# EFFECTS OF DYNAMIC PRICING OF PERISHABLE PRODUCTS ON REVENUE AND WASTE

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**Paper published by the journal: *Applied Mathematical Modelling (2017)*, Vol. 45, pp: 148-164 (doi: 10.1016/j.apm.2016.12.0249)**

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## ABSTRACT

This paper deals with dynamic price strategies to reduce food and other perishable products spoilage. A deterministic mathematical model is proposed to study the influence of a number of factors, such as price elasticity of demand, age-sensitivity of demand and age profile of initial inventory, on revenue and spoilage. A parametric, bi-objective approach is considered with the aim of estimating the existing trade-offs between revenues and spoilage. The effects of price discounting are different in each scenario and also depend on the speed at which the price is reduced as it ages. Although a dynamic price strategy helps reduce spoilage, its effect on total revenue depends heavily on the scenario. In some specific cases identified below in the paper, total revenue can slightly increase or, at least, maintain its level. In other scenarios, the spoilage reduction comes as a loss in total revenue that can go from small to significant, depending on the scenario and the speed of the price discounting strategy. The proposed approach allows the quantification of the available trade-offs for each scenario. It also allows the analysis of the age distribution of units sold and their respective revenue contribution.

**Keywords**: perishable products; dynamic pricing; food spoilage; waste reduction; price elasticity

## 1. INTRODUCTION AND MOTIVATION

In order to attract customers, large retailers are offering on their shelves a greater number of fresh products, allowing them to compete against the more traditional channels that usually specialize in those items [39]. Li et al. [35] report that more than 81% of the sales of the grocery retailing industry in the US in 2009 corresponded to food and beverages, and 63% of those were products with a limited shelf life, i.e. more than 50% of this sales channel are perishable units. Standard commercial products can be found anywhere, but perishable products are required on a daily basis and appreciated by customers looking for quality. These products introduce an additional complexity in the management of the stock. Thus, very often they require careful handling, and above all, their limited shelf life requires the implementation of some sort of strategy that avoids the spoilage of outdated units.

The problem of how to manage the inventory of perishable products has been extensively researched since the 1970s [43]. Depending on the lifetime of the products, models can be classified into three categories [45]: fixed lifetime, random lifetime, and models with a decaying lifetime. Everybody is accustomed to seeing on the shelves items marked with a fixed expiry date ("sell by" or "best before"), predetermined by the manufacturer to be valid under certain temperature, handling and storing conditions [33]. Theoretically a product is valid until that date, and many authors have considered this case in taking pricing decisions (see [38]). In spite of that, the customer often sees less utility in some products as they become aged. Goyal and Giri [24] review different decay distributions that have been considered in the literature (exponential, Gamma, Weibull, etc.).

As periodic replenishment practices give rise to the presence on the shelves of units with different expiry dates but the same price, the customer prefers to select the fresher units which provide a higher perception of quality [15]. According to Chung and Li [16], 88% of consumers frequently check expiry dates when buying. It is clear that adjusting the price to the product characteristics, instead of adopting a fixed price along its whole shelf life, could increase sales and as a consequence perhaps the revenues gained by the retailer. That is to say, instead of posting a fixed price for a long period, the seller can dynamically change the price, thus balancing supply and demand based on information such as inventory shelf life and price elasticity of demand.

For instance, when the expiry date draws near, the retailer can post a lower price as, for the same price, the client may prefer a product with a longer shelf life because it is considered to be of higher quality [47]. In addition to reducing spoilage, this measure can produce a revenue loss, although not always because the price discount can be compensated for by an increase in sales. Note that in the case of food products (which are typically perishable items), they are price inelastic, with demand elasticities in the range 0.3~0.8 for common products [7]. It would therefore be interesting to explore how elasticity can influence demand so as to compensate for the suggested dynamic price reductions.

From a historical point of view, the interest in revenue management started in the early 1970s, focusing on airline and hotel overbooking [12], industries in which capacity is difficult to change in the short term and variable costs are small. The interest was not initially in intervening in the prices by looking for higher revenues, but in the capacity, by opening or closing certain fare classes as demand evolved in a segmented market. It was in the 1990s that pricing policies became a hot research topic, with the publication of some seminal works in the field (e.g. [23]). Applications moved from the hotel and airline business to many other industries (retail, energy, etc.), also to perishable products, price-sensitive demand and finite horizons [12]. However, the quantification of the benefits of dynamic pricing over a fixed price strategy has not been extensively studied. This is mainly due, according to Sen [50], to the difficulty of efficiently calculating optimal policies, and the high operational cost of changing prices on the shelves.

In spite of that, according to Elmaghraby and Keskinocak [19], there are three main reasons for the increased interest in these policies that differentiate price by expiry date: a higher level of data availability by the retailers (evidently it is necessary to know your customers very well to make good decisions); better Decision Support Systems (DSSs) are available; and a better technology that makes changing prices on the shelves less cumbersome. Some authors [32, 39] have researched how traceability technology such as RFID can help to monitor and control time-sensitive perishables, providing data such as temperature, humidity, stock, expiry dates, or even demand trends, that help to make more founded decisions. Technology is, therefore, reducing operational costs and facilitating the implementation of this type of policy.

All these studies consider the sellers as revenue maximizers [19]. However, an inefficient stock rotation causes spoilage of expired units representing, in addition to billions of dollars of cost, an important problem in the short shelf life supply chain, with tons of items taken out of the stream and discarded [32]. Ferguson and Ketzenberg [22] report that in retailing, in some cases, up to 15% of perishables are disposed of due to spoilage or damage. Some other studies show different spoilage rates depending on the country, the channel and the product (e.g. [5]).

In any case, while the poverty rate in Europe is increasing due to the economic crisis and more people need to go to Food Banks to collect products for covering their daily necessities [49], at the same time millions of tons of edibles are landfilled in European countries every week [20, 6]. This is a concern for governments, NGOs and society at large. In fact many retail companies have included waste reduction as one of their operational targets and performance indicators [14]. Therefore, not only does a severe financial problem exist around perishable products management at the retailer level, but there also exists an environmental (and social) impact as well, which should be included in the decision framework.

Keeping this in mind, one main issue is considered in this research that has not previously been considered in detail. Here the spoilage of expired products is not to be considered as being included in part of the cost function, but as a goal in itself, given the environmental and social issues involved. Therefore, we are not using here a single, cost-based objective function, but a bi-objective approach that considers, as another objective in the decision making process, the reduction in the number of units that are discarded because they have reached their expiry date and can no longer be sold nor safely consumed.

Thus, our goal is to gain insight and shed light on the relationship between revenue and wasted units when different dynamic pricing policies are implemented under different scenarios (i.e. products with different price elasticities and different aversions of customers to acquiring perishable products with shorter remaining shelf lives). By studying the effects of different parameters on the total revenue and total waste, we can better understand and quantify the overall effects of dynamic price-discount policies under different scenarios.

The approach proposed in this paper is rather general and applies to any perishable product (food, magazines, season tickets, etc.) for which a price reduction can stimulate demand for the aged units and thus help reduce waste/spoilage/unsold units. The idea, however, is to do this without harming sales revenue. The experiments carried out are aimed at showing that, depending on the scenario considered, this can generally be achieved.

Note that although we are considering a rather simplified model (e.g. we assume deterministic demand, continuous-time price markdown, etc.), it is still able to provide valuable insights into the relationship between dynamic pricing and revenue and waste, thus giving clues for managers to handle the day by day running of this type of operation. Furthermore, differently from existing research which approaches the issue from a strictly economic viewpoint, we propose a bi-objective approach, thus placing emphasis on the environmental and social impacts of the problem, and looking for ways of harmonising profits and social responsibility.

The structure of the paper is the following. In Section 2 we review some of the existing literature, which will give us clues as to the parameters and factors that must be taken into account in our study. In Section 3 we present a mathematical model that incorporates those factors and parameters, and in Section 4 some formal analytical results based on the previous modelling. Later, in Section 5, we present and discuss the results of a number of experiments that have been carried out and finally, in Section 6, we summarize the conclusions of our study.

## 2. RELEVANT LITERATURE

When modelling products which, by their nature, are bound to become outdated, demand is conditioned (among other factors, see [18]) by the remaining shelf life of the item. Two initial assumptions can be made regarding knowledge of the demand function: uncertain demand (see for instance [35] for some comments regarding models under this assumption, or [48]) for deterministic demand.

Different models have been proposed considering the aging factor under deterministic demand. For instance, Rajan et al. [46] model this situation with a demand function which decreases as the price and age of the perishable units increase, deriving the optimal ordering cycle and price. Later Abad [3] extended this approach allowing backlogs.

Although most studies consider homogeneous units as not depending on their ages (see, for instance, [21,44]), in order to compensate for the reduction in demand, a dynamic pricing strategy appears as an interesting and cost-saving option [52]. Note that, although this is already quite a common practice in the retail industry, some stores are still reluctant to follow it as they are afraid that it could affect the company's image [35]. In some cases, in addition to the reputational risk, there is also a risk of selling already obsolete or damaged products that may put lives at risk and would require further compensation [28].

Liu et al. [39] define dynamic pricing as the assignment of different prices to items of the same category, considering their individual characteristics and changes to their status. Elmaghraby and Keskinocak [19] identify three characteristics to categorize the literature dealing with dynamic pricing: replenishment vs. no replenishment of inventory; dependent vs. independent demand; myopic vs. strategic customers (i.e. whether the customer purchases immediately when the price is acceptable, or looks forward to evaluating the price changes). Note that when the customer knows that the price will be marked down later on, additional factors have an influence on the demand for the product [51], such as delay in the buying decision (with the risk of the product being sold out), or creating the image of a lower quality product after the price has been reduced. Besanko and Winston [11] show that, in the case of potential myopic customers, it is better to set an initially high price and define a deeper decreasing price, while in the case of strategic customers a less steep decrease of prices works better.

In the literature discussions can be found about different factors affecting the modelling of the global demand and pricing alternatives [37]. As regards, for instance, the existence of salvage values, the presence of multiple liquidation channels where the seller can take a final decision on the remaining units (as well as the disposal costs in the case of many perishable products), makes it quite difficult to define a general framework for this variable.

Elmaghraby and Keskinocak [19] recognize the initial inventory decision as something very influential in the optimization of the pricing process. They mention especially questions such as how sensitive are the profits of optimal pricing policies to any deviation in the initial stock from optimal levels.

Most dynamic pricing approaches model the demand uncertainty by assuming it follows a specific distribution – estimated using historical data. However, some authors claim there is a risk in trusting those estimations, considering the observed fast technological and cultural changes and volatile market conditions, which can introduce serious errors in the solution [36]. These authors propose a fuzzy demand model where demand at period t given a price $p_t$ is defined as a random fuzzy variable $\tilde{D}_t(p_t)$. Three different fuzzy programming models were considered for determining the best prices for period t.

Moreover, price markdown can be implemented in different ways depending on the moments in the product lifetime when the price is updated. Most of the models consider pricing policies with a continuous-time approach. However, from a realistic point of view it would be very difficult and costly to apply in practice, and periodic policies would be more convenient [13]. Chung and Li [16] consider two main procedures to put dynamic pricing into practice: a fixed-discount strategy, where the seller divides the product life into several stages and announces the initial and discounted prices at each moment; and a contingent strategy, which consists of fixing the initial price and defining a point in time at which the seller will announce the new price. According to Aviv and Pazgal [8], contingent strategies lead to lower expected revenues when dealing with strategic customers.

Since the aim of this paper is to show the compatibility of reducing waste with maintaining (or increasing) revenue, the assumption of reducing the price continuously with age is not vital and could alternatively be substituted by two or any finite number of price reductions. The results for a stepwise price reduction policy would be rather similar and, of course, would depend on the steepness of the price decreases.

In this regard, deciding the rate at which the price should be reduced is not a trivial decision. Tsiros and Heilman [53] studied the willingness of customers to pay as time passes for six perishable food products, and they observed that, depending on the product, the shape of the function changed, being linear for four of them and non-linear in the other two cases (incidentally meat products). Chung and Li [14] studied the most basic case, by considering a constant reduction rate for each remaining shelf life day. There are two main advantages to their assumption: this linear pattern simplifies the approach, making it simpler to implement, and the procedure results are more transparent for customers, which would stimulate consumption patterns. Actually, the goal in Chung and Li [14] was to better understand how dynamic pricing strategies would influence consumer behaviour, after they observed that the usual policy of suddenly marking down the price when the expiry date was imminent was not appreciated by customers. According to a survey carried out by Chung and Li [16], a common practice in the Korean retail industry is to reduce the price of perishables by 30% when 30% of the product shelf life remains.

However Wang and Li [55] acknowledge that there can be some criticism of the assumption of a strictly linear demand function of the price, even though its wide use is based on an acceptable approximation of real demand. To respond to that criticism, and when the customers are aware of some deterioration through the age of the product, measured by its quality $q(t)$, they define the demand function as $D(t)=D_0-\alpha p(t)+\beta q(t)$, with $\alpha$ and $\beta$ being two parameters measuring the influence on the demand of the price and the quality. All other factors such as competitors' prices, customers' perception, etc., are included in those parameters. This is the approach followed in the next section.

Throughout the research developed so far, the usual approach was to consider the spoilage as something undesirable due to its economic impact on the general cost function; however, we are considering a new point of view in which the spoilage is something to fight against independently of the cost. To reduce waste (mainly if we are talking about something so sensitive nowadays, such as food) is an objective in itself, when taking into account social and environmental factors. In this sense, this paper covers a new way of dealing with the problem, by acting on the demand via prices.

## 3. MODELLING ASSUMPTIONS

### 3.1. Demand and price functions

Let us denote $L$ as the maximum shelf life of a perishable product being studied, and let $I(a,t)$ be the inventory of the product having an age $a \leq L$, in an instant $t \leq L$. In order to estimate the effects of a dynamic pricing policy as the product ages, let us suppose there is an initial fresh product stock, $I(0,0)$, which, together with the rest of the stock at instant 0, $I(a,0)$, defines the age profile of the initial stock of the product (see Figure 1). As mentioned above, this factor is considered to be relevant as it will have an influence both on sales and spoilage.

We assume a continuous-time, deterministic demand and no replenishment during the horizon L. Of course, in the real world, each time period the inventory may be renewed with replenished, zero-age units, will modify the inventory age profile at time t. The proposed approach can be adapted to that replenishment scenario but, in this paper, we are concerned with studying the depletion of the given initial stock and how that process may be influenced by adopting a dynamic pricing policy.

========== FIG 1 ==========

The evolution of the permanence in stock of the units available at instant 0 will depend on the scenario considered and on the dynamic pricing policy applied. Note that, at most, at time L, all the initial stock I(a,0) in the process would have been either sold or wasted. Those units that reach their maximum shelf life are retired and recorded as spoilage. The rest are sold units, but their contribution to revenue is not uniform since the sale price depends on the age of the product at the time of sale. The goal is to measure the effect on revenue and spoilage of a discount policy depending on the different factors considered.

We assume there is a pool of buyers that define an aggregate continuous-time demand function. As it usually happens in the case of perishable products, demand depends not only on the retailer price but also on the age of the product [3, 55]. Thus, a product whose age is close to its maximum shelf life is not as appealing as a fresh one, and the customer prefers to pick up products with longer remaining shelf lives. The demand function considered takes into account this demand leakage. It is implicitly done through a demand function that jointly depends on price and product age. Let D(*p*,*a*) be the demand as a function of both variables, price (denoted *p*) and age (denoted *a*), with $a \leq L$. As we are proposing to explore the effects of including that client perception about aging into the price scheme, we therefore consider that the price depends on age, p(*a*). As will be seen in the mathematical formulation, we consider three parameters in the model: $\alpha$ (for the price elasticity of demand), $\beta$ (for the influence of aging on demand) and $\gamma$ (for the influence of aging on price).

Regarding demand, we assume constant price elasticity of demand $\alpha > 0$, and, for a given price, a decreasing demand as the product loses freshness, controlled by parameter $\beta > 0$:

$$D(p,a) = D_0 \left(\frac{p}{p_0}\right)^{-\alpha} \left(1 - a/L\right)^{\beta} \tag{1}$$

Note that when β=1 the decrease in demand is linear with age, with demand for a product decreasing to zero as its age reaches the maximum shelf life. Demand decreases less than linearly for β>1 (see Figure 2), i.e. the demand reduction with age is lower at first (so demand is not much lower for relatively fresh units, and only falls as the age of the unit approaches the maximum shelf life). Note that we shall not consider β values lower than 1 because that would mean that the demand reduction with age would be steep at first, which is not generally the case in practice.

========== FIG 2 ==========

Regarding the price discount policy, we propose that the price reduction due to aging (in case it is considered) should follow a similar pattern to the demand reduction, using a parameter γ≥0 to control the speed of the price reduction. Mathematically,

$$p(a) = p_o\left[1-\left(a/L\right)^{\beta}\right]^{\gamma} \tag{2}$$

Note that the above equation assumes that for *a*=L the retail price would be zero. Note also that γ=0 means that no dynamic pricing is applied, while higher γ values accelerate the price reduction. The case with β=γ=1 coincides with the basic linear markdown practice assumed by some authors [14].

Including in (1) the dependence of variable price with age, we arrive at the following demand function:

$$D(a) = D_o\left[1-\left(a/L\right)^{\beta}\right]^{1-\alpha\gamma} \tag{3}$$

where the exponent of 1-αγ now has a special importance: when γ=1/α, demand is constant for any age (i.e. marking down the price for older units compensates for a reduction in demand); when γ<1/α demand still decreases with age; a scenario with $\gamma>1/\alpha$ means that demand increases as the product is older, an extreme-discount scenario which will not be considered.

With respect to the justification of the functional forms assumed for the demand and the price discount policy considered, we should remember that in the classic economic theory, the simplest function relating demand and price is the linear approach, with many researches modelling the demand function in this way [17]; another very common approach is to assume a constant elasticity demand, already considered in classical texts such as Koutsoyiannis [34]. In fact many papers on supply chain management (including those dealing with perishable products) consider a constant price elasticity function of the type $D=D_0p^{-\alpha}$ (e.g., [1, 30, 4, 42, 41, 31]). In order to keep some optimization properties [2], some of those papers consider elasticities $\alpha>1$. However, in order to consider the most general case, as other authors do (e.g. [54]), we allow here the possibility of an inelastic demand.

Time is another major factor that can have an influence on demand. In addition to the "best before" case we deal with in this paper, some products change the demand pattern as the season passes, with higher rates at the beginning or at the end. As regards the use of a demand function that jointly depends on price and product age we can mention, among others [52, 29, 40]. Regarding the influence of time over demand, many authors assume a linear relationship (e.g. [17]), although a more general polynomial formulation was proposed by Barbosa and Friedman [10], $D=D_0t^r$ with $r>-2$. In our case, demand decreases with time, which would correspond to the case $r<0$. However, as explained before, we are expecting a smaller decay rate at the beginning and a steeper one as time passes. Therefore the concave approach we have selected (with $\beta>1$) was considered to be more appropriate. Researchers that have also considered a polynomial dependence of demand with age include [9, 25, 26, 27].

### 3.2. Calculation of revenue and spoilage

As we assume that the inventory is not replenished in the time horizon [0,L], at time t≤L, there cannot be any inventory having age *a*<t, and the current inventory of age *a* corresponds to units that had age (*a*-t) in the initial inventory. Of course, part of the inventory that initially had age (*a*-t) must have been sold during the interval [0,t]. The other fraction remains in inventory and has age *a* at time t, i.e.

$$I(a,t)=\begin{cases} 0 & \forall a<t \\ \max\{0, I(a-t,0)-\int_0^t D(a-t')dt'\} & \forall a\geq t \end{cases} \tag{4}$$

The wasted units are those units arriving at age L without being sold. Therefore, the waste generated at instant t is

$$W(t)=I(L,t) \tag{5}$$

and the total waste generated is

$$TW=\int_0^L W(t)dt \tag{6}$$

Regarding the calculation of revenues, we need to calculate in advance the instant at which the inventory with age *a* runs out. Let us denote that instant as $\tau(a)$, i.e.

$$\tau(a)=\min\{t\in[0,L]: I(a,t)=0\} \tag{7}$$

i.e., $I(a,\tau(a))=0$, or, equivalently

$$I(a-\tau(a),0)=\int_0^{\tau(a)} D(a-t')dt' \tag{8}$$

As there is no more inventory of age *a* after instant $\tau(a)$, we can therefore calculate the number of units of age *a* sold at instant t as

$$S(a,t)=\begin{cases}\mathrm{D(a)} & \forall t\le\tau(a)\\ 0 & \forall t>\tau(a)\end{cases} \tag{9}$$

The number of units sold at time t is

$$S(t)=\int_t^L S(a,t)\,da \tag{10}$$

and the total sales is

$$TS=\int_0^L\int_t^L S(a,t)\,da\,dt=\int_0^L S(t)\,dt \tag{11}$$

In a similar way, the revenue from selling products of age *a* at instant t is

$$R(a,t)=\begin{cases}p(\mathrm{a})\cdot\mathrm{D(a)} & \forall t\le\tau(a)\\ 0 & \forall t>\tau(a)\end{cases} \tag{12}$$

The revenue from units sold at time t is

$$R(t)=\int_t^L R(a,t)\,da \tag{13}$$

and the total revenue is

$$TR=\int_0^L\int_t^L R(a,t)\,da\,dt=\int_0^L R(t)\,dt \tag{14}$$

The above mathematical expressions allow us to compute total revenue (TR), total sales (TS) and total waste (TW). By the way, TS and TW are related in the sense that their sum is equal to the total initial inventory, i.e.

$$TS + TW = \int_0^L I(a,0)\,da \tag{15}$$

Moreover, we can calculate the number of units of each age sold in the time horizon [0,L] as

$$\begin{aligned}\hat{S}(a) &= \int_0^{\mathrm{L}} S(a,t)\,dt = \int_0^{\tau(a)} S(a,t)\,dt = \\ &= \int_0^{\tau(a)} D(a)\,dt = D(a)\cdot\tau(a)\end{aligned} \tag{16}$$

This leads to another way of computing total sales as

$$TS = \int_0^L \hat{S}(a)\,da = \int_0^L D(a)\cdot\tau(a)\,da \tag{17}$$

Analogously, the amount of revenue obtained from the sale of units aged *a* along the horizon [0,L] can be computed as

$$\begin{aligned}\hat{R}(a) &= \int_0^{a} R(a,t)\,dt = \int_0^{\tau(a)} R(a,t)\,dt = \\ &= \int_0^{\tau(a)} p(a)\cdot D(a)\,dt = p(a)\cdot D(a)\cdot\tau(a)\end{aligned} \tag{18}$$

This allows an alternative expression for computing total revenue as

$$TR = \int_0^L \hat{R}(a)\,da = \int_0^L p(a)\cdot D(a)\cdot\tau(a)\,da \tag{19}$$

Note also that the ratio $\hat{S}(a)/TS$ represents the percentage of the total units sold that had age *a*. Similarly, the ratio $\hat{R}(a)/TR$ corresponds to the percentage of the total revenue that comes from units of age *a*.

## 4. SOME ANALYTICAL RESULTS

Parameter γ controls the steepness of the price reduction. In fact, this is the only parameter that can be controlled by the decision maker (the other two, price elasticity and age sensitivity of demand, depend on consumers and cannot be changed by a manager's decision). Therefore it would be interesting for our analysis to know the relationship between γ and the relevant model outputs.

It is proven below that any price reduction policy will positively affect waste generation, reducing the number of spoiled units. In other words, there is an inverse relationship between the total waste generated TW, and the price reduction rate γ. Note that this means that when the price decreases at a faster rate, sales increase, thus reducing the number of units that reach their expiry dates.

Given that according to equation (15) the value TS+TW is a constant, stating that TW decreases when γ increases is equivalent to saying that when γ increases TS also increases. We should therefore prove that dTS(γ)/d(γ) is non-negative.

**Proposition 1.** $\partial TS(\gamma)/\partial\gamma \geq 0$

*Proof.* Following the definition of TS in equation (11), and the definition of S(t) in equation (10), it holds that

$$\frac{\partial TS(\gamma)}{\partial\gamma} = \int_0^L \frac{\partial S(t,\gamma)}{\partial\gamma} dt = \int_0^L \int_t^L \frac{\partial S(a,t,\gamma)}{\partial\gamma} dt \qquad (20)$$

Also, by equation (9), it holds that

$$\frac{\partial S(a,t\gamma)}{\partial\gamma} = \begin{cases} -D_0\alpha[1-(a/L)^{\beta}]^{1-\alpha\gamma} \ln(1-(a/L)^{\beta}) & \text{if } t \leq \tau(\text{a}) \\ 0 & \text{if } t > \tau(\text{a}) \end{cases} \qquad (21)$$

Since the partial derivative in equation (21) has a non-negative value in any case, it holds by equation (20) that $\partial TS(\gamma)/\partial\gamma \geq 0$.

This result thus indicates that for the dynamic price policies considered, increasing the intensity of the price reduction (i.e. increasing $\gamma$) always leads to higher sales. Moreover, the positive effect of price reduction on waste reduction is higher the larger the price elasticity of demand, i.e., if $\alpha > \alpha'$ then $\partial TS(\gamma,\alpha)/\partial\gamma \geq \partial TS(\gamma,\alpha')/\partial\gamma$.

**Proposition 2.** $\partial(\partial TS(\gamma)/\partial\gamma)/\partial\alpha \geq 0$

*Proof.* According to (20) and (21) it follows that

$$\frac{\partial}{\partial\alpha}\left(\frac{\partial TS(\gamma)}{\partial\gamma}\right) = \frac{\partial}{\partial\alpha}\left(\int_0^L \int_t^L \frac{\partial S(a,t,\gamma)}{\partial\gamma}\, da\ dt\right) = \int_0^L \int_t^L \frac{\partial}{\partial\alpha}\left(\frac{\partial S(a,t,\gamma)}{\partial\gamma\partial\alpha}\right) da\ dt \tag{22}$$

i.e.,

$$\frac{\partial^2 S(a,t,\gamma)}{\partial\gamma\partial\alpha} = \begin{cases} -D_0\, ln\ 1-(a/L)^{\beta} \left[1-(a/L)^{\beta}\right]^{1-\alpha\gamma} 1-\alpha\gamma & \text{if } t \leq \tau(a) \\ 0 & \text{if } t > \tau(a) \end{cases} \tag{23}$$

And, again, this second partial derivative is non-negative in any case, which confirms the proposition.

This result thus states that the sales increase effect of the price reduction policy is more intense the larger the value of the demand elasticity. This means that products with elastic demand are the best candidates, i.e. those that will benefit more from, for implementing this type of dynamic pricing policy.

## 5. EFFECTS OF DYNAMIC PRICING

In order to study in more detail the effects of the proposed dynamic price strategy both on total revenue and total waste, a series of experiments considering different combinations of parameters $\alpha$, $\beta$ and $\gamma$ were carried out. The software used for all the calculations was MATLAB®, a well-known scientific computing environment. Seven different values ($\alpha \in \{1/3; 1/2; 2/3; 1; 3/2; 2; 3\}$) were considered for the price elasticity of demand. Note that these values consider different inelastic, unitarily elastic and elastic demand scenarios. For each value of $\alpha$ we explored 20 different, equally-spaced values for the price discount speed parameter $\gamma$ varying from 0 to $1/\alpha$, allowing us to gauge in detail the effect of this influential parameter. The different $\gamma$ values go from no price reduction ($\gamma$=0) to the maximum reasonable price reduction intensity. Thus we do not consider extreme-discount policies involving $\gamma > 1/\alpha$. We also use one of three possible values for parameter $\beta = \{1; 2; 5\}$. These values have been chosen to represent three different levels for this factor: $\beta$=1 represents a linear reduction in demand due to product aging, while the other two values represent a non-linear demand reduction, with the more significant effect occurring at a later date, the larger the value of $\beta$ (see Figure 2). Note that, because it is less realistic, we do not consider $\beta<1$ which would correspond to a situation where the demand reduction with age would be steeper at first than later on.

Regarding the age profile of the initial inventory, we have considered three scenarios (see Figure 3). The first one (labelled case I) implicitly assumes a random picking of units by customers, which can give rise to a uniform distribution of the number of units of each age kept in stock. The opposite case (labelled case III) corresponds to the more likely common situation in which the number of older units in inventory decreases with age (in our case, linearly). Case II is a mixture of the other two, with an initial random picking until instant L/2, and then a linear decrease. The three patterns can be seen as special cases of a single pattern with constant inventory until a certain age (0 for case I, L/2 for case II and L for case III) and then a linear decreasing trend to reach zero stock at L. No convex pattern for initial product age is considered. In all three cases the total number of units in the initial inventory, i.e. the area below the corresponding age profile, was fixed at the same value, namely 300 units, for the sake of comparison.

========== FIG 3 ==========

These three initial inventory profiles are just examples considered to illustrate and assess the effectiveness of using price reductions to reduce waste without sacrificing revenue. The methodology used to assess the effectiveness works independently of a given initial inventory profile. Actually, the initial inventory profile would be different from one company to another and, even for a given company, it would be different in different time periods. Whatever the initial inventory profile, i.e. for any arbitrary initial inventory profile, the proposed approach can empirically/numerically compute the sales and revenue in each time period and for the whole time horizon.

The specific values used for the parameters were L=10 t.u. and $p_0$=5 m.u. In order to fix the value of the demand parameter $D_0$ we carried out some preliminary experiments to assess the amount of spoilage generated in each case. Figure 4 shows the total waste and the total revenue for the no-discount scenario (i.e. $\gamma$=0) and for $\alpha$=1 and $\beta$=2. Note that TW is highest for case I and lowest for case III (with case II in between). For TR the opposite occurs.

Note that all the points plotted in Figure 4 lie on a straight line. That is not surprising, given that, according to (15), the sum of TW and TS is a constant, the same for all three initial inventory cases considered. And, since the results plotted in Figure 4 correspond to a no-discount policy $\gamma$=0, i.e. $p=p_0$, the total revenue TR is proportional to TS. Hence

$$\left.\begin{aligned} TR &= p_0 \cdot TS \\ TS + TW &= c \end{aligned}\right\} \Rightarrow \frac{1}{p_0} \cdot TR + TW = c \tag{25}$$

With respect to the value of $D_0$ chosen, in the end a value $D_0$=15 was selected so that the spoilage that results after the initial experiments (around 5%-10% depending on the case) is similar to the levels observed in practice and reported in the literature. For example, Chung and Li [14] report that disposal rates are on average 2%, sometimes reaching 10% while Ferguson and Ketzenberg [22] mention that spoilage rates may reach 15%.

Figure 4 also allows us to comment that an obvious way to reduce spoilage and, at the same time, increase revenue is by increasing the demand rate (e.g. through promotion). Doing so, however, has costs and also its effectiveness is marginally decreasing. Thus, it can be seen in Figure 4 that the effects of $D_0$ increases are marginally decreasing.

========== FIG 4 ==========

Also, before we show the results obtained for the different scenarios, it is interesting to take a look at the type of information that can be derived from the proposed approach. Thus, for example, Figure 5 shows, for α=2, β=1 and case I of the initial inventory age profile, the total sales and total revenue obtained with the different price discount policies from γ=0 (i.e. no discount) to γ=1/α (which is the maximum discount rate considered). For each value of γ, a different point is obtained. In this case, small values of γ slightly increase TR but those disappear as γ approaches 1/α. The effect on TW of varying γ is monotonic, i.e. TW always decreases as γ increases. This confirms the theoretical result presented in Section 4; that is because the price discounts offered to customers stimulate demand and thus reduce spoilage. But what we would like to focus on is that the slope of the segment joining each of these points with the origin represents the average price charged. It can be seen that the higher the discount the lower the average price and, what is more interesting, higher revenue, lower spoilage and lower average prices can be obtained using the appropriate dynamic pricing policy.

This gives the best of both worlds: profitability and corporate social responsibility. Unfortunately, as we will see, this situation, which benefits the company, consumers, the environment and society in general, does not always occur. For this to happen, certain factors must concur, among them that the price elasticity of demand is sufficiently large. It is clear that if demand is not sufficiently price elastic, then price discounts would not produce a demand increase large enough to compensate for the loss of revenue due to the price reduction. In other words, a dynamic price strategy increases its effectiveness and attractiveness as the price elasticity of demand increases. The other two factors, i.e. the age profile of the initial inventory and the sensitivity of demand to age, also have an influence, but not as significant as that of the demand elasticity.

========== FIG 5 ==========

### 5.1. Revenue loss vs. spoilage reduction

From the above discussion it follows that, although spoilage reductions are warranted with a dynamic price strategy, the total revenue may fall. In that case, the proposed approach allows us to see the trade-offs between both magnitudes. Thus, for example, Figure 6 shows, for $\alpha=\beta=2$, the reduction in total revenue versus the reduction in total waste that occurs as the discount speed parameter $\gamma$ increases. The slope of the curve represents the marginal loss of revenue for a unit waste reduction. Note that the curves are not generally monotonic. Parts of the curves have a positive slope, meaning that the reduction in waste brings about revenue losses but, also, in some parts, the slope is negative indicating increases in total revenue compatible with waste reduction. Thus, in the case of initial inventory age profile I, total revenue first slightly increases (w.r.t. the $\gamma$=0 no-discount benchmark) but, after reaching a maximum, decreases if higher price discounts are made. This type of trade-off analysis is different from the one carried out in the simulation study of Chung and Li [14], which reaches the conclusion that an additional 2% augmentation in sales is required to compensate for every 5% increase in markdown.

========== FIG 6 ==========

Figures 7 and 8 show the effects on TR and TW of changing the discount speed factor $\gamma$ for varying values of parameters $\alpha$ and $\beta$, and for the three initial inventory age profile cases. Note that the scale of the TW axis indicates that, as mentioned above, case I is the one that generates the largest spoilage while case III generates the least. The decrease in spoilage as $\gamma$ increases is significant, reaching zero total waste in the cases of initial inventory age profiles II and III. Since in age profile I there exists initially an inventory with a close to zero remaining shelf life, it is not possible to reduce TW to zero. As regards the total revenue, it can be seen that because of the high price elasticity of this scenario $\alpha$=3, it increases (or at least does not decrease) for small $\gamma$, although when one TW reaches zero further $\gamma$ increases are unnecessary and only lead to a reduction in TR. TR increases are more significant in cases I and II because they involve more spoilage than in case III.

========== FIG 7 ==========

Almost the same effects can be observed in Figure 8. Thus, again, cases I and II lead to larger spoilage in the no-discount $\gamma$=0 scenario. However, independently of the scenario, TW is always reduced when $\gamma$ increases. As regards TR, although, as we mentioned above, for high price elasticities TR can be increased, for lower price elasticities TR decreases as $\gamma$ increases, because the increases in demand induced by the price discounts are not large enough to compensate for the lower unit price. It is also evident that faster price discounts (i.e. larger $\gamma$ values) than the one required to reduce TW to a small value (close to zero) are not recommended since they only contribute to further reduce TR.

========== FIG 8 ==========

Note that the proposed approach can be related to a single-variable bi-objective optimization problem since we consider two objective functions, namely total revenue and total sales volume (or equivalently, total waste) as well as a single decision variable, namely the discount intensity $\gamma$. Thus, when plotting the value of the two objective functions for different values of $\gamma$ (as in Figures 5, 7 and 8) we can visualize the corresponding Pareto Front, i.e. the non-dominated section of the plot. We have labelled the above proposed approach a "parametric, bi-objective approach" rather than a bi-objective optimization approach because we are not trying to optimize the objective function in the sense of computing the best value of parameter $\gamma$. What the proposed approach does is to assess, by evaluating the two objective functions for different values of $\gamma$, what the effects on the two objective functions are. These are related to the shape of the corresponding Pareto Front and, as the results show, generally depend on the scenario considered, i.e. on the price elasticity, the age sensitivity of demand and the profile of the initial inventory.

**5.2. Sales and revenue as a function of age and time**

Figure 9 shows the detailed results on the time and age distribution of the number of sold units (respectively, S(t) and $\hat{S}(a)$) and their corresponding revenue R(t) and $\hat{R}(a)$. Although only the results of a sample scenario are shown, the proposed approach allows us to study the distribution patterns for any dynamic pricing scenario. For the scenario shown in Figure 9, corresponding to $\alpha$=1, $\beta$=2 and γ=0.5, although there is little difference in the average age of units sold between the three cases (5.09, 5.16 and 4.79 for cases I, II and III, respectively), it can be seen that there are differences in their respective shapes, with cases I and II involving a larger share of aged units and a smaller share of fresher units than case III. The area below each of these curves represents the TS of these scenarios, which are, respectively, 234.6 p.u., 290.2 p.u. and 297.7 p.u. As regards the revenue contribution of units of different ages, the distribution has a similar shape to that for the units sold, except that the curves reflect that, slightly at the beginning but more pronounced with age, the more aged units are sold at a lower price and therefore their relative revenue contribution is decreasing with age. Similarly, the area below each of these curves represents the TR of these scenarios, which are, respectively, 940.7 u.m., 1136.2 u.m. and 1246.4 u.m.

========== FIG 9 ==========

As regards the evolution of sales and revenue with time, it can be noted that the sales and revenue at time zero, i.e. S(0) and R(0), are the same for the three initial inventory profiles. This is due to the fact that those two values do not depend on the initial inventory profile. Thus, according to (9), (10) and (13),

$$S(0)=\int_0^L S(a,0)da=\int_0^L D(a)da=\int_0^L D_0\left(1-\left(\frac{a}{L}\right)^{\beta}\right)^{1-\alpha\gamma}da \tag{26}$$

For the α=1, β=2 and γ=0.5 scenario considered in Figure 9, this leads to

$$S(0)=\int_0^L D_0\left(1-\left(\frac{a}{L}\right)^{2}\right)^{1/2}da=\frac{D_0}{L}\int_0^L\sqrt{L^2-a^2}\,da=\frac{D_0 L\pi}{4} \tag{27}$$

For the parameter values assumed to be $D_0$=15 and L=10, a value of S(0)=117.81 units sold results, which is the same for all three initial inventory cases considered.

Analogously, according to (12), (13), (2) and (3),

$$R(0)=\int_0^L R(a,0)da=\int_0^L p(a)D(a)da=\int_0^L p_0 D_0\left(1-\left(\frac{a}{L}\right)^{\beta}\right)^{1-\alpha\gamma+\gamma}da \qquad (28)$$

For the α=1, β=2 and γ=0.5 scenario considered in Figure 9, this leads to

$$R(0)=\int_0^L p_0 D_0\left(1-\left(\frac{a}{L}\right)^{2}\right)da=\frac{p_0 D_0}{L^2}\int_0^L \left(L^2-a^2\right)da=\frac{2p_0 D_0 L}{3} \qquad (29)$$

For the parameter values $p_0$=5, $D_0$=15 and L=10, this results in a value of R(0)=500 m.u., independent of the initial inventory case considered.

Finally, note that if there is no price reduction, i.e. if γ=0, the shape of both functions $S(t)$ and $R(t)$ is the same since they are proportional, i.e. $R(t)=p_o\,S(t)$. The difference in shape that can be observed in Figure 9, however small, reflects the existence of price reductions in the γ=0.5 scenario considered.

### 5.3. Effects of demand elasticity on total revenue

In order to measure the effect of demand elasticity, the reduction in total revenue corresponding to a reduction of 50% in spoilage will be considered. Let $\gamma_{50\%}$ be the smallest value of the discount speed factor γ that leads to a reduction larger than 50%. Such a value is variable and depends on the scenario considered. Figure 10 shows the loss in total revenue (w.r.t. the γ=0 no-discount scenario) that corresponds to the $\gamma_{50\%}$ policy.

========== FIG 10 ==========

It can be seen that the loss of total revenue clearly depends on the demand elasticity α, although that dependence is smaller as β increases. Note also that, actually, total revenue losses generally exist for demand elasticities below 1.5. Above that threshold, total revenue is practically maintained or even slightly increased. Interestingly, for high α values, the improvements in total revenue are higher as β decreases.

The scenarios in which the (negative) economic impact of spoilage reduction is higher, correspond to an inelastic demand (i.e. $\alpha<1$). This is not surprising since, when demand is price inelastic, the effectiveness of price discounts, as a means to achieve demand increases, is rather limited. Actually, as α decreases below the unity threshold, the loss in total revenue increases at an exponential rate. This behaviour occurs for the three initial inventory age profiles, although it is somewhat less acute for case III. As a rule of thumb, using the middle scenario (i.e. case II and $\beta=2$) as the reference, it can be estimated that the total revenue loss for a 50% reduction in spoilage can be around 20% for the worst case of a rather low demand elasticity ($\alpha<0.5$).

## 6. CONCLUSIONS

In recent times the interest in reducing spoilage, especially of food products, has increased not only because of its economic significance but also because of its social and environmental impact. Dynamic price strategies, i.e. offering aged units at a lower price than fresh units, are an effective way of reducing spoilage since customers are thus encouraged to demand less fresh but cheaper units. There is the risk, however, that if the price reduction is too steep or the price elasticity of demand too low, the price dynamic strategy may lead to a lower total revenue.

In order to measure and quantify the effects of these and other factors (such as the initial inventory age profile or the sensitivity of demand to the product age) on sales, revenue and spoilage, this paper proposes a continuous-time mathematical model that allows studying the depletion of a given initial inventory. The aim of this model is to gain insight and learn about the interactions between the dynamic price strategy and different factors considered.

In addition to the total sales and revenue, the corresponding age distribution can also be computed and analysed. And not only can the total sales and revenue for the whole horizon be computed but also the value of those magnitudes in each time period. It has also been proven that an age-dependent price discount policy, such as the one considered, always reduces the number of units spoiled/wasted so that the higher the discount rate, the fewer the number of units reaching their end of life. Moreover, this effect is more evident as the price elasticity of demand increases.

A number of experiments have been carried out considering many different scenarios and the first remark to make here is that the behaviour is different, depending on the scenario considered. In all of them, however, it is confirmed that the dynamic price strategy can significantly reduce total waste, often theoretically to a value close to zero. However, the effect on total revenue is not always positive. In some scenarios with high price elasticity of demand and large potential spoilage, total revenue can be slightly increased if the price is discounted. In some other cases, total revenue may be kept more or less constant, provided the price-discount speed is not too high. And in some other cases, generally involving low price elasticity, an age-insensitive demand or a not too aged initial inventory, there can be a substantial revenue loss that increases as the price-discount is performed faster.

The numerical results depend on the initial inventory profile and indicate, in some specific cases (e.g. those in which the price elasticity and/or the age sensitivity of the demand is small), that reducing the price reduces revenues without hardly increasing sales or reducing waste. In such cases the proposed approach would warn against (and discourage) the use of dynamic pricing. Thus, the advantage of the proposed approach is its ability to assess and compute the effects of a given price reduction policy in any specific situation.

Regarding the managerial consequences of the study carried out, we observed that the effects on revenue of marking down the products as they are approaching their expiry date, are very dependent on the demand elasticity, but, even when the product is very price inelastic, we have estimated that, in general, reducing the spoilage by 50% would have an impact on total revenue loss not exceeding 20%. In a more desirable scenario, if the demand elasticity is between 1 and 2, the loss in total revenue compatible with spoilage reduction is, in general, very small or non-existent. And, if the demand elasticity is above or equal to 2, it is possible to combine a significant reduction in spoilage with a small increase in revenue. As a conclusion, we can expect the dynamic price policy to be very effective in reducing spoilage without large reductions in total revenue (or even with small increases) for a large fraction of possible demand and consumer behaviour scenarios, excluding those cases of low to very low values of price elasticity, especially when combined with insensitivity of demand to product age. In those unfavourable cases, some other alternative, different from price discounts, should be sought.

As regards limitations, there are some on which we would like to comment. Thus, for example, we have not considered the feasibility or the cost of implementing a continuous-time dynamic price strategy. Moreover, some researchers [35] argue that, in practice, prices are not generally changed smoothly, since only significant discounts will increase demand significantly. In addition, other issues that also affect demand and that can interact with the dynamic price strategy, such as displayed quantity effects, i.e. how the display of larger quantities of products can attract customers and increase demand, have not been considered. Also, although we have considered a deterministic scenario in our analysis, in principle, that price reduction can eliminate or reduce waste without hurting sales also applies to the case of stochastic demand. The experiments would have to be replicated a number of times to carry out a Monte Carlo simulation of the effects on total revenue and on total waste of each discount intensity value $\gamma$. The fact that we would have confidence intervals would complicate the analysis but similar conclusions might be expected, i.e. that, depending on the scenario, it is feasible to achieve waste reduction and revenue increase/preservation.

In any case, the present research can be considered as a first step, a proof of concept that the best of both worlds (more revenue and less spoilage) can be achieved through this type of dynamic pricing. The theoretical and empirical results obtained in this simplified approach are encouraging to further study more realistic and complex approaches. Thus, for example, it is possible to extend the analysis to an infinite horizon approach with successive cycles of a given length T (not necessarily equal to L) so that the initial inventory of each cycle is endogenously determined by the replenishment decisions and pricing policy used. Thus, for a given order size, at the beginning of each cycle (e.g. the first day of the week), the existing stock is replenished with fresh product, which ages along the week so that the inventory profile at the start of the following cycle depends on the order size and the dynamic pricing policy used (which determines the sales level). The decision problem is to determine the optimal values (in a Pareto sense) of the order size and the price reduction rate. Revenue and waste would be the two objective functions considered.

The proposed approach can also be extended so as to consider a discrete-time finite horizon with replenishment in every period and with fixed initial and final inventory profiles. The idea is to compute the amount to order each period and the price reduction rate (which can be the same or different in each period) that are optimal, in the bi-objective sense of maximizing revenue and minimizing waste. Studying those more realistic scenarios is, however, left for further research, provided that the advantages of the dynamic pricing policies studied in this paper have been well established.

Finally, although the proposed approach adopts a simplified, aggregate perspective in which differences in customers' behaviour are not distinguished, from a micro perspective these differences between customers can be essential and should be modelled. Other factors, such as operating costs (including ordering and holding costs), lead times and reliability of suppliers, etc., would also increase the realism and practicality of the approach, which in turn would contribute to its implementability.

**Acknowledgements.** This research was carried out with the financial support of the Spanish Ministry of Science grant DPI2013-41469-P and the European Regional Development Fund (ERDF).

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**Figure 1.** Inventory evolution with time. As no replenishment is considered, I($a$,t)=0 $\forall a$<t. I(L,t) represents the product wasted at instant t≤L.

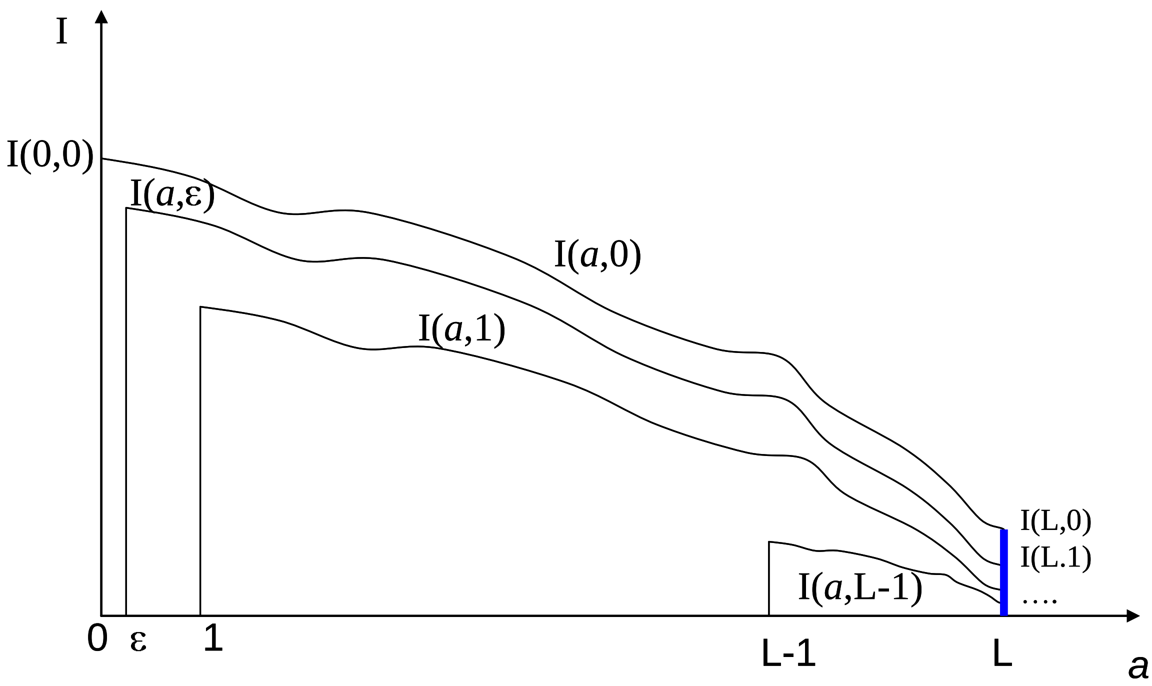



**Figure 2.** Decreasing demand as the units age, depending on the β parameter

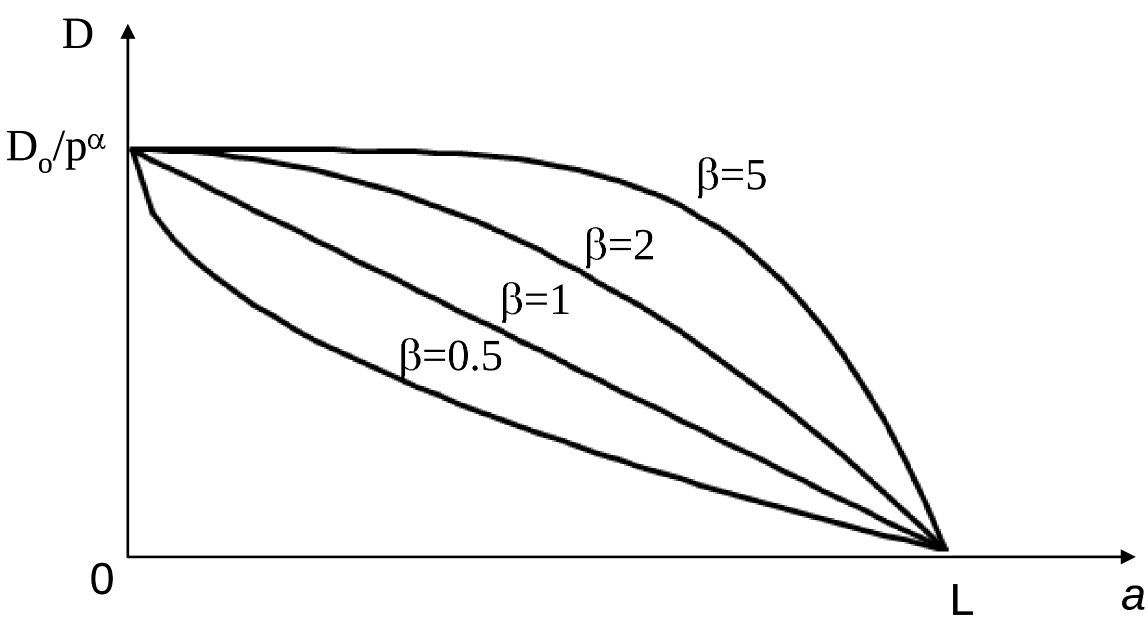


**Figure 3.** Age profile of the initial inventory I($a$,0) in the three cases considered, all with the same total initial stock

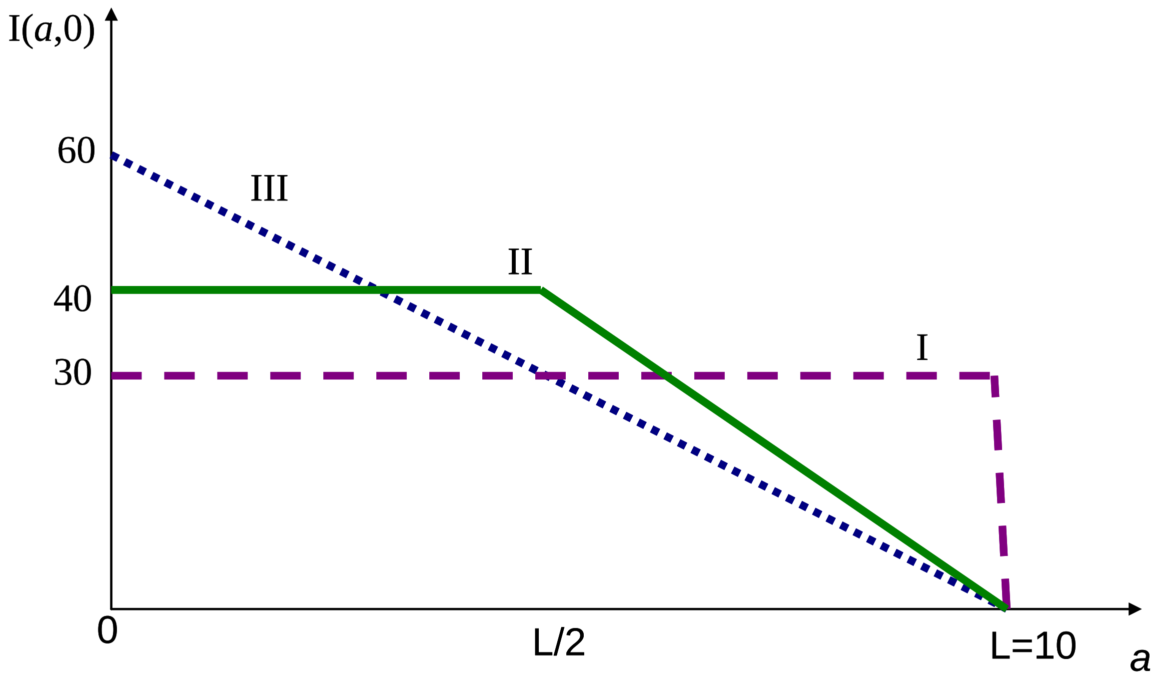



**Figure 4.** Change in total revenue and total waste, for a no-discount scenario ($\alpha$=1, $\beta$=2, $\gamma$=0), as $D_0$ increases

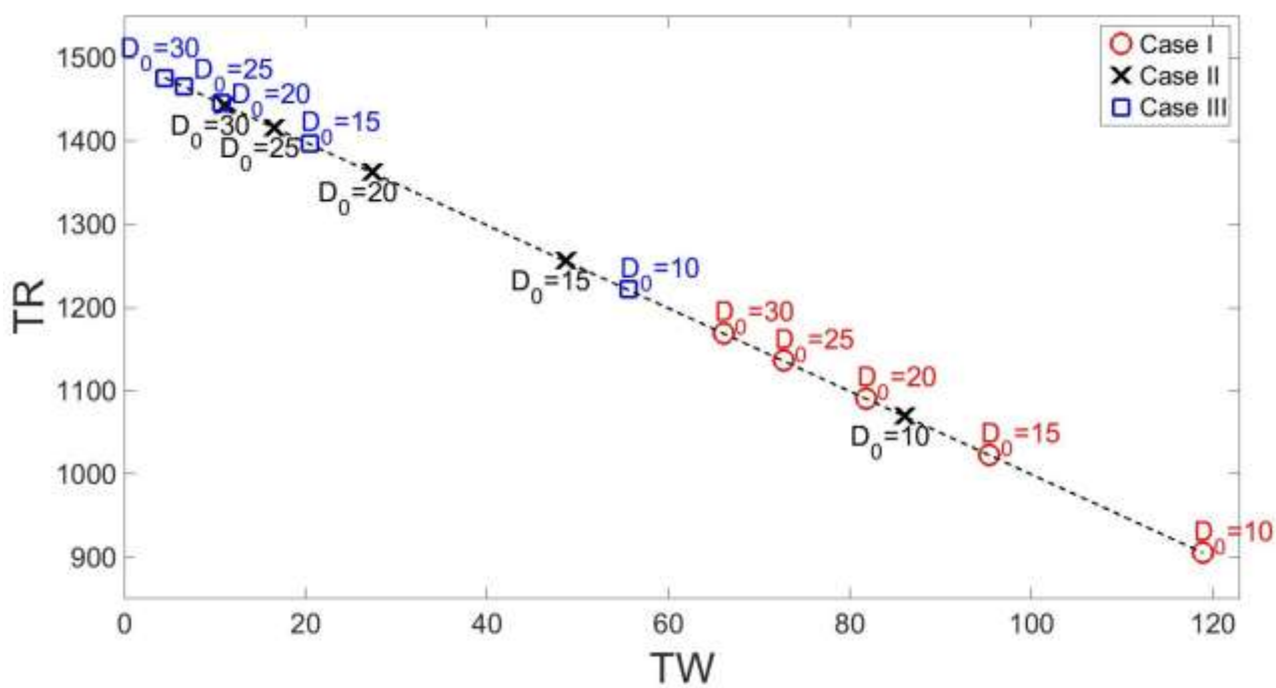


**Figure 5.** Change in total revenue and total sales, for a sample scenario (case I, $\alpha$=2, $\beta$=1), as $\gamma$ increases. Slope of segment represents resulting average price charged.

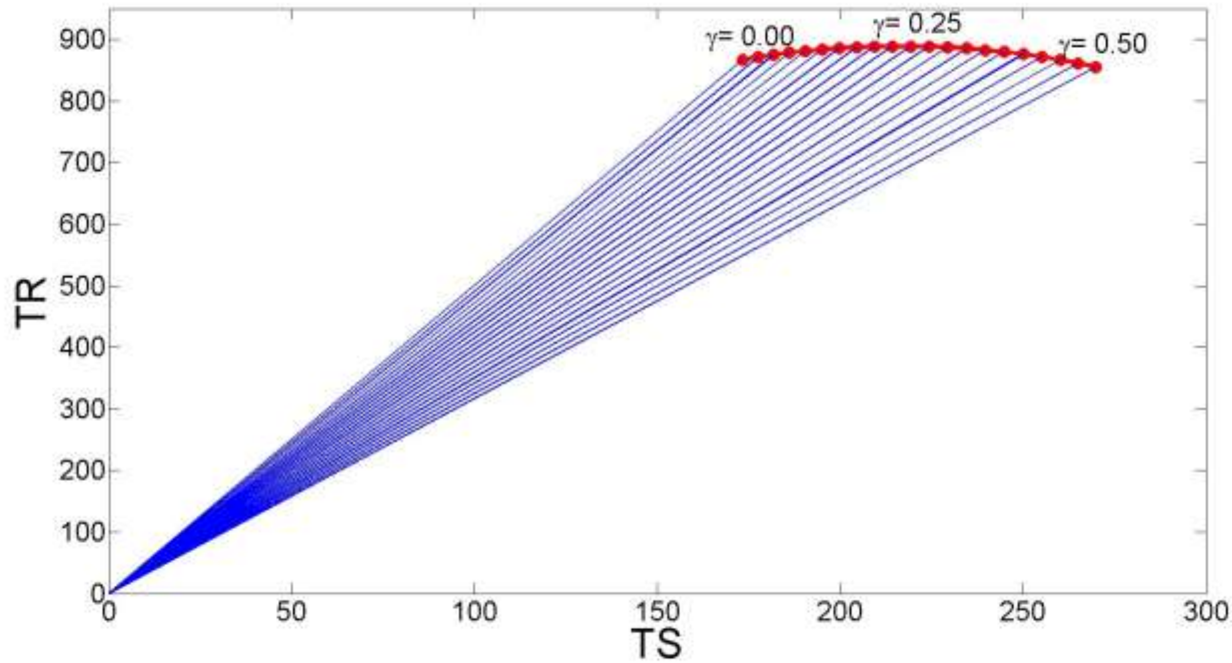



**Figure 6.** Revenue loss (w.r.t. no-discount benchmark) versus total waste reduction ($\alpha$=2, $\beta$=2)

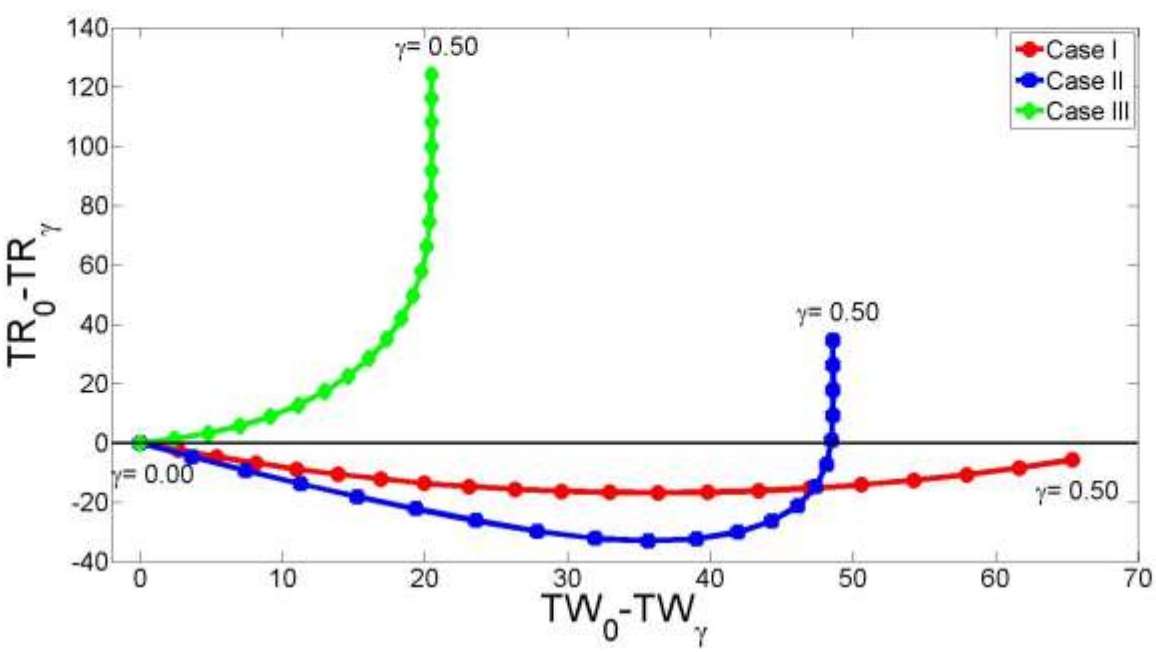
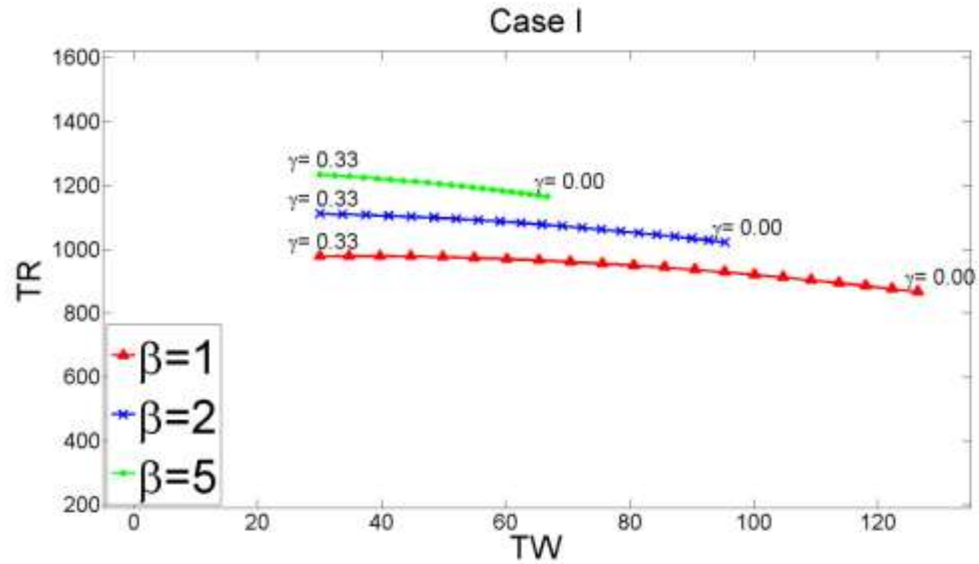
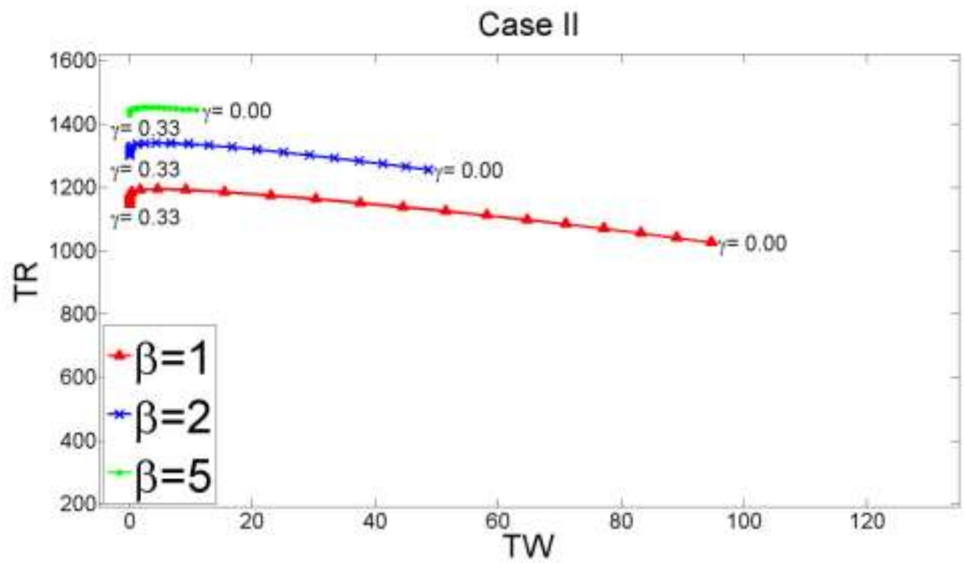


**Figure 7.** Change in total revenue and total waste, for α=3 and different β values, as γ increases



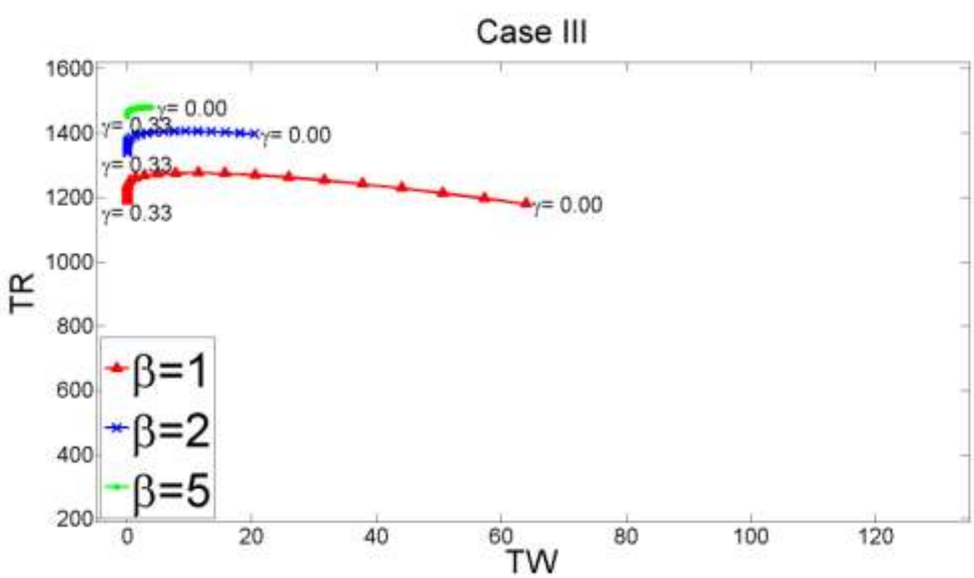



**Figure 8.** Change in total revenue and total waste, for β=2 and different α values, as γ increases

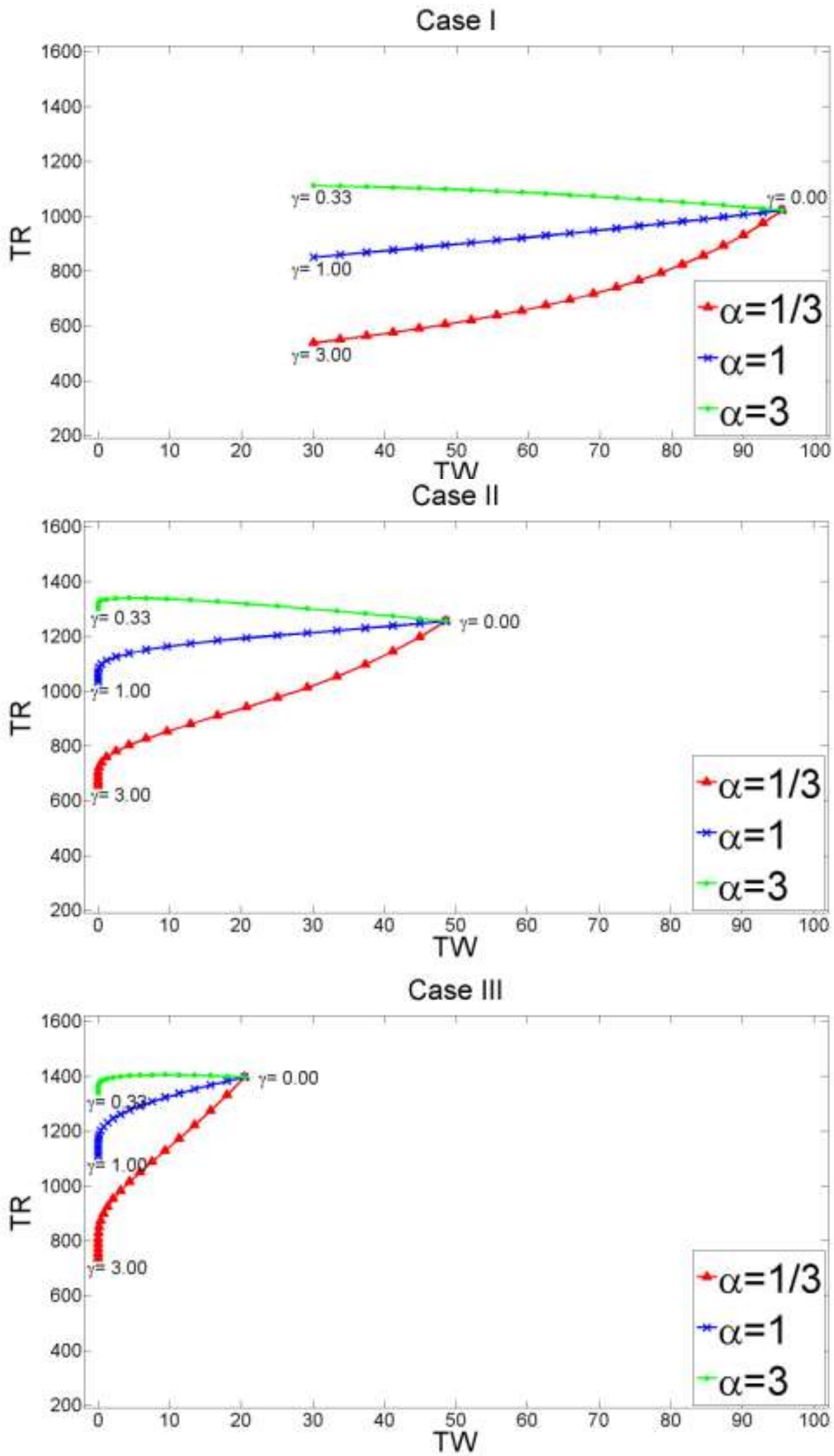


**Figure 9.** Age and time distribution of number of sold units and corresponding revenue for a sample scenario (α=1, β=2 and γ=0.5)

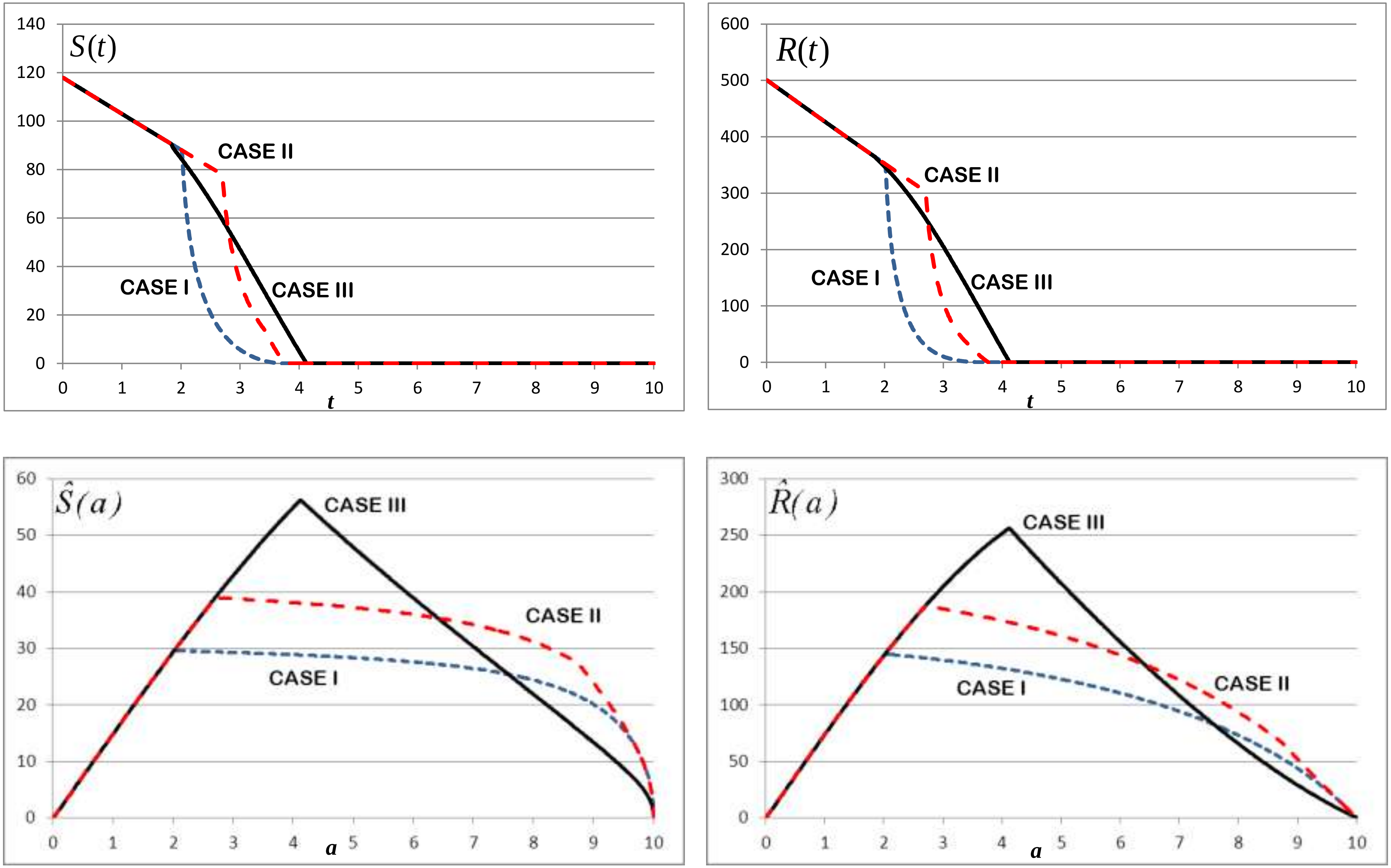


**Figure 10.** Loss in total revenue corresponding to a discount speed factor $\gamma_{50\%}$