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Influence of forecasting electricity prices in the optimization of complex hydrothermal systems

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ABSTRACT

This paper proposes a new method for addressing the short-term optimal operation of a generation company, fully adapted to represent the characteristics of the new competitive markets. We propose an efficient and highly accurate novel method for next-day price forecasting. We model the functional time series with a linear autoregressive functional model which formulates the relationships between each daily function of prices and the functions of previous days. For the optimization problem (formulated within the framework of nonsmooth analysis using Pontryagin's Maximum Principle), we propose a new method that uses diverse mathematical techniques (the Shooting Method, Euler's Method, the Cyclic Coordinate Descent Method). These techniques are well known for the case of functions, but are adapted here to the case of functionals and are efficiently combined to provide a novel contribution. Finally, the paper presents the results of applying our method to a price-taker company in the Spanish electricity market.

1. Introduction

Short-term hydrothermal scheduling (STHS) is known as one of the most important optimization problems of the power systems. STHS has long been an object of interest in the scientific community. Hundreds (and thousands) of papers have been published, and several techniques have been applied to solve this problem such as: Dynamic Programming, Lagrangian Relaxation Methods, Bender's Decomposition Methods, Heuristic Decomposition Methods, Genetic Algorithms, etc. We refer the readers to the following list of excellent books: [1–7], which include numerous references to the published works.

In traditional centralized markets the aim of STHS is to determine the optimal operation schedule of thermal units and hydro-plants which minimizes the total thermal production cost over a short-term period, taking into account the system-wide (coupling) and unit-wise (local) operating constraints. In the new competitive deregulated electricity market, the objective function of a company can be defined as one of maximizing profits over a period of up to one week.

This paper presents the new short-term problems that one firm faces when preparing its offers for the day-ahead market, taking into account the expected market clearing price. Since next-day price forecasting [8–10] is a crucial aspect, the paper proposes an efficient and highly accurate novel next-day price forecasting method. We model the series using a functional approach that considers hourly prices as observations of daily functions of prices. We model this functional time series with a linear autoregressive functional model which formulates the relationships between each daily function of prices and the functions of previous days.

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For the optimization problem, we propose a new method that employs diverse mathematical techniques which are well known for the case of functions. However, they are adapted here to the case of functionals and are efficiently combined to provide a novel contribution. Our technique makes maximum use of the special characteristics of the new competitive markets, leading to an algorithm that is very easily implemented besides being flexible to modeling. The algorithm is not affected by the size of the problem and is guaranteed to converge.

We shall set out our problem as an optimal control problem (OCP) in continuous time, a Lagrange-type functional, with non-regular Lagrangian and non-holonomic inequality constraints [11]. The problem shall be formulated within the framework of nonsmooth analysis [12,13] using Pontryagin's Maximum Principle (PMP). The associated variational problem will be related to solving a two-point boundary value problem (TPBVP). The construction of the solution can be performed with an adapted version of the shooting method [14,15], in combination with a discretized and adapted version of Euler's method. When the problem involves various hydro-plants, we have developed an algorithm of its numerical resolution prompted by the so-called method of cyclic coordinate descent (CCD) [16–18].

In the present paper, we generalize several previous studies in which we analyzed particular aspects of the technique. In [19], we presented the adapted CCD and in [20] we tested its convergence (under certain conditions). The non-regular nature of the Lagrangian of the OCP was analyzed in [21,22], presenting its application to the study of valve points in [23]. These studies focused on traditional, centralized markets. But the deregulated electricity market is where the technique is found to be even more advantageous. Three previous papers by the authors studied this problem under certain simplifications. In [24], our model of the spot market explicitly represents the price of electricity as a known exogenous variable. In [25], the volatility of the spot market price of electricity is represented by a *stochastic model* using *clustering techniques*. Finally, in [26], we propose a very efficient next-day price forecasting method based on a functional approach, but considering a hydrothermal system with only one hydro-plant. In the present paper, we shall show that the solution of hydrothermal systems with hydropredominance is significantly more complex.

The paper is organized as follows. In Section 2 we present the Hydrothermal Problem. In Section 3, we give some basic definitions and preparatory mathematical results which are necessary for our approach. Sections 4 and 5 present the optimal solution and the description of the algorithm. In Section 6, we expound the bases of the self-regressing functional models and present the results of applying the complete model in the Spanish electricity market. In Section 7, we discuss the results of numerical experiments. Finally, Section 8 summarizes the main conclusions of our research.

2. Statement of the hydrothermal problem

The optimization problem of one company is described in this section. Let us assume that our hydrothermal system accounts for n hydro-plants and m thermal plants: the $(H_n - T_m)$ Problem. Let us assume that the thermal subproblem has previously solved a standard unit commitment (UC) problem. We shall thus address the economic dispatch (ED) problem directly.

Let $\Psi_i(P_i(t)) : D_i \subseteq \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ $(i = 1, \ldots, m)$ be the quadratic cost functions of the m thermal plants

$$\Psi_i(P_i(t)) = \alpha_i + \beta_i P_i(t) + \gamma_i P_i^2(t) \tag{1}$$

where P_i is the power generated, and we consider technical restrictions of the type

$$P_i^{\min} \le P_i(t) \le P_i^{\max}; \quad \forall t \in [0, T]. \tag{2}$$

$[0, T]$ being the optimization interval. In prior studies [27,28], it was proven that the problem with m thermal plants may be reduced to the study of a hydrothermal system made up of one single thermal plant, called the*thermal equivalent*: the $(H_n - T_1)$ Problem. We shall denote as the thermal equivalent of $\{\Psi_i\}_1^m$, the function Ψ with $P(t)$ being the power generated by said thermal equivalent.

Besides, our system accounts for n hydro-plants that have pumping capacity. Let $H_i(t, z_i(t), \dot{z}_i(t)) : \Omega_{H_i} = [0, T] \times \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$ $(i = 1, \ldots, n)$ be the function of *effective hydraulic contribution* of the ith hydro-plant, and we consider technical restrictions of the type

$$H_i^{\min} \le H_i(t, z_i(t), \dot{z}_i(t)) \le H_i^{\max}; \quad \forall t \in [0, T]. \tag{3}$$

$z_i(t)$ being the volume that is discharged up to the instant t by the ith hydro-plant, and $\dot{z}_i(t)$ the rate of water discharge at the instant t by the ith hydro-plant. If we assume that b_i is the volume of water that must be discharged by the ith hydro-plant during the entire optimization interval $[0, T]$, the following boundary conditions will have to be fulfilled:

$$z_i(0) = 0, \qquad z_i(T) = b_i. \tag{4}$$

We shall denote $b = (b_1, \ldots, b_n)$, $z = (z_1, \ldots, z_n)$, $\dot{z} = (\dot{z}_1, \ldots, \dot{z}_n)$, and $H(t, z(t), \dot{z}(t)) = \sum_{i=1}^n H_i(t, z_i(t), \dot{z}_i(t))$ as the function of effective hydraulic contribution of the set of hydro-plants. From the viewpoint of a power generation company, and within the framework of the new deregulated electricity market, transmission losses are not relevant, and will not be considered.

In the $(H_n - T_1)$ problem, the objective function can be defined as the profit maximization during $[0, T]$

$$F(P, z) = \int_0^{\mathrm{T}} L(t, P(t), z(t), \dot{z}(t))\mathrm{d}t \tag{5}$$

$$= \int_0^{\mathrm{T}} [p(t)(P(t) + H(t)) - \Psi\,(P(t))]\,\mathrm{d}t. \tag{6}$$

Revenue is obtained by multiplying the total production of the company by the clearing price $p(t)$ in each hour t, and cost is given by Ψ, the cost function of the thermal equivalent. With this statement, our objective functional in continuous time form is

$$\begin{array}{ll} \max\limits_{P,z}: & \int_0^{\mathrm{T}} L(t, P(t), z(t), \dot{z}(t))\mathrm{d}t \\ \text{subject to:} & z_i(0) = 0, z_i(T) = b_i \\ & H_i^{\min} \le H_i(t, z_i(t), \dot{z}_i(t)) \le H_i^{\max} \\ & \forall i = 1, \ldots, n; \forall t \in [0, T] \end{array} \tag{7}$$

with $z \in \left(\widehat{C}^1[0, T]\right)^n$, where $\widehat{C}^1$ is the set of piecewise C^1 functions.

3. Mathematical formulation

As can be seen, this paper addresses a very complex problem of hydrothermal optimization with pumped-storage plants. In the next section we shall present this problem as an optimal control problem (OCP) with non-regular Lagrangian and non-holonomic inequality constraints (differential inclusions). In the present section, we introduce the basic mathematical tools that are necessary to obtain a necessary minimum condition.

A standard Lagrange-type OCP can be mathematically formulated as follows:

$$\max_{u(t)} \int_0^{\mathrm{T}} L\,(t, x(t), u(t))\,\mathrm{d}t \tag{8}$$

subject to satisfying:

$$\dot{x}(t) = f\,(t, x(t), u(t)) \tag{9}$$

$$x(0) = x_0; \qquad x(T) = x_T \tag{10}$$

$$u(t) \in U(t), \quad 0 \le t \le T \tag{11}$$

where L is an objective function, $x = (x_1(t), \ldots, x_n(t)) \in \mathbb{R}^n$ is the state vector, with initial conditions x_0 and final conditions x_T, $u = (u_1(t), \ldots, u_m(t)) \in \mathbb{R}^m$ is the control vector, U denotes the set of admissible control values, and t is the operation time that starts from 0 and ends at T. The state variables (or simply the states) must satisfy the state equation (9) with given conditions (10). Let $\mathbb{H}$ be the Hamiltonian function associated with the problem

$$\mathbb{H}(t, x, u, \lambda) = L\,(t, x, u) + \lambda \cdot f\,(t, x, u) \tag{12}$$

where $\lambda = (\lambda_1(t), \ldots, \lambda_n(t)) \in \mathbb{R}^n$ is called the costate vector. The classical approach involves the use of Pontryagin's Minimum Principle (PMP) [11], which results in a two-point boundary value problem (TPBVP). In order for $u \in U$ to be optimal, a nontrivial function λ must necessarily exist, such that for almost every $t \in [0, T]$

$$\dot{x} = \mathbb{H}_\lambda = f; \qquad x(0) = x_0; \qquad x(T) = x_T \tag{13}$$

$$\dot{\lambda} = -\mathbb{H}_x \tag{14}$$

$$\mathbb{H}(t, x, u, \lambda) = \max_{v(t)\in U} \mathbb{H}(t, x, v, \lambda). \tag{15}$$

We now introduce some preliminary ideas of a generalized theory of differentiation: Nonsmooth Analysis [12,13]. Nonsmooth Analysis works with locally Lipschitz functions f that are differentiable almost everywhere (the set of points at which f fails to be differentiable is denoted by Ω_f). The generalized (or Clarke's) gradient ∂f can be calculated as a convex hull of (almost) all converging sequences of the gradients

$$\partial f(x) = \mathrm{co}\left\{\lim \nabla f(x_i) : x_i \longrightarrow x, x_i \notin \Omega_f\right\}. \tag{16}$$

At the points of smoothness of $f(x)$, it is essential for the generalized gradient to coincide with its gradient and, for a convex function, with its subgradient. The nonsmooth version of PMP for the above OCP states that:

$$\dot{x} = \mathbb{H}_\lambda = f; \qquad x(0) = x_0; \qquad x(T) = x_T \tag{17}$$

$$-\dot{\lambda} =\in \partial_x\mathbb{H} \tag{18}$$

$$\mathbb{H}(t, x, u, \lambda) = \max_{v(t)\in U} \mathbb{H}(t, x, v, \lambda). \tag{19}$$

4. Optimal solution

Section 2 presented the problem from the Electrical Engineering point of view; we now proceed to resolve the mathematical problem formulated. We begin the development in this section by presenting the simple $(H_1 - T_1)$ Problem, with one pumped-storage hydro-plant. In this case, we have $z = z_1$.

When pumped-storage plants are considered, the derivative of H with respect to $\dot{z}(H_{\dot{z}})$ presents discontinuity at $\dot{z} = 0$, which is the border between the power generation zone ($\dot{z} \geq 0$) and the pumping zone ($\dot{z} \leq 0$). According to the definition (16), $\partial_{\dot{z}} H(t, z(t), \dot{z}(t)) = [H_{\dot{z}}^+, H_{\dot{z}}^-]$ *a.e.* is fulfilled. We shall assume that Ψ is strictly increasing and strictly convex, that H verifies $H_{\dot{z}} > 0$ and $H_z(t, z(t), 0) = 0$, as well as the strictly increasing nature of $L_{\dot{z}}(t, z, \cdot)$. The real models meet these assumptions.

In several previous papers (see [21–26] for its complete development), this problem was formulated as an optimal control problem (OCP). We considered the state variables to be $z(t)$ and $P(t)$ and the control variables $u_1(t) = H\,(t, z(t), \dot{z}(t))$ and $u_2(t) = \dot{P}(t)$. The OCP

$$\begin{aligned} & \max_{u_1(t), u_2(t)} \int_0^{\mathrm{T}} L(t, P(t), u_1(t))\mathrm{d}t \\ \text{s. t.} \quad & \dot{z} = f(t, z, u_1); \qquad \dot{P} = u_2 \\ & z(0) = 0; z(T) = b \\ & u_1(t) \in \{x | H^{\min} \leq x \leq H^{\max}\} \end{aligned} \tag{20}$$

was formulated within the framework of nonsmooth analysis, using PMP, the following result being proven:

Theorem of Coordination. *If (z^*, P^*) is a solution of our problem, then $\exists K \in \mathbb{R}^+$ such that:*

$$\begin{aligned} &\text{(i)} \quad \textit{If } \dot{z}^*(t) = 0 \Longrightarrow \yen_{z^*}^+(t) \leq K \leq \yen_{z^*}^-(t) \\ &\text{(ii)} \quad \textit{If } \dot{z}^*(t) \neq 0 \Longrightarrow \yen_{z^*}(t) \textit{ is } \begin{cases} \geq K & \textit{if } H(t) = H^{\min} \\ = K & \textit{if } H^{\min} < H(t) < H^{\max} \\ \leq K & \textit{if } H(t) = H^{\max} \end{cases} \\ &\text{(iii)} \quad \dot{\Psi}\left(P^*(t)\right) = p(t). \end{aligned} \tag{21}$$

The function

$$\yen_z(t) = L_{\dot{z}}(t, P(t), z(t), \dot{z}(t)) \cdot \exp\left[-\int_0^t \frac{H_z(s)}{H_{\dot{z}}(s)}\mathrm{d}s\right] \tag{22}$$

was called the *coordination function* of z, and we denoted by $\yen_z^+(t)$ and $\yen_z^-(t)$ the expressions obtained when considering the lateral derivatives with respect to $\dot{z}$.

It is very important to stress that, with our technique (and without the need to use other decomposition techniques), the problem is easily broken down into the two subproblems: Thermal and Hydro.

– To calculate the optimum power $P^*(t)$ of the thermal equivalent plant in the Thermal subproblem, we solve Equation (iii) in (21). $P^*(t)$ is distributed, (as we see in [27,28]), between the m thermal plants, and hence is completely resolved.

– To obtain the optimum operating conditions of the hydro-plant in the Hydro-subproblem, we shall use the *coordination equation*

$$\yen_z(t) = K, \quad \forall t \in [0, T]. \tag{23}$$

The problem consists in finding for each K the function z_K that satisfies $z_K(0) = 0$ and the conditions of the Theorem of Coordination, and from among these functions, the one that gives rise to an admissible function ($z_K(T) = b$). From the computational point of view, the construction of z_K can be performed using the same procedure as in the shooting method, with the use of a discretized version of the coordination equation (23) (adapted Euler's method). The exception is that at the instant when the values obtained for z and $\dot{z}$ do not obey the constraints, we force the solution z_K to belong to the boundary until the moment when the conditions of leaving the domain (established in the Theorem of Coordination) are fulfilled.

For the adapted simple shooting method, we consider the function $\varphi(K) := z_K(T)$ and calculate the root of $\varphi(K) - b = 0$, which may be done approximately using elemental procedures. The secant method was used in previous papers, and the algorithm converges rapidly for a wide range of $K_{\min}$ and $K_{\max}$.

5. The optimization algorithm

In this section, we study the general case in which the system consists of n hydro-plants: the $(H_n - T_1)$ Problem. In several previous papers (see [19,20]), the authors developed a very efficient algorithm for solving the problem numerically. The algorithm uses a strategy inspired by the Cyclic Coordinate Descent (CCD) method. The classic CCD method minimizes a function of n variables cyclically with respect to the coordinate variables. With our method, the $H_n - T_1$ problem could be

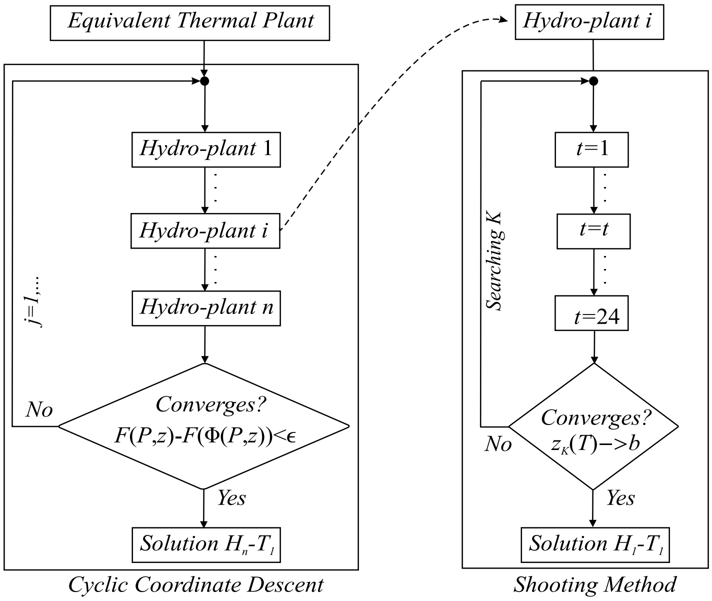

Fig. 1. The optimization algorithm.

solved like a sequence of problems of the type $H_1 - T_1$. At each *j-th* iteration of the algorithm for the $H_n - T_1$ problem, we calculate n stages, one for each hydro-plant; and at each stage, we calculate the optimal functioning of a hydro-plant, while the behavior of the rest of the plants is assumed fixed (see Fig. 1).

For every $z = (z_1, \ldots, z_n)$, we consider the functional $F_z^i(P, v_i)$ defined by

$$\int_0^{\mathrm{T}} \left[p(t)(P(t) + H_z^i(t, v_i(t), \dot{v}_i(t))) - \Psi\,(P(t))\right] \mathrm{d}t \tag{24}$$

where $H_z^i(t, v_i, \dot{v}_i)$ is

$$H(t, z_1, \ldots, z_{i-1}, v_i, z_{i+1}, \ldots, \dot{z}_{i-1}, \dot{v}_i, \dot{z}_{i+1}, \ldots, \dot{z}_n) \tag{25}$$

and represents the power generated by the hydrosystem as a function of the ith plant, while the behavior of the rest of the plants is assumed to be fixed. We call the ith *minimizing mapping* the mapping ϕ_i, defined as:

$$\phi_i(P, z_1, \ldots, z_i, \ldots, z_n) = (P^*, z_1, \ldots, z_i^*, \ldots, z_n) \tag{26}$$

where (P^*, z_i^*) minimizes F_z^i. If we set

$$\Phi = (\phi_n \circ \phi_{n-1} \circ \cdots \circ \phi_2 \circ \phi_1) \tag{27}$$

$$(P^j, z^j) = \Phi(P^{j-1}, z^{j-1}) \tag{28}$$

beginning with some admissible (P^0, z^0), we construct a sequence of (P^j, z^j) via successive applications of $\{\phi_i\}_{i=1}^n$ and the algorithm will search

$$\lim_{j\to\infty} (P^j, z^j). \tag{29}$$

The convergence (under certain conditions) of this algorithm has been studied in [20].

It is most important to note that in the particular case of hydraulic coupling not existing between the n hydro-plants, no more than one iteration is required, since the K optimum are obtained directly as a result of the fact that the behavior of one hydro-plant does not affect the rest. This is proof of the power of the method in addressing large-sized systems without coupling.

6. Next-day price forecasting using functional linear models

The formulation of the Hydrothermal Problem and the expression of its solution depend on the hourly clearing price of electricity. Logically, however, this price is unknown beforehand and hence it is crucial to have the best forecasts at one's disposal. A fundamental characteristic of hourly clearing price series is their daily seasonality. Later, we shall see that this daily pattern differs for each day of the week, although in all cases it presents a very strong structure that cannot be ignored by any forecasting model.

This fact suggested to the authors of the present paper that functional techniques might be ideal for price forecasting insofar as they could model these highly stable daily structures by means of the specification of an appropriate functional space for their time series.

On the other hand, to the best of our knowledge, no studies existed with real data on the forecasting capacity of functional techniques compared to classical scalar techniques or hence any indications as regards under which conditions certain techniques might be more favorable than others.

6.1. Functional linear models

A functional linear regression model [29] is an extension of the multivariate linear regression model to the case of infinite-dimensional or functional data:

$$E(Y(t)|\mathbf{X}) = \alpha(t) + \langle \mathbf{X}, \boldsymbol{\beta} \rangle = \alpha(t) + \sum_{j=1}^{m} \int X_j(s)\beta_j(s,t)\mathrm{d}s \tag{30}$$

where $\mathbf{X} = (X_1, \dots, X_m)$ are the functional predictors, Y is the response function, $\boldsymbol{\beta}$ is a vector of parameter functions and α is the mean response function.

Note that in the above model, the value $E(Y(t)|\mathbf{X})$ depends on the complete profile of each covariate functional X_j; i.e. the values $X_j(s)$ of these functions at all the points $s \in \mathcal{T}$ influence the value of this expectation. This general model includes the following particular pointwise case:

$$E(Y(t)|\mathbf{X}) = \alpha(t) + \left\langle \mathbf{X}, \boldsymbol{\beta}' \right\rangle = \alpha(t) + \sum_{j=1}^{m} X_j(t)\beta_j'(t) \tag{31}$$

in which, in contrast with the previous model (30), the value $E(Y(t)|\mathbf{X})$ does not depend on the values of the covariables at other points $s \in \mathcal{T}$ and, accordingly, does not depend on the functions X_j each considered as a whole, but rather on their values at each forecasting point t. Actually, the model (31) is a traditional multivariate model at each point t.

In the functional model, we do not see the sample functions f_i, $i = 1, \dots, n$ in the majority of applications, but only their values $f_i(t_j)$ at a set of n_p points $t_j \in \mathbb{R}$, $j = 1, \dots, n_p$. For the sake of simplicity, we shall assume that these are common to all the functions f_i, $i = 1, \dots, n$. These observations may, moreover, be subject to noise; in which case they take the form: $g_{ij} = f_i(t_j) + \varepsilon_{ij}$, where we assume that ε_{ij} is random noise with zero mean, $i = 1, \dots, n$, $j = 1, \dots, n_p$.

Therefore, the functional approach first requires estimating each sample function $f_i \in \mathcal{F}$, $i = 1, \dots, n$. One way to do so is to assume that $\mathcal{F} = \mathrm{span}\{\phi_1, \dots, \phi_{n_b}\}$ with $\{\phi_k\}$ sets of basic functions [29]. For our research, we chose a family of B-splines as the set of basic functions, given their good local behavior. If, for the sake of simplicity, we represent as f any of the functions f_i, $i = 1, \dots, n$ in the sample, we have:

$$f(t) = \sum_{k=1}^{n_b} c_k \phi_k(t). \tag{32}$$

Hence, the smoothing problem consists in determining the solution f to the following regularization problem:

$$\min_{x \in \mathcal{F}} \sum_{j=1}^{n_p} \left\{ g_j - f\left(t_j\right) \right\}^2 + \lambda \Gamma(f) \tag{33}$$

where $g_j = f(t_j) + \varepsilon_j$ is the result of observing f at point t_j, Γ is an operator that penalizes the complexity of the solution, and λ is a regularization parameter that regulates the intensity of this penalization. In our case, we used the operator $\Gamma(f) = \int_{\mathcal{T}} \left\{D^2 f(t)\right\}^2 \mathrm{d}t$, where $\mathcal{T} = [t_{\min}, t_{\max}]$ and D^2 is the second-order differential operator. Bearing in mind the expansion (32), the above problem (33) may be written as:

$$\min_{\mathbf{c}} \left\{ (\mathbf{g} - \boldsymbol{\Phi}\mathbf{c})^{\mathrm{T}}(\mathbf{g} - \boldsymbol{\Phi}\mathbf{c}) + \lambda \mathbf{c}^{\mathrm{T}}\mathbf{R}\mathbf{c} \right\} \tag{34}$$

where $\mathbf{g} = (g_1, \dots, g_{n_p})^{\mathrm{T}}$, $\mathbf{c} = (c_1, \dots, c_{n_b})^{\mathrm{T}}$, $\boldsymbol{\Phi}$ is the $n_p \times n_b$ matrix with elements $\boldsymbol{\Phi}_{jk} = \phi_k(t_j)$ and $\mathbf{R}$ is the $n_b \times n_b$ matrix with elements $R_{kl} = \left\langle D^2\phi_k, D^2\phi_l \right\rangle_{L_2(\mathcal{T})} = \int_{\mathcal{T}} D^2\phi_k(t) D^2\phi_l(t)\mathrm{d}t$.

The solution to this problem is given by $\hat{\mathbf{c}} = (\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{\Phi} + \lambda\mathbf{R})^{-1}\boldsymbol{\Phi}^{\mathrm{T}}\mathbf{g}$, such that the estimated values of the true function f at the observation points are obtained by means of $\hat{\mathbf{f}} = \mathbf{S}\mathbf{g}$, where $\mathbf{S} = \boldsymbol{\Phi}(\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{\Phi} + \lambda\mathbf{R})^{-1}\boldsymbol{\Phi}^{\mathrm{T}}$ and $\hat{\mathbf{f}} = (\hat{f}(t_1), \dots, \hat{f}\left(t_{n_p}\right))^{\mathrm{T}}$.

The selection of λ forms part of the model selection problem and is usually performed using cross validation.

When faced with a functional linear model such as that of Eq. (30), the aforementioned smoothing process must be applied both to the regression functions and the functional response. We shall thus assume two functional spaces expanded by two sets of basis functions: $\mathcal{F}_X = \text{span}\{\phi_1, \ldots, \phi_{n_X}\}$ and $\mathcal{F}_Y = \text{span}\{\eta_1, \ldots, \eta_{n_Y}\}$ such that the example functions admit the following types of expansions (32) in terms of the respective bases:

$$x_{ji}(t) = \mathbf{c}_{ji}^{\mathrm{T}} \boldsymbol{\phi}(t), \quad i = 1, \ldots, n, j = 1, \ldots, m \tag{35}$$

$$y_i(t) = \mathbf{d}_i^{\mathrm{T}} \boldsymbol{\eta}(t), \quad i = 1, \ldots, n \tag{36}$$

where $\boldsymbol{\phi}(t) = (\phi_1(t), \ldots, \phi_{n_X}(t))^{\mathrm{T}}$ and $\boldsymbol{\eta}(t) = (\eta_1(t), \ldots, \eta_{n_Y}(t))^{\mathrm{T}}$.

For ease of exposition, we shall henceforth assume a single covariate functional and that the example functions (x_i, y_i), $i = 1, \ldots, n$ have been centered with the aim of avoiding the independent term $\alpha(t)$:

$$E(Y(t)|X) = \int X(s)\beta(s, t)\mathrm{d}s. \tag{37}$$

Within this framework, we assume that the above smoothing process on the example functions produces the expansions $x_i(t) = \mathbf{c}_i^{\mathrm{T}} \boldsymbol{\phi}(t)$, $y_i(t) = \mathbf{d}_i^{\mathrm{T}} \boldsymbol{\eta}(t)$, $i = 1, \ldots, n$, which may be jointly written by means of:

$$\mathbf{x}(t) = \mathbf{C}\boldsymbol{\phi}(t) \tag{38}$$

$$\mathbf{y}(t) = \mathbf{D}\boldsymbol{\eta}(t) \tag{39}$$

where the matrices $\mathbf{C}$ and $\mathbf{D}$ are of dimension $n \times n_X$ and $n \times n_Y$ and their rows are the vectors $\mathbf{c}_i^{\mathrm{T}}$ and $\mathbf{d}_i^{\mathrm{T}}$, $i = 1, \ldots, n$, respectively.

We likewise assume that the coefficients function β admits an expansion in terms of the aforementioned bases:

$$\beta(s, t) = \sum_{k=1}^{n_X} \sum_{l=1}^{n_Y} b_{kl}\phi_k(s)\eta_l(t) = \boldsymbol{\phi}(s)^{\mathrm{T}} \mathbf{B} \boldsymbol{\eta}(t) \tag{40}$$

where $\mathbf{B}$ is an $n_X \times n_Y$ matrix of coefficient b_{kl}. This may be written in functional form as:

$$\beta = \boldsymbol{\phi}^{\mathrm{T}} \mathbf{B} \boldsymbol{\eta}. \tag{41}$$

From all the above, for each observation $i = 1, \ldots, n$, the model (37) becomes:

$$\hat{y}_i(t) = \int X(s)\beta(s, t)\mathrm{d}s = \mathbf{c}_i^{\mathrm{T}} \mathbf{J}_{\boldsymbol{\phi}} \mathbf{B} \boldsymbol{\eta}(t) \tag{42}$$

or equivalently, in compact functional form: $\hat{y}_i = \mathbf{c}_i^{\mathrm{T}} \mathbf{J}_\phi \mathbf{B} \boldsymbol{\eta}$, where $\mathbf{J}_\phi = \int \boldsymbol{\phi}(s)\boldsymbol{\phi}(s)^{\mathrm{T}} \mathrm{d}s$ is an $n_X \times n_X$ matrix with $(\mathbf{J}_\phi)_{kl} = \langle \phi_k, \phi_l \rangle = \int \phi_k(s)\phi_l(s)\mathrm{d}s$ elements.

Applying the above to all the example observations, we obtain the following functional-type vectorial expression:

$$\hat{\mathbf{y}} = \mathbf{C}\mathbf{J}_\phi \mathbf{B}\boldsymbol{\eta} = \hat{\mathbf{D}}\boldsymbol{\eta} \tag{43}$$

where $\hat{\mathbf{D}} = \mathbf{C}\mathbf{J}_\phi B$ is an $n \times n_Y$ matrix.

The fit is carried out by means of regularized minimum squared errors:

$$\min \left\{ \sum_{i=1}^{n} \left\| y_i - \hat{y}_i \right\|^2_{L_2(\mathcal{T})} + \lambda_s \Gamma_s(\beta) + \lambda_t \Gamma_t(\beta) \right\} \tag{44}$$

where Γ_s, Γ_t are regularized operators that penalize the complexity of β as a function of s and as a function of t, respectively. As regards s:

$$\Gamma_s(\beta) = \int \int [L_s\beta(s, t)]^2 \, \mathrm{d}s\mathrm{d}t = \text{trace}\{\mathbf{B}^{\mathrm{T}} \mathbf{R}_s \mathbf{B} \mathbf{J}_\eta\} \tag{45}$$

where $\mathbf{J}_\eta = \int \boldsymbol{\eta}(t)\boldsymbol{\eta}(t)^{\mathrm{T}} \mathrm{d}t$ is an $n_Y \times n_Y$ matrix and R_s an $n_X \times n_X$ matrix with $(\mathbf{R}_s)_{k,l} = \langle L_s\phi_k, L_s\phi_l \rangle_{L_2(\mathcal{T})}$. Analogously, with respect to t, we have that:

$$\Gamma_t(\beta) = \text{trace}\{\mathbf{B}^{\mathrm{T}} \mathbf{J}_\phi \mathbf{R}_t \mathbf{B}\} \tag{46}$$

where $\mathbf{R}_t$ is an $n_Y \times n_Y$ matrix with $(\mathbf{R}_t)_{k,l} = \langle L_t\eta_k, L_t\phi\eta_l \rangle_{L_2(\mathcal{T})}$.

Bearing in mind that the squared error for the ith example observation is:

$$\left\| y_i - \hat{y}_i \right\|^2 = \left\langle y_i - \hat{y}_i, y_i - \hat{y}_i \right\rangle = \left\{ (\mathbf{D} - \hat{\mathbf{D}})\mathbf{J}_\eta (\mathbf{D} - \hat{\mathbf{D}})^{\mathrm{T}} \right\}_{ii} \tag{47}$$

$$= \left\{ (\mathbf{D} - \mathbf{C}\mathbf{J}_\phi \mathbf{B})\mathbf{J}_\eta (\mathbf{D} - \mathbf{C}\mathbf{J}_\phi \mathbf{B})^{\mathrm{T}} \right\}_{ii} \tag{48}$$

the resulting fitting criterion is:

$$\min_{\mathbf{B}} \begin{Bmatrix} \text{trace}\left\{(\mathbf{D}-\mathbf{C}\mathbf{J}_\phi\mathbf{B})\mathbf{J}_\eta(\mathbf{D}-\mathbf{C}\mathbf{J}_\phi\mathbf{B})^{\mathrm{T}}\right\} \\ +\lambda_s\,\text{trace}\{\mathbf{B}^{\mathrm{T}}\mathbf{R}_s\mathbf{B}\mathbf{J}_\eta\} + \lambda_t\,\text{trace}\{\mathbf{B}^{\mathrm{T}}\mathbf{J}_\phi\mathbf{R}_t\mathbf{B}\} \end{Bmatrix} \tag{49}$$

from which, deriving with respect to **B**, we obtain the normal equations for this matrix:

$$\mathbf{J}_\phi\mathbf{C}^{\mathrm{T}}\mathbf{C}\mathbf{J}_\phi\mathbf{B}\mathbf{J}_\eta + \lambda_s\mathbf{R}_s\mathbf{B}\mathbf{J}_\eta + \lambda_t\mathbf{J}_\phi\mathbf{B}\mathbf{R}_t = \mathbf{J}_\phi\mathbf{C}^{\mathrm{T}}\mathbf{D}\mathbf{J}_\eta. \tag{50}$$

6.2. Application of functional linear models for next-day price forecasting

Given a functional time series, i.e. a series of regular functions $\{Y_\tau\}$ with $Y_\tau : \mathcal{T} \to \mathbb{R}$, $\tau \in \mathbb{N}$, the above functional framework can be used to formulate an autoregressive functional linear model for the conditional functional mean $E(Y_\tau(t)|\mathbf{X}_\tau)$ at time τ given the vector of observed functions $\mathbf{X}_\tau = (Y_{\tau-1}, \dots, Y_{\tau-m})$.

Electricity prices show a strong daily seasonality structure that can be modeled through daily functions defined on the interval [0, 24] h. Consequently, the series of prices can be seen as a sequence of daily functions $\{Y_\tau\}$, where $Y_\tau : [0, 24] \to \mathbb{R}$ is the function of day τ such that prices observed on that day are observations $Y_\tau(t)$, $t \in [0, 24]$ of the function Y_τ.

However, this series also shows strong weekly seasonality. With the aim of also taking this weekly structure into account, we finally postulate a mixed model that combines this functional autoregressive daily model with a functional ANOVA model that incorporates the specific effect of each week day. For the case of the linear model, the resulting mixed model has the following general form:

$$\hat{Y}_\tau(t) = \alpha_0(t) + \mathbf{a}^{\mathrm{T}}\boldsymbol{\alpha}(t) + \sum_{j=1}^{m}\int Y_{\tau-j}(s)\beta(s,t)\mathrm{d}s \tag{51}$$

where $\mathbf{a} = (a_1, \dots, a_7)^{\mathrm{T}}$ with $a_k = 1$ if the day is the kth day of the week and $d_l = 0$ for $l \neq k$, and $\boldsymbol{\alpha}(t) = (\alpha_1(t), \dots, \alpha_7(t))^{\mathrm{T}}$ with $\alpha_k(t)$ being the specific mean effect of each day of the week, and where $s, t \in [0, 24]$.

In the above model, the complete profile of the price function on day $\tau - j$, $Y_{\tau-j}$ influences the price forecast of moment t on day τ, $Y_\tau(t)$. A particular case is the pointwise model (Eq. (31)):

$$\hat{Y}_\tau(t) = \alpha_0(t) + \mathbf{a}^{\mathrm{T}}\boldsymbol{\alpha}(t) + \sum_{j=1}^{m} Y_{\tau-j}(t)\beta(t) \tag{52}$$

in which the price forecast at the moment t only depends on the price at the same moment t of preceding days (apart from the mean overall effect (α_0) and the effect α_k of the specific week day to which day τ belongs).

With the aim of assessing the advantages of functional linear models (FLMs) in forecasting daily electricity prices within the context of the Hydrothermal Problem, we used these models to predict said price on the Spanish market [30] during the period from 01/04/1998 to 31/07/2007 (3409 days with 24 hourly items per day), comparing their results with those of other reference techniques.

For the sake of comparison, the period under consideration was divided up into two subperiods: one to train the different models, from 01/04/1998 to 31/03/2005, comprising 2550 days; and another as the test set to determine the contributions and utility of the different techniques under consideration, comprising 859 days.

The following models were considered in the comparison:

1. A basic functional ANOVA model of the following form:

$$\hat{Y}_\tau(t) = \alpha_0(t) + \mathbf{a}^{\mathrm{T}}\boldsymbol{\alpha}(t) \tag{53}$$

with which a grand mean effect α_0 and a mean effect α_k are measured for each week day $k = 1, 2, \dots, 7$ in the daily price function.

2. A functional ANOVA like the prior one, though also incorporating a time component:

$$\hat{Y}_\tau(t) = \alpha_0(t) + \mathbf{a}^{\mathrm{T}}\boldsymbol{\alpha}(t) + \rho\cdot\tau. \tag{54}$$

3. AR$(24 \times m)$ + GARCH(1, 1) scalar models, where m is the number of lagged days that influence each hourly price, i.e. if $\{Z(r); r \in \mathbb{N}\}$ is the hourly prices series (considered as a series of scalars):

$$\hat{Z}(r) = \gamma_0 + \sum_{j=1}^{m\times 24}\gamma_j\hat{Z}(r-j) + \varepsilon_r \tag{55}$$

$$\varepsilon_r = \sqrt{h_r}\vartheta_r \tag{56}$$

Table 1
Average squared errors of the different models.

Model	ASEtrain	ASEtest
ANOVA	1.1960	6.2300
ANOVA + $\rho \cdot$ t	1.1302	4.4063
AR(24) + GARCH(1, 1)	0.6947	1.4414
AR(24 × 7) + GARCH(1, 1)	0.4478	0.8990
ANOVA + FP − AR(1)	0.4104	1.0098
ANOVA + FP − AR(7)	0.3407	0.7537
ANOVA + FF − AR(1)	0.3627	0.8488
ANOVA + FF − AR(7)	0.3293	0.7373

with:

$$h_r = \kappa_0 + \kappa_1 h_{r-1} + \theta_1 \varepsilon_{r-1}^2 \tag{57}$$

where: $\{\varepsilon_r; r \in \mathbb{N}\}$ is a process with $\mathbb{E}_{\varepsilon_r|\mathcal{A}_{r-1}}(\varepsilon_r) = 0$ and $\mathrm{Var}(\varepsilon_r) = \sigma_\varepsilon^2$, $\{\vartheta_r, r \in \mathbb{N}\}$ is a process with independent zero mean variables and $\mathrm{Var}(\vartheta_r) = 1$ for all $r \in \mathbb{N}$ and which satisfy suitable restrictions on the parameters $\kappa_0, \kappa_1, \theta_1$ so that $\mathrm{Var}_{\varepsilon_r|\mathcal{A}_{r-1}}(\varepsilon_r) = \mathbb{E}_{\varepsilon_r|\mathcal{A}_{r-1}}(\varepsilon_r^2) > 0$ and the stationarity hypothesis for $\{\varepsilon_r^2\}$ are satisfied.

4. Pointwise autoregressive functional linear models according to Eq. (52).
5. Fully functional autoregressive linear models according to Eq. (51).

In the case of the functional models, the choice of the normalizing parameters (λ in the smoothing model and λ_s, λ_t in the estimation of the functional coefficients β) was carried out by crossed validation across a values grid.

The forecasting capacity of the functional models was determined by calculating the average squared error of the forecast in the period under consideration (training or test):

$$\mathrm{ASE} = \frac{1}{n-q} \sum_{r=q}^{n} (y_r - \hat{y}_r)^2 \tag{58}$$

where $y_r, \hat{y}_r$ are respectively the hourly price and its forecast for each model in the rth hour, commencing the count at the first hour q of the period in which this may be done, depending on the autoregressive nature of the model. For the training period, $n = 2550 \times 24$ and q is the greatest number of hourly lags for each model. If the model is functional in nature with m daily lags, then $q = 24 \times m$ hourly lags. For the test period, $n = 859 \times 24$ and $q = 1$ always, since the models were only trained with the training period.

It should be borne in mind that, on account of their nature, functional models forecast the hourly price at daily intervals, i.e. they forecast the price function of the following day using, at the most, the information from the previous day. They thus forecast the prices corresponding to the following 24 h all at once. Scalar models, on the other hand, due to their hourly nature, forecast the following hour on the basis of the preceding ones.

To carry out an equitative comparison of both types of models, they must all have the same information at their disposal when forecasting each hourly price. Hence, in order to enable comparison between functional and scalar models, the hourly forecasts of the latter were always carried out 24 h ahead of the last hour of the previous day, just as functional models do. However, their estimation process was naturally hourly in character, using the series $\{Z(r); r \in \mathbb{N}\}$ of hourly data (not considering any daily structure).

Table 1 shows the ASE in the training and test periods of different models under comparison. The ANOVA+FP-AR models are autoregressive pointwise functional ANOVA models, whereas the ANOVA+FF-AR are autoregressive full functional ANOVA models. The results of the training period refer to the period from the 22nd to the 2550th day so as to allow comparison of all the models regardless of the different number of lags used for each one. For the autoregressive model (scalar or functional), the results shown are for $m = 1$ and 7 daily lags ($\times 24$ hourly lags for the scalar models).

From this table, it is evident that the functional models produce better results than the other models, thereby indicating that the functional approach better captures and exploits the daily structure of electricity prices. To illustrate this, Fig. 2 shows this structure for each day of the week obtained by the ANOVA model without any autoregressive structure. The plots for Tuesday, Wednesday and Thursday are virtually indistinguishable.

7. Example

A program that solves the optimization problem was written using the Mathematica package and then applied to a real system: the hydrothermal system that the electricity company *HC* [31] has in Asturias (Spain). This system is made up of 4 classic thermal plants: *Aboño 1*, *Aboño 2*, *Soto 1* and *Soto 2* and 3 variable-head hydro-plants: *Salime*, *Tanes* (pumped-storage) and *La Barca*. Let us see the models of different subsystems used in our study.

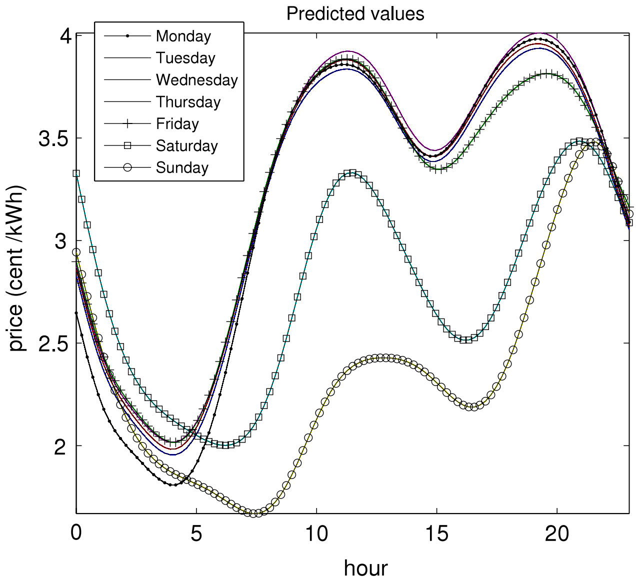

Fig. 2. Plots of the daily price function on each day of the week.

Table 2
Coefficients of the thermal plants.

Plant	1 Aboño 1	2 Aboño 2	3 Soto 1	4 Soto 2	Eq.
α_i	2762.6	1673.5	174.9	3634.5	5594.4
β_i	39.647	46.894	47.873	37.521	28.380
γ_i	0.0298	0.0025	0.0064	0.0373	0.0028
$P_i^{\min}$	50	50	50	50	200
$P_i^{\max}$	360	543	254	350	1507

7.1. Thermal model

For the thermal cost functions, we use the classic (see [1]) quadratic model

$$\Psi_i(P_i(t)) = \alpha_i + \beta_i P_i(t) + \gamma_i P_i^2(t). \tag{59}$$

The data on the plants and the thermal equivalent plant are summarized in Table 2. The units for the coefficients are: α_i in $(euro/h)$, β_i in $(euro/h.MW)$, γ_i in $(euro/h.MW^2)$, and the technical limits for thermal power generation $P_i^{\min}$ and $P_i^{\max}$ in (MW).

7.2. Hydro-model

HC's hydro-network in Asturias has three hydro-plants on different rivers, so the hydraulic system has no hydraulic coupling. We use a *variable head* model (a model perfectly described by El-Hawary [1]), and H (for a conventional hydro-plant, $\dot{z}(t) \geq 0$) is given by

$$H(t) = \frac{B_y(S_0 + t.i)}{G}\dot{z}(t) - \frac{B_y}{G}\dot{z}(t)z(t) - \frac{B_t}{G}\dot{z}^2(t). \tag{60}$$

For the pumped-storage plant, H is defined piecewise, taking in the pumping zone ($\dot{z}(t) < 0$): $H^- = M \cdot H^+$. The parameters that appear in this formula are: the efficiency G in (m^4/h.MW), the restriction on volume b in (m^3), the natural inflow i in (m^3/h), the initial volume S_0 in (m^3), and the coefficients B_y in (m^{-2}) and B_t in (hm^{-2}), parameters that depend on the geometry of the reservoir. The data on the hydro-plants is summarized in Table 3. The limits $H^{\min}$ and $H^{\max}$ are in (MW) and the efficiency of the Tanes hydro-plant is $M = 1.15$.

7.3. Results of forecasting

First, we consider the aim of the company to be that of maximizing profits during an optimization interval of one day, and we consider a discretization of 24 subintervals: $[0, T] = [0, 24]$. The constraint on the volume b refers to each day and we

Table 3
Hydro-plant coefficients.

Plant	1 Salime	2 T anes	3 La Barca
G	519 840	137 542	163 950
b	12.1×10^6	2.3×10^6	1.85×10^6
i	504 167	95 833.3	77 083.3
S_0	189.5×10^6	19.3×10^6	20.2×10^6
B_y	0.4341×10^{-6}	3.0656×10^{-6}	2.6171×10^{-6}
B_t	2.94×10^{-5}	3.12×10^{-5}	2.35×10^{-5}
H^{min}	0	−114.5	0
H^{max}	112	123	57.7

Table 4
Profits.

	Profits (10^8 Euro)	% Loss
Potential profits	5.5502	–
ANOVA	4.1061	−26.02
ANOVA + $\rho \cdot$ t	4.8887	−11.92
AR(7 × 24) + GARCH(1, 1)	5.3403	−3.78
ANOVA + FP − AR(7)	5.4580	−1.66
ANOVA + FF − AR(7)	5.4512	−1.78

Table 5
Profits.

	Profit
1 day horizon	3.19261×10^6
1 week horizon	3.35076×10^6

assume that the natural flow compensates the daily consumption of water. Likewise, the initial volume S_0 is assumed not to vary from one day to the next.

For each model and each hourly price estimation $p(t)$, the proposed algorithm provides us with the optimal $P(t)$ and the optimal $H_i(t)$. Let us assume that these optimum powers are those that the company offers on the next day and that these powers are accepted by the market operator in the effected clearance (to do so, it suffices to choose a conservative strategy). By means of the optimization algorithm, using different models to forecast hourly prices will produce different optimum powers which, in turn, will produce different profit levels for the company. If these profit levels are compared with the potential profits that would be obtained if the optimum power were determined using the real clearing price $p(t)$ of the following day at each hour t, we shall obtain a real measure of the forecasting effectiveness of each technique within the context of the Hydrothermal Problem. Table 4 shows the profits (10^8 Euro) that would be obtained when using the different forecasting models (autoregressive models with 7 daily lags) to solve the Hydrothermal Problem compared with the potential profits that would be obtained if the real next-day hourly prices, if known beforehand, were used. Calculations refer to the test period (01/04/2005– 31/07/2007).

As can be seen from this table, the functional models produce results that are clearly superior to the rest of the models, obtaining a 50% reduction in produced loss when using the AR + GARCH model.

7.4. *Influence of hydro-scheduling*

Finally, let us see the major influence that the correct allocation of water, and hence the hydraulic component of the system, has on optimization. To do so, we now consider the aim of the company to be that of maximizing profits during a optimization interval of one week, and we consider a discretization of 168 subintervals: $[0, T] = [0, 168]$. The constraint on the volume b is now 7 times that corresponding to each day and we assume that the remaining parameters remain the same. If we compare the optimum powers obtained with this time horizon and with that considered previously (day to day), the results (shown in Figs. 3 and 4) are most clarifying. The allocation of water is very different, the plants generally waiting to the end of the interval to obtain maximum use of the water (which is the same as the maximum equivalent height for our variable load hydro-plants).

The profits for the first week of the test period (from 01 /04/ 2005 to 07 /04/ 2005) are given in Table 5. As can be seen, they are very disparate, the weekly horizon being much more effective.

Note that both time horizons belong to the so-called short-range, and hence there is no sense in considering stochastic problems for the natural inflow i (we estimate it as known in the whole week). Furthermore, the term $(S_0 + t.i)$ is also assumed not to exceed the maximum volume during the optimization period.

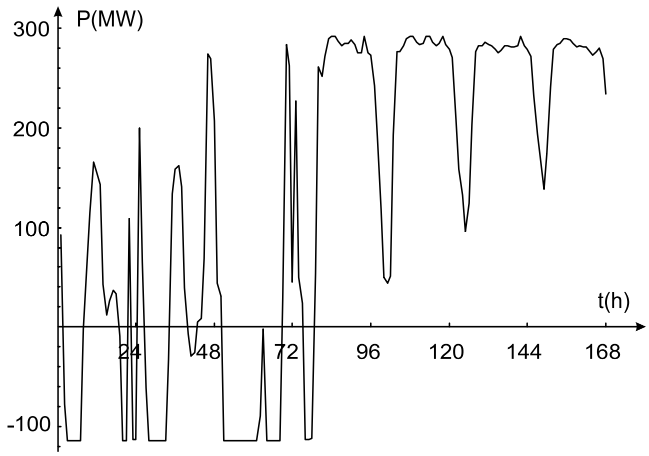

Fig. 3. Optimal Hydro-power 1 week horizon.

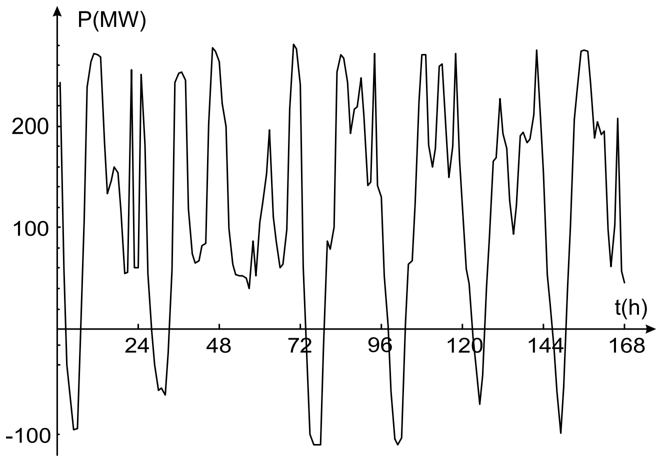

Fig. 4. Optimal Hydro-power 1 day horizon.

8. Conclusions

This paper presents a novel method for developing the economic dispatch algorithm in a hydrothermal system within the framework of a competitive deregulated electricity market.

On the one hand, we propose an efficient, yet highly effective novel next-day price forecasting method based on a functional approach that considers the hourly price time series as a series of hourly observations of daily price functions. Diverse pointwise and fully functional autoregressive models were tested using real data from the Spanish market for the period 1998–2007, comparing their predictive power to that of classical ANOVA and linear heteroskedastic models (AR + GARCH). Furthermore, the impact of this forecasting improvement on the profits of a power generation company was assessed, finding that the functional models achieved a 50% reduction in losses produced by the AR + GARCH model with respect to the potential profits that would be obtained if the real next-day hourly prices were used in the Hydrothermal Problem, should they be known beforehand.

On the other hand, insofar as the underlying mathematical problem is concerned, our technique makes maximum use of the special features of the new competitive markets, leading to an algorithm that is very easy to implement and which is highly flexible to modeling of the physical subsystems (considering models with a high degree of detail and with hardly any simplifications). The algorithm is not affected by the size of the problem (since it is easily broken down into its various subproblems) and convergence is guaranteed, thus allowing us to carry out studies of a very diverse nature with a minimum effort. To demonstrate this, we have shown in this paper how the the solution of hydrothermal systems with hydropredominance varies significantly when the time horizon of short-range scheduling is varied.

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