

A STATISTICAL APPROACH TO PREDICTION OF THE CMM DRIFT BEHAVIOUR USING A CALIBRATED MECHANICAL ARTEFACT

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Abstract

This paper presents a multivariate regression predictive model of drift on the Coordinate Measuring Machine (CMM) behaviour. Evaluation tests on a CMM with a multi-step gauge were carried out following an extended version of an ISO evaluation procedure with a periodicity of at least once a week and during more than five months. This test procedure consists in measuring the gauge for several range volumes, spatial locations, distances and repetitions. The procedure, environment conditions and even the gauge have been kept invariables, so a massive measurement dataset was collected over time under high repeatability conditions. A multivariate regression analysis has revealed the main parameters that could affect the CMM behaviour, and then detected a trend on the CMM performance drift. A performance model that considers both the size of the measured dimension and the elapsed time since the last CMM calibration has been developed. This model can predict the CMM performance and measurement reliability over time and also can estimate an optimized period between calibrations for a specific measurement length or accuracy level.

Keywords: multivariate regression models, Coordinate Measuring Machine, drift behaviour, calibration.

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1. Introduction

Coordinate Measuring Machines (CMMs) are one of the most common measuring instruments used to verify production quality with high precision. They are high cost machines which usually require controlled environments and require regular assessment and calibration processes with expensive means. In fact, interim performance verification tests following standards like ISO 10360-2:2009 [1] are periodically carried out in order to evaluate the overall volumetric performance of the CMM. Into this field, the concept of “maximum permissible error of indication” for a given L length measurement, and the “maximum permissible single stylus form error” are the main parameters that define the CMM performance, capability and even the necessity of recalibration. When a verification procedure is carried out, these parameters are obtained and compared with manufacturer specifications. However, for a given calibration process, these parameters must be compared with an uncertainty value. In both cases, determination of a correct verification – or calibration – period can save a huge amount of time and money.

CMMs and their uncertainty have been extensively studied in the past and even nowadays. The common thread about CMM uncertainty consists in carrying out performance tests following International Standards [2] and experimental tests using calibrated objects [3, 4]. Basically, these tests involve measuring five lengths of a well-known master gauge in several positions within the working volume of the CMM. ISO standards propose an uncertainty budget that considers several contributions, such as the calibrated uncertainty of the master gauge, the

uncertainty of the coefficient of thermal Expansion of the master gauge, the uncertainty due to the input temperature of the master gauge, the uncertainty due to the misalignment of the master gauge, or the uncertainty due to the used fixture method. Afterwards, errors of indication for the five length sizes are plotted in function of the length size together with the measurement uncertainty and the maximum permissible error stated by the CMM manufacturer. This way, it can be deduced whether the CMM performance meets the specifications or not.

Some authors develop their own methodologies supported by a master gauge. Among these master gauges different three-dimensional features can be found, like small cylinders and rectangular prisms [5], a combination of cylinders & cones [6], and other geometric characteristics [7, 8], Ball plates [9, 10], ring gauges [11] or Ball Bar devices [12].

It is also common to apply GUM principles to the uncertainty determination in these tests [7, 12–14]. Some authors develop analytical models to calculate uncertainty taking into account different tolerances in specific tasks [9, 15, 16]. From a different approach to the matter, others researchers start from the Virtual CMM concept [17] and propose models that attempt to predict the CMM uncertainty by modelling the CMM behaviour [18–20], using factorial design [12] or taking into account the effects of several contributions [21–23].

The evaluation of the CMM used in this paper looks for a drift model on a different level of performance than other researchers do. The main objective is to determine a suitable period between CMM calibrations, both economically as functionally. Another aim of this survey is to establish the “CMM Capability Chart” and how this chart evolves over time. This chart is used to determine which CMM specification (MPE_E) is required in order to measure a distance or a diameter with a given tolerance.

However, the measurement uncertainty of a specific measurement task (GD&T) or of a given geometric feature is not considered here. This kind of research requires a task-oriented and dedicated budget and demands very different processes and techniques than the one presented here. This is because the uncertainty evaluation task depends on the machine parameters, like the head probe’s speed [22], the probing points technique [23], the spatial distribution [16, 24], as well as on the differences of coordinates of probing points used to calculate a particular geometric characteristic [15, 25]. Moreover, the uncertainty of coordinate measurements also depends on the software for analytical evaluation [22, 24, 26].

Given the complexity of the CMM measurement system and the quantity of factors involved, there are also many works that study the effects of these factors on the CMM performance at a time. Design of Experiments (DOE) and Analysis of Variance (ANOVA) techniques broaden this study by defining several levels for each factor and performing measurements corresponding to selected combinations of factors [12, 27–31].

Even though CMM uncertainty sources have been extensively analysed in many ways, the CMM performance evolution over time (known here as a drift) has been relegated to the background. It is generally assumed that the CMM performance is progressively worse, but almost nobody knows how. For this reason, periodic CMM re-verification or interim tests are carried out as a means to know when the CMM needs to be calibrated.

This work presents a study of the CMM measurement’s drift by measuring a step gauge artefact through more than twenty-seven weeks. The available information allowed the creation of a predictive model of the drift that can be used to avoid non-productive and time consuming re-verification tests. From the point of view of the authors the main interest of the present research is that it presents a novel and easy to implement methodology to control the CMM machine drift based on simple statistical techniques. This research is in line with others recently published that use machine learning and statistical techniques for modelling and prediction in manufacturing. Some of the most remarkable papers published in the last years are those that model the errors found in the machine tools in order to improve their productivity, taking into account possible error sources such as time [32], those that from a three dimensional statistical

approach are able to determine the tolerance for manufacturing [33] or even applications of artificial neural networks for predicting surface roughness in a milling process or a tool life in machining.

2. The experimental set-up. Materials and methods

The method is based on repeating an extended version of the ISO 10360-2 test for determining the error of length measurement during more than five months and in intervals between five and ten days. In the period between tests, the CMM was functioning constantly during an average working time of 2–3 hours daily. This CMM is designed for educative and research purposes and is not intended to be employed for intensive production use.

So, at least one day a week, the step gauge (Fig. 1-left) was measured in seven different orientations (denoted as X, Y, Z, XY, XZ, YX and XYZ location) into the CMM measurement volume.

For each orientation, seven length standards (with dimensions about 20, 60, 150, 240, 330, 420 and 500 mm) were measured three times for each of two opposite directions. Every length size was materialized between two points of a standalone step gauge. Once the spatial orientation was established, the gauge was first aligned manually and subsequently automatically following the well-known 3–2–1 procedure. Afterwards, the measurement points for evaluating each length error were chosen so that they were located along the neutral axis of the gauge. A typical setup for one of the seven gauge orientations is shown in Fig. 1a. Fig. 1b shows a screenshot of the PC-DMIS software program used to measure the gauge. Besides, a function for axis-linear thermal compensation (available in PC-DMIS for the CMM) was always activated to minimize the influence of temperature variations.

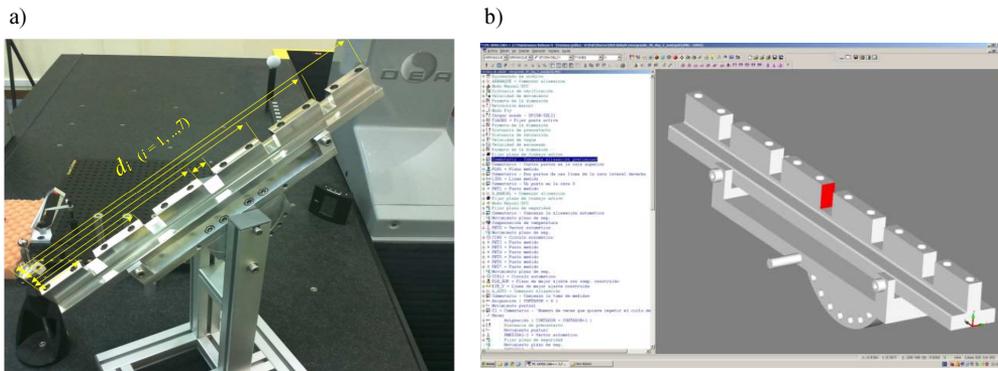


Fig. 1. a) The step gauge artefact with its seven standard lengths and the test setup for one of the 1st volume measuring orientation; b) a PCDMIS screen-capture for the off-line CAD programming.

The step gauge and the multi-orientation fixture were specially designed and manufactured for this purpose. The gauge is made of stainless steel (Austenitic AISI 304) alloy with the total length of about 500 mm. It has a T-inverted section (50x50 mm), inscribed in an equilateral triangle, thus ensuring a low centre of gravity. The measurement surfaces of the crenels (60 mm length) were machined by milling and grinding, materializing the seven standard lengths (d_i in Fig. 1). Finally, a heat treatment, on a stress relief bake, was carried out in order to relieve residual stresses after machining. Regarding the measurement strategy, the seven measured distances (d_i) were materialized using always the same points. Such points are close to the neutral axis, located into the laterals of the crenels and with opposite normal vectors, so all the lengths consist of two probes in opposite directions. The fixture supports this master gauge with

the Bessel points (very close to the Airy points). The nominal values of the seven standard lengths were obtained from calibration carried out by an accredited external laboratory in order to maintain enough traceability during this procedure. The uncertainties of the nominal values were between 0.4 and 0.8 μm .

Although the period during which the tests were performed is considered representative enough for evaluating evolution of the CMM drift, the own step gauge drift has not been considered as relevant in this study. The dimensions of the step gauges did not change during the test period, as the artefact was robustly designed, heat-treated before its first calibration and was always kept in controlled ambient conditions inside the metrology laboratory since the beginning of the study.

The CMM used was a DEA Global Image model (Hexagon Metrology), with a PH10MQ indexed head and an SP25 probe (Renishaw). A stylus made of 25 mm long carbon fibre with a spherical tip made of synthetic ruby of 6 mm diameter was attached to the sensor. The environment temperature of the laboratory has been controlled to be within the range of $20 \pm 1^\circ\text{C}$. The experiment began just after the CMM was recalibrated (Fig. 2 left) and was being adjusted throughout the full CMM performance test, thus verifying linear, squareness, volumetric accuracy and repeatability. These tests were carried out by the Hexagon Metrology laboratory (which is an ENAC certified laboratory; ENAC is the Spanish National Accreditation Body for ISO 17025) ensuring the maximum permissible error for a length measurement of $MPE_E = 2.2 + 3L/1000 \mu\text{m}$, where L is in mm.

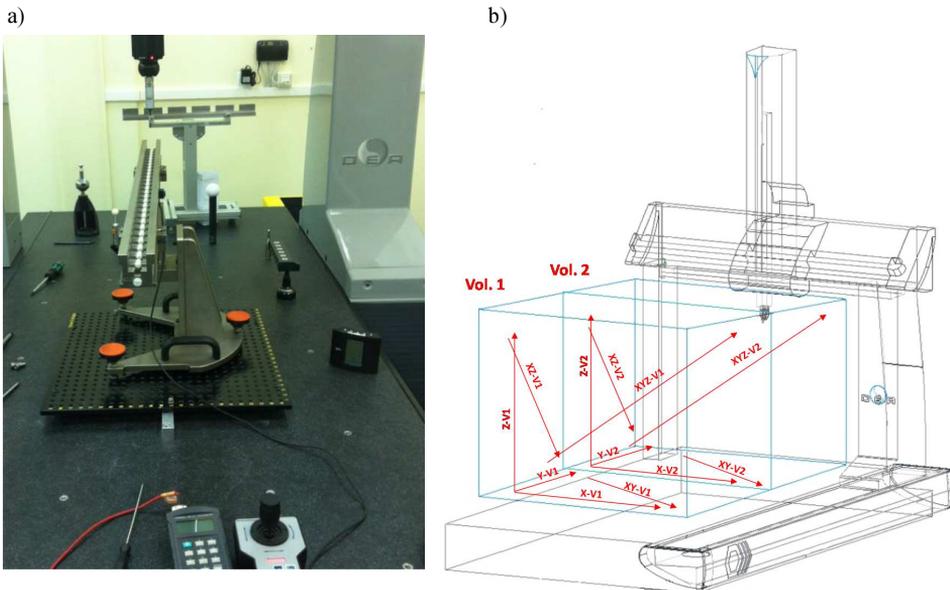


Fig. 2. a) The CMM during preliminary calibration (with KOBA® gauge) before the beginning of the experiment; b) Some volumes and orientations considered for posterior interim tests.

Although the overall length of the step gauge does not cover the whole range of the CMM ($X900\text{-}Y1500\text{-}Z800 \text{ mm}$), the consideration of two volumes (Vol. 1 and Vol. 2) makes possible a global CMM evaluation. Fig. 2-Right shows the volumes and the nomenclature used for the tests. Every volume is defined in such a way that it covers the width and the height of the original CMM working volume and the length is roughly half of the original length. Due to the probe accessibility, approach distances and collision avoidance movements, those dimensions

must be reduced in size. Therefore, the length of these volumes (along the Y axis) is approximately 700 mm, and the two volumes overlap along 100 mm.

Table 1. The description of the variables used in the experiment.

| Variable name | Description |
|---------------------------------|---|
| Volume | The volume of the machine in which the master gauge is placed and the measurement is taken. |
| Length (Measurement identifier) | The total of six measurements were performed in each test for each of the seven nominal measurements. Please note, that each measurement is the distance between two parallel planes, having opposite normal vectors. |
| Days | The elapsed time, expressed in number of days, since the date of the last CMM adjustment and calibration till the measurement date. |
| Nominal | The nominal value of each dimension (the range of values from 19.996 to 499.934 mm according to the external gauge calibration test). |
| Actual value | The actual value of the measurement taken by the CMM machine. |
| Deviation | The difference between the nominal and the actual values. |

2.1. Collected data and variables

The entire survey covers the total of 24 ISO-10360 tests, each one with 588 standard length measurements; thus: two CMM volumes per test, seven orientations per volume, seven lengths per orientation, three repetitions per length and two directions per repetition (different sequence in probing end points of each length). In order to elaborate a statistical model and taking into account the large amount of generated information, a particular nomenclature adopted during the research is listed in Table 1. The step gauge was placed in two volumes of the machine, materializing the total of fourteen different spatial positions (combining X, Y, Z, XY, XZ, YZ and XYZ axes with the aforementioned volumes Vol. 1 and Vol. 2), as shown in Fig. 2b. The X, Y and Z positions mean locating the gauge in parallel to the axes of the CMM, the orientations denoted with two letters, *i.e.*, XY, XZ or YZ, mean the in-plane diagonals, while the XYZ orientation corresponds to the volumetric diagonal, the longest space diagonal.

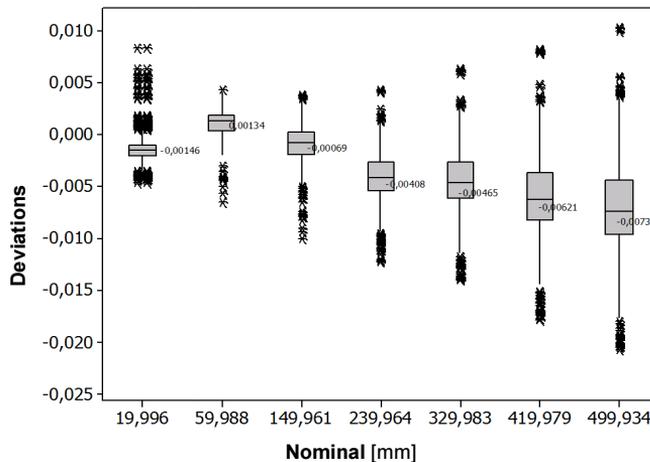


Fig. 3. A boxplot of differences between the Nominal and Actual values of all the measurement results.

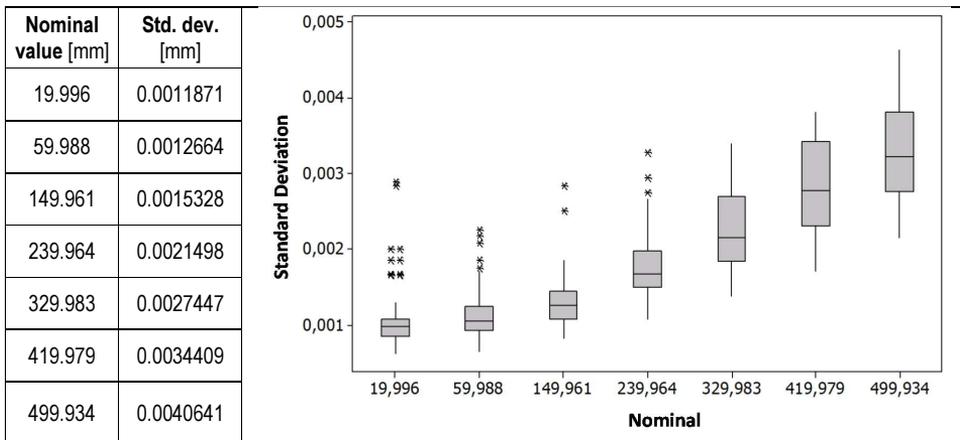
The measurements were taken on twenty-four different days, starting in December 2012 (on the date of the last CMM calibration) and finishing in April 2013, using always the same interim procedure.

3. Statistical implementation

All the data were analysed using R Statistical software. With this tool, several models were tested in order to find the best relation – seeking an equation – between the variables considered. The first model looks for the differences between the nominal and the measurement values. Although other variables, like the master gauge location (volumes Vol. 1 or Vol. 2) and orientation (X, Y, Z, XY, etc.) were taken into account, those variables were not found relevant or their relevance was negligible compared to Days and Length variables when the systematic error or standard deviation was computed.

Figure 3 shows a boxplot of the differences between the Actual and the Nominal values of all the measurements taken in this survey, ordered according to the Nominal range value. The Shapiro-Wilk normality test ($W = 0.8685$, P-Value = $1.708 \text{ E-}11$) clearly indicates that the differences between the Nominal and Actual values of all the measurements did not follow a normal distribution. This lack of normality led us to employ the Kruskal-Wallis test in order to check whether the medians of the deviations were the same for all the nominal Lengths or not. According to the results, the null hypothesis can be rejected ($p = 0.000$), so it can be stated that medians are different depending on the nominal values.

Table 2. The standard deviation of each one of the measurement deviations.



Please also note, that for the Nominal values equal to or greater than 59.988 mm (Fig. 3), as the Nominal value increases, the absolute value of the median of the differences between the Nominal and Actual values also increases (please note, that in our case the median values are negative). In order to check the equality of variances, the Levene test was applied. The results ($W = 684.01$, $p = 0.000$) provided evidence to claim that the measurements have unequal variances, depending on the Length nominal value. According to the results obtained it can be also stated that the larger the nominal value, the greater the standard deviation, with a significant Pearson’s correlation coefficient of 0.990 (Table 2).

For each calibrated length, the systematic error was calculated using the difference between the mean (average) value of all measured (Actual) values and the Nominal value.

3.1. Statistical model proposal

In order to check the influence of the time and length of the nominal value on the systematic error, a linear model using both variables and their interaction was implemented firstly. The systematic error was calculated using the difference between the mean (average) value of all measured (Actual) values and the Nominal value. The results are presented in Table 3 which shows the results of the linear model for the systematic error using Days, Nominal value and the interaction of Days and Nominal values as input variables. According to the obtained p-values, the term that represents the elapsed number of days since the last calibration is not significant ($p = 0.9609$), while the Nominal value of the measurements and the interaction of both Days and Nominal values are all of them significant, considering the statistical significance α level of 10% for the p-value. This adjusted R-squared value of the model is 0.7962.

Table 3. The first linear model of the systematic error behaviour.

| | Estimate | Std. error | p-value |
|--------------|------------------------|-----------------------|------------|
| Intercept | $-5.552 \cdot 10^{-4}$ | $3.92 \cdot 10^{-4}$ | 0.1584 |
| Days | $2.463 \cdot 10^{-7}$ | $5.016 \cdot 10^{-6}$ | 0.9609 |
| Nominal | $1.389 \cdot 10^{-5}$ | $1.321 \cdot 10^{-6}$ | 10^{-16} |
| Days:Nominal | $3.196 \cdot 10^{-8}$ | $1.691 \cdot 10^{-8}$ | 0.0603 |

According to the previous results, and in order to improve the performance of the model, it was repeated removing the Days variable, but maintaining the interaction between this variable and the measurement nominal values. Table 4 shows the results of this new linear model of the systematic error for given Nominal values and the interaction between the Nominal values of the measurements and the value of Days. The adjusted R-squared value of this model is 0.7973, and all the coefficients are significant.

Table 4. The second linear model of the systematic error behaviour.

| | Estimate | Std. error | p-value |
|---------------|------------------------|-----------------------|------------|
| Intercept | $-5.381 \cdot 10^{-4}$ | $1.803 \cdot 10^{-4}$ | 0.00324 |
| Nominal | $1.384 \cdot 10^{-5}$ | $8.979 \cdot 10^{-7}$ | 10^{-16} |
| Days: Nominal | $3.265 \cdot 10^{-8}$ | $9.530 \cdot 10^{-9}$ | 0.00076 |

3.2. Safe modelling of the real systematic error

Therefore, according to the results of Table 4, and considering the confidence interval of 95%, the most unfavourable linear model of the Systematic error would be:

$$\text{Systematic Error} = -1.823017 \cdot 10^{-4} + 1.561597 \cdot 10^{-5} \cdot L + 5.145359 \cdot 10^{-8} \cdot L \cdot D, \quad (1)$$

where L is the value of the length to be measured and D is the number of days since the last CMM adjustment and calibration.

The coefficients employed in the (1) are those values that guarantee that – according to the results of Table 4 – in 95% of cases the systematic error value will be lower than the value predicted by the model. So, and in order to achieve this aim, the upper limit of the confidence interval 2.5% – 97.5% of coefficients Intercept, Nominal and Nominal: Days was chosen for the (1). Fig. 4 shows a Box-Behnken design of the variables Length and Days versus Systematic Error combining all the empirical data, without any extrapolation.

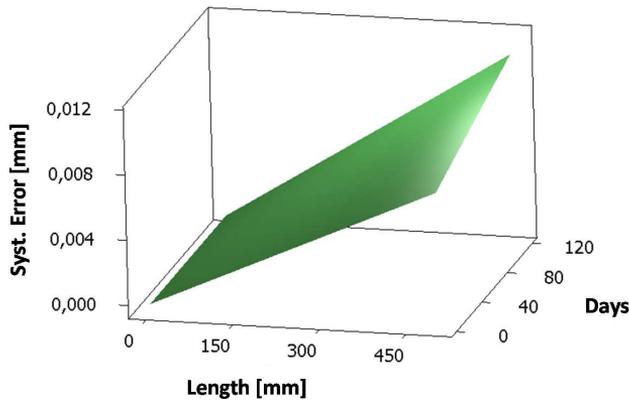


Fig. 4. A 3D surface plot of the variables Length and Days versus Systematic error for all the experimental values.

3.3. Safe modelling of the total error

In the case of the available CMM, and according to the results of the full calibration test performed before (when the present research began), the maximum permissible error (MPE_E), given by external calibration and adjustment (2), allows for expressing, in millimetres, the systematic error among other random errors.

$$MPE_E \leq 2.2 \cdot 10^{-3} + 3 \cdot 10^{-6} \cdot L. \quad (2)$$

In the worst possible case, the total error of the measurement could be considered as the sum of the aforementioned MPE_E plus the above systematic error model, which leads us to the following equation:

$$\text{Total Error} = 2.117698 \cdot 10^{-3} + 1.861597 \cdot 10^{-5} \cdot L + 5.145359 \cdot 10^{-8} \cdot L \cdot D. \quad (3)$$

The (3) expresses the Total Error of the CMM, *i.e.*, the maximum possible combination of MPE_E of measurement plus the equation that covers the systematic error. This equation adjusts statistically the total error of the measurement deviations according to the measurement nominal length and the elapsed time from the previous CMM adjustment. Fig. 5 presents the total error of measurements between 0 and 500 mm and for the period of one year since the last calibration. As it can be observed, the larger the measured length, the larger the total error and, for the same length, the total error increases as the “time since the previous calibration” increases.

In the model presented in Fig. 5, the points of the first six curves to the left correspond to the empirical data perfectly fitted by the implemented statistical model, whereas the right lines are derived from an extrapolation for the period of up to one year since the last calibration. The boundary curve between the green and blue points represents the limit between measurements with the total maximum error below 0.010 mm and measurements with the total maximum error above that value. It can be estimated that, at an instant immediately after the calibration date, errors of measurements performed for lengths of up to 420 mm do not exceed that value. However, after one year since the calibration date, only the measurements of lengths below 210 mm are affected for such low errors.

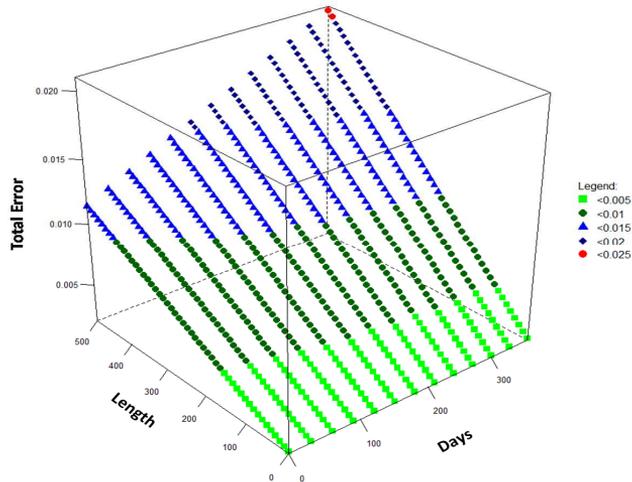


Fig. 5. A 3D Plot of the variables Length and Days versus the total error. The green area shows combinations of Days and Lengths corresponding to Total Errors below 0.010 mm.

Consequently, a model for the total error has been obtained in function of the elapsed time since the last calibration date and the nominal value of the dimension to be measured. Thus, as it can be seen in Fig. 6, for a specific nominal value of the dimension to be measured, the increase of the total error can be predicted taking into account the elapsed time since the last calibration. Furthermore, for a given elapsed time since the calibration date, the increase of the total error can be also predicted based on the nominal value of the dimension to be measured, and therefore the need of CMM adjustment and subsequent recalibration can be evaluated.

The model allows to adjust the period between calibrations and to decide if measurements are still valid after the period since the last calibration. For instance (Fig. 6), there is no need to calibrate the analysed CMM during the period of two years if the machine is limited to measure dimensions of up to 250 mm, the period being increased for dimensions of up to 200 mm, and so on. It is obvious that the extrapolation over time (beyond 2 or 3 years) would require repeating the measurements during a longer period, nevertheless this model is supposed to be a first step to describe the evolution of a CMM drift.

In any case, the model also allows to establish a “safety zone” (Fig. 6) for a dimensional tolerance defined over a work piece. If the maximum tolerance specification of the work pieces to be measured is set to 0.012 mm, the region under the red dotted curve in Fig. 6 indicates the working zone where the work pieces can be measured meeting the specified requirements.

The obtained data, both experimental and extrapolated ones, are referred to the real values measured with a CMM aimed at production inspection. The CMM is not a high precision machine dedicated to calibration tasks. As a matter of fact, the daily use of the CMM during the periods between tests consisted of diverse tasks, such as measurements performed in different volumes of the machine, on work pieces of different geometries and sizes, with different touch probes, stylus, *etc.* Nevertheless, although values of the total error around 0.004 mm in the dimension of 500 mm could be considered acceptable, these error values force to calibrate and adjust the machine almost every year.

As the final check, a few months after the end of the experiment, the full calibration was carried out by the same external laboratory (ENAC accreditation) that had performed the first calibration, finding that the machine was out of manufacturer specifications, providing the value of $MPE_E = 2.2 + 7.5 \cdot L/1000 \mu\text{m}$ (with L in mm). This calibration was performed using the same KOBAR[®] step Gauge Bar 1020 mm long in a single volume, like the first calibration performed at the beginning of this survey. The agreement between the value of the maximum permissible

error obtained by the last calibration test and the value predicted by the developed model was very high.

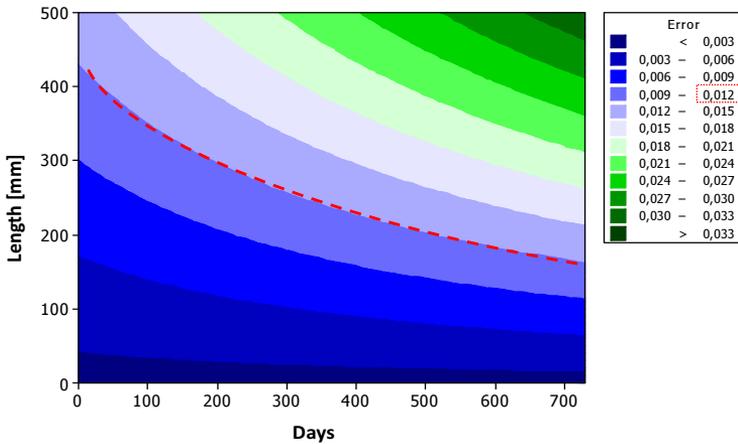


Fig. 6. A graphical 2D extrapolation for two years in measurement of distances over 500 mm. The red dashed line shows the security limit for the CMM total error < 0.012 mm.

4. Conclusions

This work has been focused on creating a statistical model that fits the measurement results corresponding to the measurement of the same artefact with the same CMM over an established period, allowing to find an equation that describes the evolution of the systematic error and the standard deviation of the measurement results over time. Several conclusions can be extracted, such as: the use of two volumes for our CMM re-verification has no significance in our study, the time and length variables have a greater influence. Therefore, they define and govern the model.

The model could also be used to estimate an appropriate value for the calibration or re-verification period to adapt this period to the measured item size and to the tolerance range to verify (ensuring the reliability of measurements for a specific length to be measured or for a given tolerance range).

It must be noted, that the presented model has been developed with the statistical analysis from an experimentation that guarantees repeatability and traceability of results. In this work the same test has been repeated under realistic conditions and controlling the magnitudes of influence. So, the variability is constrained to that corresponding to the normal working conditions of the CMM allowing to construct a model of behaviour of the CMM over time.

The extrapolation of this model allows to predict the evolution of the CMM performance, which saves both time and cost, when the specifications are not very exigent (tolerance range in the order of tenths of millimetre) or when the volume of the work piece to be measured does not cover the entire volume of the CMM or an important part of the CMM longer axis. On the other hand, if the behaviour of our CMM indicates that measurements are out of the conformance zone for a given significance level, the developed model can be used to establish how much the calibration/re-verification period must be reduced in order to ensure measurements.

It is evident that the period employed in performing this study is much shorter than that proposed for extrapolating the model results. These results will not be valid in the case of abrupt changes in the CMM performance tendency, which will be only detected by regularly monitoring the CMM by means of interim checks or re-verification tests. The CMM used for

developing this study is adjusted and calibrated each 2 years, so intermediate re-verification tests are performed regularly. The results of these tests provide an important feedback in order to improve the adjustment of the model and validate it.

Finally, the developed statistical model is intended to be applied to the analysis of drift or behaviour of Articulated Arm Coordinate Measuring Machines (AACMMs or CMAs). In this case, the study is even more justified due to two main reasons. On one hand, the behaviour of this type of equipment over time is not being currently studied and, on the other hand, it is evident that a correlation cannot be established with regard to the Cartesian linear axes, because their structure includes rotary encoders. In fact, in order to implement this model for AACMMs, it must be taken into account that the influence of the length on the measurement uncertainty is not as strong as it is in the case of CMMs. Possibly the model should consider such variables as the relative position between the CMM and gauge, the quadrant of the working space where the measurement is carried out, the fixture method type or the geometrical features being measured, as well as the influence of the operator that handles the AACMM.

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