in nT \\ \forall c \in C \end{cases} \quad (3.21)$$

Constraints (3.21) limits one specific retailer $c \in C$ to be only visited once on a given day $d \in nT$ by no more than one vehicle and no more than one route. Therefore, split deliveries are not allowed

Chapter 4

Computational results

This section illustrates the behavior of our solution approach in two different ways. First, the model is tested on a set of randomly generated problem instances. Next, this approach is compared to the solution proposed by Raa and Dullaert [13] for the cyclic inventory routing problem.

4.1 Design of instances

In the literature of the cyclic inventory routing problems, some datasets are available, e.g. the paper of Sindhuchao et al., [8], the one of Aghezzaf et al., [9] or the paper of Birger Raa [3].

However, these datasets cannot be used in this thesis for several reasons. In Sindhuchao et al., [8], no fixed vehicle cost is considered; in Aghezzaf et al., [9], a single vehicle is assumed so fleet sizing is not an issue; in Birger Raa [3], the route design phase is not considered. Moreover, it does not take inventory capacity constraints at the retailers into consideration.

However, due to similarities with this master thesis, the instances of this paper are based on the instances proposed in Raa and Aghezzaf [12]. The mentioned instances have been generated according to a 5×2^4 Factorial Design in which the different factors that have been analyzed are illustrated in table 4.1.

The first factor is the customer storage capacity restriction (CCAP), with levels “No” and “Yes” indicating if this restriction is taken into account or not. The second factor is the holding cost rate (H), which can be either 8 or 80 eurocents per unit per day. This holding cost factor (H) is assumed to be the same for all the customers and the depot. The third factor is the Fixed Vehicle Cost (F), which can be either 100 and 400 euros per day. Finally, the fourth and last factor is the number of customers (NR). The two levels that have been considered for this factor are 10 and 15 customers.

Factor	Shorthand	Level ' - 1'	Level ' 1'
Customer capacity restriction	CCAP	No	Yes
Holding cost rate	H	0.08 €/ (u. day)	0.8 €/ (u. day)
Fixed Vehicle cost	F	100 €/ (u. day)	400 €/ (u. day)
Number of customers	NR	10 Cust.	15 Cust.

Table 4.1: Key factors considered in the Factorial Design

The test instances were introduced in Raa and Aghezzaf [12] are generated as follows. First, the different retailers are located randomly within a service area circle. The radius of this circle is randomly generated between 75 and 100 km and the depot is always placed in the center of the circle. Euclidean distances are used. Customer demand rates are randomly generated between 1 and 10 units per day. Furthermore, their customer storage capacity is generated randomly such that it can hold between 2 and 10 days of supply.

Loading and dispatching the vehicles is assumed to take half an hour ($t_j = 0.5 \text{ hour } \forall j$) and cost 20 euro, while deliveries at the customers are assumed to take 15 minutes and cost 10 euro ($t_j = 0.25 \text{ hour } \forall j$). The vehicles have a capacity of 100 units, a vehicle speed of 50 km per hour and a fixed vehicle cost that can be either 100 or 400 euro per day. Furthermore, a variable cost of 1.2 euro per km is considered.

The total cost of each route option is composed out of the variable transportation cost of each route multiplied by the length of each route, the loading and dispatching cost and the delivery cost at each customer.

The duration of each route is calculated by taking into account the total length of the route, considering a vehicle speed of 50 km/h and the dispatching time at the retailers as well as the loading time at the depot (*15 and 30 minutes respectively*).

A fixed shipment cost of 35 euro per order is also charged to the incoming products at the depot. The total driving time limit is 8 h and the maximum number of vehicles that can be used is set to 5 and the considered time horizon is equal to 12 days.

Finally, 50 route options are considered for each of the instances. These options, have been designed by making sub-tours of the optimal solutions provided by the algorithm of Raa and Dullaert [13] for the same set of instances. In this way, a group of 50 good solutions that contain the optimal ones are always taken into account for each instance. Furthermore, the periodicity requirements of each customer have been extracted from this solutions. The working principles of the heuristic algorithm developed in Raa and Dullaert [13], are explained in appendix A.

The different models presented in this paper have been programmed with IBM ILOG CPLEX Optimization Studio 12.7. Computational testing was done on a 2 GHz Intel(R) Core (TM) i7-2630 QM with 4 GB of RAM limiting the maximum running time to 300 seconds.

The results for all $240 = (3 \times 5 \times 2^4)$ instances are summarized in tables 4.2, 4.3 and 4.4, displaying the following solution characteristics, that will be used to explain the various cost trade-offs and the way in which they are obtained:

- Total cost rate and its five different cost components;
- The maximum GAP of the solution after 300 seconds of running time;
- Number of vehicles used and number of tours;
- Utilization of the vehicle as the percentage of time that it is being used;
- Cumulative average stock level of all customers and depot;
- Average number of customers per tour.

After obtaining the mentioned results, it has been demonstrated that despite the fixed vehicle cost F has an important effect on the total cost of the solution, it does not have any real effect on the solutions.

Therefore, in order to evaluate the effects and the interactions of the different factors on the total cost of the solution, a linear regression analysis is performed on each of the versions of the model excluding the fixed vehicle cost factor F. Therefore, this analysis tests the rest of the factors as well as their two-way interactions. Fig. 4.1 and fig. 4.2 show the results of this analysis, provided by the statistical software R Studio 3.4.0.

CCAP	H	F	NR	Total cost	Fleet	Transport	Depot holding cost	Retailers holding cost	Shipment cost	GAP	nr Veh	nr Tours	Utilization	Stock	Cust/tour
Yes	0,8	400	15	9.712,46	4.800,00	3.484,34	6,46	1.001,66	420,00	0,00%	1,00	3,80	67,94%	73,29	4,10
Yes	0,8	400	10	8.339,18	4.800,00	2.478,67	12,93	739,58	308,00	0,00%	1,00	3,20	47,75%	76,31	3,33
Yes	0,8	100	15	6.112,46	1.200,00	3.484,34	6,46	1.001,66	420,00	0,00%	1,00	3,80	67,94%	73,92	4,10
Yes	0,8	100	10	4.739,18	1.200,00	2.478,67	12,93	739,58	308,00	0,00%	1,00	3,20	47,75%	77,10	3,33
Yes	0,08	400	15	8.771,25	4.800,00	3.484,34	15,75	100,17	371,00	0,00%	1,00	3,80	67,94%	87,79	4,10
Yes	0,08	400	10	7.607,21	4.800,00	2.478,67	16,58	73,96	238,00	0,00%	1,00	3,20	47,87%	92,42	3,33
Yes	0,08	100	15	5.171,25	1.200,00	3.484,34	15,75	100,17	371,00	0,00%	1,00	3,80	67,94%	87,79	4,10
Yes	0,08	100	10	4.007,21	1.200,00	2.478,67	16,58	73,96	238,00	0,00%	1,00	3,20	47,75%	92,11	3,33
No	0,8	400	15	9.712,46	4.800,00	3.484,34	6,46	1.001,66	420,00	0,00%	1,00	3,80	67,94%	73,25	4,10
No	0,8	400	10	8.339,18	4.800,00	2.478,67	12,93	739,58	308,00	0,00%	1,00	3,20	47,75%	77,10	3,33
No	0,8	100	15	6.112,46	1.200,00	3.484,34	6,46	1.001,66	420,00	0,00%	1,00	3,80	67,94%	73,90	4,10
No	0,8	100	10	4.739,18	1.200,00	2.478,67	12,93	739,58	308,00	0,00%	1,00	3,20	47,75%	76,13	3,33
No	0,08	400	15	8.771,25	4.800,00	3.484,34	15,75	100,17	371,00	0,00%	1,00	3,80	67,94%	89,33	4,10
No	0,08	400	10	7.607,21	4.800,00	2.478,67	16,58	73,96	238,00	0,00%	1,00	3,20	47,87%	91,37	3,33
No	0,08	100	15	5.171,25	1.200,00	3.484,34	15,75	100,17	371,00	0,00%	1,00	3,80	67,94%	89,44	4,10
No	0,08	100	10	4.007,21	1.200,00	2.478,67	16,58	73,96	238,00	0,00%	1,00	3,20	47,75%	94,70	3,33

Table 4.2: Results for Model 1

CCAP	H	F	NR	Total cost	Fleet	Transport	Depot holding cost	Retailers holding cost	Shipment cost	GAP	nr Veh	nr Tours	Utilization	Stock	Cust/tour
Yes	0,8	400	15	9.712,46	4.800,00	3.484,34	3,58	1.004,54	420,00	0,00%	1,00	3,80	67,94%	74,00	4,10
Yes	0,8	400	10	8.319,35	4.800,00	2.461,91	9,86	739,58	308,00	0,00%	1,00	3,20	47,75%	75,98	3,33
Yes	0,8	100	15	6.112,46	1.200,00	3.484,34	0,70	1.007,42	420,00	0,00%	1,00	3,80	67,94%	73,08	4,10
Yes	0,8	100	10	4.719,35	1.200,00	2.461,91	9,86	739,58	308,00	0,00%	1,00	3,20	47,75%	77,31	3,33
Yes	0,08	400	15	8.771,25	4.800,00	3.484,34	12,17	103,74	371,00	0,00%	1,00	3,80	67,94%	86,54	4,10
Yes	0,08	400	10	7.586,79	4.800,00	2.457,09	13,00	78,69	238,00	0,00%	1,00	3,20	47,75%	93,56	3,33
Yes	0,08	100	15	5.171,25	1.200,00	3.484,34	12,72	103,19	371,00	0,00%	1,00	3,80	67,94%	89,12	4,10
Yes	0,08	100	10	3.986,79	1.200,00	2.457,09	15,32	76,38	238,00	0,00%	1,00	3,20	47,75%	93,24	3,33
No	0,8	400	15	9.712,46	4.800,00	3.484,34	6,46	1.001,66	420,00	0,00%	1,00	3,80	67,94%	74,00	4,10
No	0,8	400	10	8.319,35	4.800,00	2.461,91	6,78	742,66	308,00	0,00%	1,00	3,20	47,75%	77,14	3,33
No	0,8	100	15	6.112,46	1.200,00	3.484,34	6,46	1.001,66	420,00	0,00%	1,00	3,80	67,94%	72,98	4,10
No	0,8	100	10	4.719,35	1.200,00	2.461,91	9,86	739,58	308,00	0,00%	1,00	3,20	47,75%	76,46	3,33
No	0,08	400	15	8.771,25	4.800,00	3.484,34	6,59	109,32	371,00	0,00%	1,00	3,80	67,94%	86,86	4,10
No	0,08	400	10	7.586,79	4.800,00	2.457,09	14,69	77,00	238,00	0,00%	1,00	3,20	47,75%	94,70	3,33
No	0,08	100	15	5.171,25	1.200,00	3.484,34	12,67	103,25	371,00	0,00%	1,00	3,80	67,94%	89,73	4,10
No	0,08	100	10	3.986,79	1.200,00	2.457,09	14,14	77,56	238,00	0,00%	1,00	3,20	47,75%	94,71	3,33

Table 4.3: Results for Model 2

CCAP	H	F	NR	Total cost	Fleet	Transport	Depot holding cost	Retailers holding cost	Shipment cost	GAP	nr Veh	nr Tours	Utilization	Stock	Cust/tour
Yes	0,8	400	15	9.432,96	4.800,00	2.932,18	7,33	1.315,46	378,00	0,82%	1,00	6,80	57,57%	93,16	2,36
Yes	0,8	400	10	7.957,28	4.800,00	2.081,11	9,86	793,31	273,00	0,25%	1,00	4,60	40,38%	76,22	2,33
Yes	0,8	100	15	5.833,52	1.200,00	2.919,27	7,49	1.321,76	385,00	1,57%	1,00	6,80	57,38%	93,24	2,34
Yes	0,8	100	10	4.359,72	1.200,00	2.085,09	10,24	791,39	273,00	1,56%	1,00	4,80	39,98%	82,14	2,23
Yes	0,08	400	15	8.089,31	4.800,00	2.741,97	3,94	172,40	371,00	0,13%	1,00	7,20	53,70%	129,90	2,12
Yes	0,08	400	10	7.116,48	4.800,00	1.958,24	6,63	113,61	238,00	0,08%	1,00	6,00	40,94%	123,43	1,81
Yes	0,08	100	15	4.490,82	1.200,00	2.741,97	5,13	172,73	371,00	0,47%	1,00	7,40	53,70%	130,59	2,07
Yes	0,08	100	10	3.517,19	1.200,00	1.958,24	7,97	112,98	238,00	0,69%	1,00	6,00	37,71%	124,33	1,81
No	0,8	400	15	9.321,51	4.800,00	2.583,54	5,22	1.554,75	378,00	1,83%	1,00	7,20	50,89%	110,49	2,33
No	0,8	400	10	7.668,54	4.800,00	1.633,11	3,58	979,84	252,00	1,23%	1,00	5,40	32,15%	99,22	1,92
No	0,8	100	15	5.724,24	1.200,00	2.597,64	4,03	1.537,57	385,00	6,13%	1,00	6,80	51,14%	107,65	2,45
No	0,8	100	10	4.061,31	1.200,00	1.567,51	5,86	1.042,94	245,00	6,52%	1,00	5,60	30,91%	106,92	1,80
No	0,08	400	15	7.486,97	4.800,00	1.963,14	0,71	352,12	371,00	1,64%	1,00	11,20	38,18%	251,74	1,35
No	0,08	400	10	6.400,54	4.800,00	1.158,61	0,00	203,93	238,00	0,85%	1,00	6,40	32,30%	211,59	1,65
No	0,08	100	15	3.889,77	1.200,00	1.960,24	2,28	356,25	371,00	3,75%	1,00	11,20	38,11%	253,58	1,35
No	0,08	100	10	3.016,81	1.200,00	1.383,81	1,29	193,72	238,00	1,44%	1,00	6,20	26,70%	200,71	1,67

Table 4.4: Results for model 3

```

Call:
lm(formula = Total_cost ~ CCAP + NR + H + CCAP:NR + CCAP:H +
NR:H)

Residuals:
    Min       1Q   Median       3Q      Max
-1097.96 -623.71    1.21   727.01   980.44

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.095e+04  1.210e+03   9.047 1.87e-10 ***
CCAP         -2.412e-12  1.321e+03   0.000  1.000
NR           -2.282e+02  9.416e+01  -2.423  0.021 *
H            1.888e+03  1.818e+03   1.039  0.306
CCAP:NR      -2.273e-13  1.008e+02   0.000  1.000
CCAP:H       9.851e-13  6.997e+02   0.000  1.000
NR:H         -5.812e+01  1.399e+02  -0.415  0.681
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 796.5 on 33 degrees of freedom
Multiple R-squared:  0.5257,    Adjusted R-squared:  0.4394
F-statistic: 6.096 on 6 and 33 DF,  p-value: 0.0002241

```

(a) Model 1

```

Call:
lm(formula = Total_cost2 ~ CCAP + NR + H + CCAP:NR + CCAP:H +
H:NR)

Residuals:
    Min       1Q   Median       3Q      Max
-1097.96 -603.73    1.21   697.24   990.01

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.099e+04  1.192e+03   9.216 1.2e-10 ***
CCAP         4.391e-12  1.301e+03   0.000  1.0000
NR           -2.323e+02  9.278e+01  -2.503  0.0174 *
H            1.887e+03  1.791e+03   1.053  0.2998
CCAP:NR      -4.143e-13  9.928e+01   0.000  1.0000
CCAP:H       2.783e-12  6.894e+02   0.000  1.0000
NR:H         -5.796e+01  1.379e+02  -0.420  0.6770
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 784.9 on 33 degrees of freedom
Multiple R-squared:  0.5385,    Adjusted R-squared:  0.4546
F-statistic: 6.418 on 6 and 33 DF,  p-value: 0.0001487

```

(b) Model 2

```

Call:
lm(formula = Total_cost3 ~ CCAP + NR + H + CCAP:NR + CCAP:H +
NR:H)

Residuals:
    Min       1Q   Median       3Q      Max
 -997.25 -345.43   39.14  405.09  991.01

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  4164.01     835.24   4.985 1.93e-05 ***
CCAP         1073.77     911.54   1.178  0.24724
NR           208.59     64.99   3.210  0.00296 **
H            297.99    1254.61   0.238  0.81373
CCAP:NR      -29.09     69.54  -0.418  0.67842
CCAP:H       -637.56    482.90  -1.320  0.19583
NR:H         148.52     96.58   1.538  0.13362
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 549.7 on 33 degrees of freedom
Multiple R-squared:  0.7896,    Adjusted R-squared:  0.7514
F-statistic: 20.64 on 6 and 33 DF,  p-value: 7.029e-10

```

(c) Model 3

Figure 4.1: Output from the linear regression analysis for the total cost rate taking into account the two-interaction analysis

```

Call:
lm(formula = Total_cost ~ CCAP + NR + H)

Residuals:
    Min       1Q   Median       3Q      Max
-1133.84 -618.16   1.21   717.14  1032.75

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.127e+04  6.453e+02  17.461 < 2e-16 ***
CCAP         8.198e-14  2.418e+02   0.000  1.00000
NR          -2.537e+02  4.836e+01  -5.247  7.05e-06 ***
H           1.162e+03  3.358e+02   3.460  0.00141 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 764.6 on 36 degrees of freedom
Multiple R-squared:  0.5232,    Adjusted R-squared:  0.4835
F-statistic: 13.17 on 3 and 36 DF,  p-value: 5.867e-06

```

(a) Model 1

```

Call:
lm(formula = Total_cost2 ~ CCAP + NR + H)

Residuals:
    Min       1Q   Median       3Q      Max
-1113.86 -597.96   1.21   694.92  1042.18

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.131e+04  6.359e+02  17.782 < 2e-16 ***
CCAP         5.145e-13  2.383e+02   0.000  1.00000
NR          -2.578e+02  4.765e+01  -5.409  4.28e-06 ***
H           1.162e+03  3.309e+02   3.512  0.00122 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 753.4 on 36 degrees of freedom
Multiple R-squared:  0.5361,    Adjusted R-squared:  0.4974
F-statistic: 13.87 on 3 and 36 DF,  p-value: 3.626e-06

```

(b) Model 2

```

Call:
lm(formula = Total_cost3 ~ CCAP + NR + H)

Residuals:
    Min       1Q   Median       3Q      Max
-1115.33 -395.27   10.73   347.19  1242.27

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  3669.20    472.18   7.771 3.29e-09 ***
CCAP         429.62    176.91   2.428  0.0203 *
NR          259.40     35.38   7.331 1.21e-08 ***
H          1835.76    245.72   7.471 7.99e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 559.5 on 36 degrees of freedom
Multiple R-squared:  0.7623,    Adjusted R-squared:  0.7425
F-statistic: 38.49 on 3 and 36 DF,  p-value: 2.522e-11

```

(c) Model 3

Figure 4.2: Output from the linear regression analysis for the total cost rate taking into single influence of each factor

The results of the mentioned analysis show that any of the two-way interactions have a significant effect on the solution for any of the three different models. Furthermore, the customer storage capacities are not significant for any of the first two versions but have a significant effect on the third one. The rest of the factors have all a significant influence on the final solutions.

4.2 Effect of the customer storage capacity restriction

In table 4.5, the global effect of the customer storage capacity restriction on the total cost is shown for the three versions of the model.

	CCAP	Total cost	Fleet	Transport	Depot holding cost	Retailers holding cost	Shipment cost	GAP	nr Veh	nr Tours	Utilization	Stock	Cust/tour
Model 1	No	6.807,53	3.000,00	2.981,50	12,93	478,84	334,25	0,00	1,00	3,50	57,86%	83,15	3,72
	Yes	6.807,53	3.000,00	2.981,50	12,93	478,84	334,25	0,00	1,00	3,50	57,86%	82,59	3,72
	Diff	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	-0,67%	0,00%
Model 2	No	6.797,46	3.000,00	2.971,92	9,71	481,59	334,25	0,00	1,00	3,50	57,85%	83,32	3,72
	Yes	6.797,46	3.000,00	2.971,92	9,65	481,64	334,25	0,00	1,00	3,50	57,85%	82,85	3,72
	Diff	0,00%	0,00%	0,00%	-0,58%	0,01%	0,00%	0,00%	0,00%	0,00%	0,00%	-0,56%	0,00%
Model 3	No	5.946,21	3.000,00	1.855,95	2,87	777,64	309,75	0,03	1,00	7,50	37,55%	167,74	1,82
	Yes	6.349,66	3.000,00	2.427,26	7,32	599,20	315,88	0,01	1,00	6,20	47,67%	106,63	2,13
	Diff	6,79%	0,00%	30,78%	155,04%	-22,95%	1,98%	-76,24%	0,00%	-17,33%	26,96%	-36,43%	17,43%

Table 4.5: Effect of the customer storage capacity restriction for the three different models

As mentioned above in the solutions of the linear regression analysis, the customer storage capacity has no influence on the solution for models 1 and 2. However, it has a significant effect on model 3. It is mainly due to the fact that the customer storage capacities are always bigger than the demand rate multiplied per the customer periodicity restriction in order to avoid infeasibilities in the optimal solutions extracted from the model of Raa and Dullaert[13]. Hence, taking this constraints into account or not does not make any difference as the periodicity constraints of each retailer have to be compulsory considered.

However, if customers impose a storage capacity restriction, when applying the third version of the model, the total cost of the solution is increased by 6,79 %. This effect is caused by a significant increase of 155,04 % in the depot holding cost and an important increase in the transportations cost.

Furthermore, introducing the customer storage capacities results in smaller, more frequent deliveries and thus a lower global stock level(-36,43 %) while the transportation cost increases by 30,78 %. Because of the smaller delivery quantities, more customers are replenished per tour (17,43 %). Moreover, the increased replenishment frequencies result in an increase in the utilization of vehicles (26,96 %).

The interaction between the customer storage capacities and the rest of the factors is not significant for any of the models. Therefore, this interactions are not analyzed.

4.3 Effect of the holding cost rate

In table 4.3, the global effect of the holding cost rate on the total cost is shown for the three different versions of the model.

	H	Total cost	Fleet	Transport	Depot holding cost	Retailers holding cost	Shipment cost	GAP	nr Veh	nr Tours	Utilization	Stock	Cust/tour
Model 1	0,8	7.225,82	3.000,00	2.981,50	9,70	870,62	364,00	0,00	1,00	3,50	57,85%	75,12	3,72
	0,08	6.389,23	3.000,00	2.981,50	16,17	87,06	304,50	0,00	1,00	3,50	57,88%	90,62	3,72
	Diff	-11,58%	0,00%	0,00%	66,73%	-90,00%	-16,35%	0,00%	0,00%	0,00%	0,05%	20,63%	0,00%
Model 2	0,8	7.215,91	3.000,00	2.973,12	6,70	872,09	364,00	0,00	1,00	3,50	57,85%	75,12	3,72
	0,08	6.379,02	3.000,00	2.970,71	12,66	91,14	304,50	0,00	1,00	3,50	57,85%	91,06	3,72
	Diff	-11,60%	0,00%	-0,08%	89,13%	-89,55%	-16,35%	0,00%	0,00%	0,00%	0,00%	21,22%	0,00%
Model 3	0,8	6.794,88	3.000,00	2.299,93	6,70	1.167,13	321,13	0,02	1,00	6,00	45,05%	96,13	2,22
	0,08	5.500,99	3.000,00	1.983,28	3,49	209,72	304,50	0,01	1,00	7,70	40,17%	178,23	1,73
	Diff	-19,04%	0,00%	-13,77%	-47,85%	-82,03%	-5,18%	-54,52%	0,00%	28,33%	-10,84%	85,41%	-22,11%

Table 4.6: Effect of the holding cost rate for the three different models

The holding cost rate has a notorious impact on the total overall cost of the solutions for the three different algorithms. Moreover, the results of the linear regression test show that this factor has a significant influence on the solutions for the three models. However, its two-way interactions are not significant and therefore, will not be analyzed.

When changing from high holding costs to low holding costs but no other factors are changed, the total cost decreases by 11,58 % for model 1, 11,60 % for model 2 and 19,04 % for model 3.

Furthermore, when holding costs are low, larger deliveries are made. There is indeed an increase of 20,63 % in the global stock level for model 1, an increase of 21,22 % for model 2 and an increase of 85,41 % for model 3.

The larger delivery quantities imply that for model 3 less customers are visited per tour (22,11 %) and it does not have any effect for the other two models. Finally, deliveries are also made less frequent. As a result, transportation costs does not change for model 1, but decrease by 0,08 % for model 2 and by 13,77 % for model 3.

4.4 Effect of the number of customers

In table 4.7, the global effect of the number of customers on the overall cost for the three models of this paper is illustrated.

	NR	Total cost	Fleet	Transport	Depot holding cost	Retailers holding cost	Shipment cost	GAP	nr Veh	nr Tours	Utilization	Stock	Cust/tour
Model 1	10	6.173,20	3.000,00	2.478,67	14,76	406,77	273,00	0,00	1,00	3,20	47,78%	84,65	3,33
	15	7.441,86	3.000,00	3.484,34	11,11	550,92	395,50	0,00	1,00	3,80	67,94%	81,09	4,10
	Diff	20,55%	0,00%	40,57%	-24,72%	35,44%	44,87%	0,00%	0,00%	18,75%	20,16 %	-4,21%	23,00%
Model 2	10	6.153,07	3.000,00	2.459,50	11,69	408,88	273,00	0,00	1,00	3,20	47,75%	85,39	3,33
	15	7.441,86	3.000,00	3.484,34	7,67	554,35	395,50	0,00	1,00	3,80	67,94%	80,79	4,10
	Diff	20,95%	0,00%	41,67%	-34,37%	35,58%	44,87%	0,00%	0,00%	18,75%	20,19%	-5,39%	23,00%
Model 3	10	5.512,23	3.000,00	1.728,22	5,68	528,96	249,38	0,02	1,00	5,63	35,14%	128,07	1,90
	15	6.783,64	3.000,00	2.554,99	4,52	847,88	376,25	0,02	1,00	8,08	50,08%	146,29	2,05
	Diff	23,07%	0,00%	47,84%	-20,46%	60,29%	50,88%	0,00%	0,00%	43,56%	14,95%	14,23%	7,48%

Table 4.7: Effect of the number of customers

As table 4.7 shows, there exist a small economy of scale when servicing customers. The total cost increases by around 20 % when the number of customers increases by 50 % in the three different models. Moreover, the cost trade-off being made is different for small and large instances. In large instances, more customers are served per tour while at the same time larger quantities are delivered. This seems contradictory but it is possible because of the increased utilization of the vehicles.

The interaction between the number of customers and the rest of the considered factors does not have a significant influence on the solutions and therefore, they will not be analyzed.

4.5 Comparison to another heuristic

In this section, the three different models developed in this paper are compared with an metaheuristic algorithm on a specific set of instances.

Raa and Dullaert [13] presented in 2017 this metaheuristic approach for the cyclic inventory routing problem under constant customer demand rates. In this framework, they made some simplifying assumptions such as not allowing split delivery or imposing a constant time between consecutive deliveries.

As in most of the papers found in the literature on cyclic inventory routing problems, the incoming products to the depot are not considered and thus, neither the holding cost at this depot. As a result, the coordination of inbound products is not considered in the solution of Raa and Dullaert [13]. For the routes design, each of the several customers are grouped in a tour with specific tour frequencies and cycle times. This means that, one customer which has been inserted into a specific tour will always receive the product with the cycle time of that specific tour even if it is not the optimal cycle time for that specific customer on a given day.

To compare our solution approach to that of Raa and Dullaert, the fixed shipment cost ($S = 0$) and the holding cost at the depot ($H_d = 0$) have been ignored .

The problem instances used for comparing both approaches are detailed in table 4.8.

Factor	Shorthand	Level Considered
Customer capacity restriction	CCAP	Yes
Holding cost rate	H	0.08 €/ (u. day)
Fixed Vehicle cost	F	400 €/ (u. day)
Number of customers	NR	10 - 15 Cust.

Table 4.8: Value of the different factors used to compare the three solutions of this paper to the Raa and Dullaert metaheuristic

In table 4.9, the results of the three different models developed in this thesis are listed in comparison to the Raa and Dullaert heuristic results for each of the instances. The different cost rates are expressed in €/day.

For all problem instances, solutions are found that are cheaper than those proposed by Raa and Dullaert [13]. It is worth mentioning that the cost decrease for the first two versions of the model is not significant (On average less than 1 for low instances%).

However, for the third version of the model, this cost decrease is on average 4,93 % for small instances and 10,34 % for large instances. This significantly better behavior is mostly due to the more freedom in the development of tours for the third version of this model.

In table 4.10, the different components of the total cost rates are expressed in detail.

CCAP	F	H	NR	Raa and Dullaert	Ver. 1	Δ	Ver. 2	Δ	Ver. 3	Δ
Yes	400	8c	10	615,2626	612,72	-0,41%	610,92	-0,71%	584,93	-4,93%
Yes	400	8c	15	716,6962	698,71	-2,51%	698,71	-2,51%	642,59	-10,34%

Table 4.9: Comparing the solution characteristics

These results show that taking into account many tour possibilities instead of one specific tour possibility to serve each customer, gives the opportunity to use the vehicles capacity much more efficiently and find much better cost trade-offs.

	Total Cost	Fixed veh. cost	Transportation Cost	Holding Cost Ret	CPU-Time (s)	nr veh	nr tour	Cust/tour
B. Raa/W. Dullaert	665,98	400,00	255,99	9,99	10,01	1,0	3,5	3,72
Version 1	655,71	400,00	248,46	7,26	8,08	1,0	3,5	3,72
diff	-1,54%	0,00%	-2,94%	-27,36%	-19,25%	0,00%	0,00%	0,00%
Version 2	654,81	400,00	247,56	7,26	7,65	1,0	3,5	3,72
diff	-1,68%	0,00%	-3,29%	-27,36%	-23,57%	0,00%	0,00%	0,00%
Version 3	613,76	400,00	202,22	11,54	267,78	1,0	6,9	1,88
diff	-7,84%	0,00%	-21,01%	15,58%	2576,54%	0,00%	97,14%	-49,43%

Table 4.10: Comparing the solution characteristics

Despite the better results in the solutions for model 3, the running times are on average 267,78 seconds, while the running times for the model of Raa and Dullaert are on average 10 seconds. It is an evidence of slow performance which should be improved by creating a heuristic in order to extend the scope of this problem to real instances.

Chapter 5

Conclusions

In this section, a set of technical and personal considerations are given. These conclusions have been extracted after analyzing the three different mathematical algorithms developed in this master dissertation and understanding the behavior of this models.

5.1 Technical outcomes

First of all, this paper analyses a cyclic planning approach for a two-echelon inventory system. In addition, a first mathematical model optimizes the fleet vehicle size according to the costs involved and the trade-off between the rest of expenses.

In this model, the vehicle fleet is used for periodic replenishments taking into account the optimal cycle time of each route with regular quantities. A second mathematical model with irregular quantities is presented. Finally, a third non-periodic model is developed in order to see if there exists any worth improvement in increasing the number of possible solutions by removing the periodicity constraints at the retailers. This assumption revealed that some deliveries should be performed with a non-periodic frequency and irregular quantities in order to achieve a notorious cost decrease.

A $3 \times 5 \times 2^4$ factorial design of experiments is repeated for both periodic and non-periodic models. Two levels were defined for each of the four presented factors (Customer storage capacity restrictions, number of customers, fixed vehicle cost and holding cost rate).

The results of this experiments have been analyzed and some factors were noticed to influence more than others in the schedule resolution:

- The first studied factor is the customer storage capacity restrictions with the levels “Yes” or “No”. It has been noticed that this kind of restrictions are not significant for any of the two first models. However, it has been noticed a significant effect on the third model where the total cost of the solution is increased by 6,79 % when applying these restrictions. Moreover, introducing the customer storage capacities results in smaller, more frequent deliveries, causing a lower global stock level and an increase in the transportation costs. These smaller delivery quantities also cause an increased utilization of the vehicles and more customers visited per tour.
- Secondly, the holding cost rate was analyzed. In this case, the holding cost has a notorious influence on the total cost for the three different models. Furthermore, when holding costs are low, larger deliveries are performed causing a significant increase on the average stock level. These larger delivery quantities also imply that less customers are visited per tour.
- The last considered factor is the number of customers. It has been noticed that there is a small economy of scale when servicing customers. Moreover, in larger instances, more customers are served per tour while at the same time larger quantities are delivered. This seems contradictory but it is possible because of the increased utilization of the vehicles.
- Finally, the two-way interactions of this factors are not significant and therefore have not been analyzed

Despite the character of this abstract, further applications can arise from the model and a personalization of them can be performed. More constraints can be added such as warehouse capacities, driving regulations or time windows for deliveries. However, they have not been implemented as the objective of this paper is to analyze how the transport costs are spread through the considered cycle time and it does not add significant value to this analysis.

5.2 Suggestions for further research

In this problem, several delivery options to visit the different customers, are considered. For the problem instances of this dissertation, they have been made by randomly creating subtours of the optimal solutions provided by the Raa and Dullaert algorithm [13] . One interesting improvement would be the design of a heuristic algorithm to find 50 good delivery options which contains the optimal solutions for each period in different situations more efficiently.

Another interesting improvement since the version 3 of this dissertation provides really good results compared to another existing algorithms but has excessive running times for real instances, would be the development of a heuristic that finds near-optimal solutions in smaller running times. Therefore, this solution could be applied to real instances more easily.

5.3 Personal outcomes

The objective of this master's dissertation was to find a solution for a specific cyclic inventory routing problem. In this particular case, a big scenario was decomposed in many little goals easy to fix.

For me, the most difficult part was the analysis of the solutions of the different problem instances and the process of extracting conclusions from them. It is not always easy to interpret the reasons of a specific behavior in the models when changing one key factor.

However, I am glad to overcome that challenge. I take with me a large number of new cross skills and competences learned while carrying out this project as well as the desire to continue with this research in the future.

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Appendix A

Metaheuristic approach for the design of delivery options

In this section, the metaheuristic approach in which this paper is based for the design of the 50 different delivery options to each retailer will be explained in detail. This algorithm was published in 2017 by Raa and Dullaert [13].

This metaheuristic is composed out of three different building blocks: the route design subproblem, the fleet design subproblem and finally, a metaheuristic framework that is built around these elements.

A.1 Route design phase

In this first phase of the algorithm, cyclic routes are designed in such a way that customer sequencing minimize the travel distance and costs. Furthermore, the cycle time has to be determined by trading-off the costs of making the route and the inventory holding costs at the customers. This cycle time is also constrained by the vehicles capacity and the customers storage capacities.

Suppose a set S of customers to be replenished and that route r visits a subset of these customers S_r . Every customer $i \in S_r$ has a constant demand rate d_i and a storage capacity of k_i units. Furthermore, a truck with a capacity of k units is available for making the route from which the cycle time T_r has to be determined.

The maximum cycle time of the route r will be restricted by the vehicle and customer storage capacity and will follow the equation:

$$T_{max,r} = \left[\min \left(\frac{k}{\sum_{i \in S_r} d_i}, \min \frac{k_i}{d_i} \right) \right] \quad (\text{A.1})$$

Furthermore, the optimal cycle time T_r^* for which the cost rate is minimal follows the equation:

$$T_r^* = \min \left(\sqrt{\frac{2(C_r + t_r \cdot \psi)}{\sum_{i \in S_r} \eta_i \cdot d_i}}, T_{max,r} \right) \quad (\text{A.2})$$

For the partitioning the set of customers in different subsets each covered by a different route, a two-phase heuristic is developed. The first phase constructs an initial solution using a savings-based heuristic and the second phase improves that solution using local search operators. Figure A.1 gives an overview of how it works.

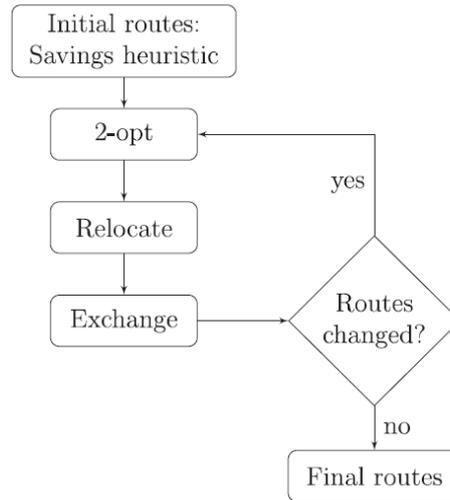


Figure A.1: Flowchart of the two-phase route design heuristic. From Raa and Dullaert [13]

The savings-based construction heuristic that Raa and Dullaert propose is an adaptation of the Clarke-and-Wright heuristic to the situation of the cyclic IRP. For standard vehicle routing, the evaluation of a merge of two routes consists of checking the vehicle capacity constraint and calculating the distance saving.

Furthermore, the merged route's maximal and optimal cycle time have to be calculated in order to minimize the route's cost rate.

The second phase of the route design is a local search based improvement phase. The local search procedure can be considered as a variable neighborhood descent algorithm (VND), consisting of various well-known algorithms.

In this second phase, the following standard local search operators are being used:

1. 2-Opt: Remove two arcs and replace them by two other arcs such that no subtours are formed and all routes are closed again.
2. Relocate: Remove a customer and try to reinsert it into another position.
3. Exchange: Switch the position of two customers from either the same or different routes.

As shown in figure A.1, the same local search operator is reiterated until no further improvements are achieved before moving on to the next operator.

Further, a modified best-accept strategy per operator is adopted in which nodes affected by a move, are marked as affected in order to achieve a better computational efficiency.

A.2 Fleet design

When a set of cyclic routes is selected, the solution for all of them is also cyclic with a cycle time equal to the least common multiple of all the individual route cycle times. To limit this least common multiple, route cycle times are limited to $T = 120$ days and all its divisors. However, in the solutions obtained for the design of the different delivery options of this dissertation, this cycle times are limited to 12 days and all its divisors i.e. $\{1,2,3,4,6,12\}$. In doing so, the infinite horizon is limited to a horizon of 12 days and the same solution will be repeated cyclically.

The second step is to build a schedule within this horizon that indicates which vehicle makes each route, taking into account that there shouldn't be any vehicle that exceeds the total driving limit of time H (e.g. 8 hours) and such that the sum of the cost rates and the vehicle cost rate is minimized.

During this fleet design phase, better solutions can often be obtained if the individual route cycle times are modified in order to align route cycle times to reduce the required fleet size. However, deviating from the optimal route cycle times will increase the individual route cost rates and is therefore only justified if that cost rate increase does not exceed the saving of reducing the fleet. Thus, in this phase, the composition of the routes is not changed, but their cycle times can still be adjusted to minimize the required fleet.

If a route r is made on day t , then it is also made on days $t + T_r, t + 2 T_r$, etc. Thus, the cycle time T_r has to be selected along with a day t in the first T_r days of the schedule for each route. Then, for all days $t + kT_r, \forall k \in \{0, 1, \dots, \frac{T}{T_r} - 1\}$, a vehicle must be selected that has enough time left to make the route on that day.

At this point, the fleet design subproblem makes use of the algorithm proposed by Raa in 2015 [3] which consists of two phases: a construction and an improvement phase.

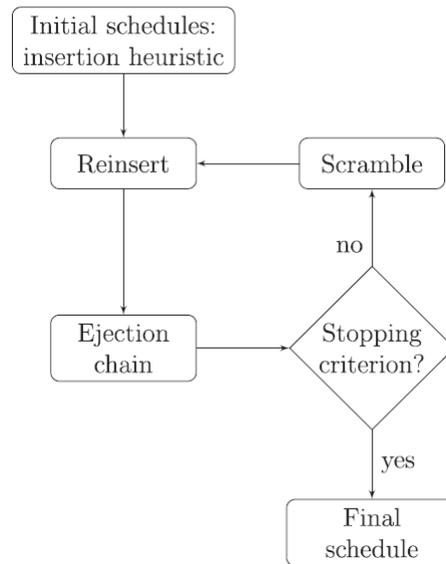


Figure A.2: Flowchart of the two-phase fleet design heuristic. From Raa and Dullaert [13]

The construction heuristic is a best-fit insertion heuristic in which each route is inserted in the way that the cumulative remaining time of the vehicles to which the routes are assigned is minimal.

Within the construction heuristic, two cycle time selection rules are used. One where routes are inserted with cycle times as close as possible to their optimal cycle times, and another one where routes are inserted with cycle times as close as possible to their maximal cycle times to minimize the fleet cost rate.

This leads to two initial schedules that are subsequently passed on to the improvement step which consists of two local-search operators:

1. Remove and reinsert any single route in the cheapest possible way.
2. Remove all routes made by the vehicle with the lowest utilization and reinsert them in the cheapest possible way.

To escape the local optimum, the schedule is *scrambled* by shuffling route allocations among the vehicles according to a random permutation for every day in the schedule horizon. The stopping criterion is a predefined number of scrambles.

Figure A.2 gives an overview on how the fleet design phase works. Further details on this algorithm can be found in the paper of B. Raa [3]

A.3 Metauristic framework

With some adjustments, the algorithm of the fleet design phase is reused as the building blocks in a metaheuristic framework that generates multiple solutions and retains the best ones.

The first adjustment to the savings heuristic is made when two routes p and q are merged into a single route and the savings are calculated as follows: $S_{pq} = TCR_p + TCR_q - \lambda$. Varying the parameter λ leads to different solutions. This metaheuristic varies this parameter between 0.8 and 1.5.

A second adjustment to lead the heuristic to different solutions is when parameterizing the utilization in the computation of the total cost rate:

$$TCR_r = \frac{C_r}{T_r} + \frac{T_r}{2} \sum_{i \in S_r} \eta_i d_i + \alpha \frac{t_r}{T_r} \psi \quad (\text{A.3})$$

The parameter α represents the relative importance of the vehicle utilization versus the actual route costs and is randomized with its value ranging between 0 and 2.

The first framework generates different solutions by randomizing the parameters α and λ . A second framework is a ruin-and-recreate heuristic that iteratively destroys and rebuilds a given solution. Ruining is done by iteratively removing a random number of customers while recreation is carried out by applying the route and fleet design heuristics to the partial solution.

In the third and final framework, the ruin-and-recreate heuristic is also adopted. It is referred as “memetic algorithm” (MA).

This MA lies in the fact that it only uses the basic building blocks mentioned above and it does not require complicated chromosome encodings.

The crossover operator used in this MA also works in a ruin-and-recreate manner. An offspring solution is generated by removing a random number of customers from one of the parent solutions, not only customers which are not incident to common arcs in both parents can be removed. The resulting partial solutions after removing some customers are optimized again using the route and fleet design heuristics outlined before.

Finally, another operator choice is population management. The population management is used to take care of the diversification of the search process, this is done by measuring the diversity of the individuals in the population and selecting individuals for crossover with the well-known roulette wheel mechanism based on these diversity scores. Further, the diversity scores are also used in composing the next generation of the population. This next generation is composed of the best solutions from the population and the most diverse among the offspring.

For further details, I refer to Raa and Dullaert (2017) [13]

Appendix B

Table annex

In the following page, a table with the different numerical results for all of the different instances is attached.

Moreover, some calculations like the utilization of the vehicles, the number of vehicles per tour and the average cumulative stock level at the customers and depot are performed.

Coordination of inbound and outbound deliveries in a distribution center

Daniel Velasco Lastra

Supervisor: Prof. dr. Birger Raa

Master's Dissertation submitted in partial fulfillment of the requirements for the student exchange program

Department of Industrial Systems Engineering and Product Design

Chair: Prof. dr. El-Houssaine Aghezzaf

Faculty of Engineering and Architecture

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